Optimal Design of Charitable Fundraising Platforms

(Authors' names blinded for peer review)

Online charitable fundraising platforms connect donors and beneficiaries, balancing participation on both sides of the market. We consider a stylized model of a profit-maximizing platform deciding its commission on donations and how many beneficiaries to admit. The platform earns commission revenue but incurs onboarding costs, while beneficiaries participate only if expected donations cover a fixed entry cost. Donors contribute only when they find a sufficiently aligned cause. We characterize equilibrium outcomes and show that both platform decisions depend on market coverage, defined as the fraction of participating donors. Importantly, the two costs shape coverage through different channels: onboarding costs raise the cost of expanding the beneficiary base, whereas entry costs impose an endogenous ceiling on the commission. Our analysis yields four main findings. First, the profit-maximizing commission can decrease as onboarding costs increase. Higher commissions reduce each beneficiary's reach, requiring more beneficiaries to reach all donors; when onboarding is costly, the platform instead lowers its commission and admits fewer beneficiaries. Second, increases in beneficiary entry costs can initially raise net donations because the platform lowers its commission to sustain beneficiary participation. However, beyond a threshold, admission declines and net donations fall. Third, a donation-maximizing commission cap weakly increases net donations but may reduce beneficiary admission. Finally, we show that policymakers must correctly anticipate a platform's strategic response when choosing a commission cap. A cap that ignores these responses can reduce both donations and beneficiary admission, highlighting the importance of considering platform incentives and market participation.

Key words: charitable fundraising, platform operations, market design, nonprofit operations

1. Introduction

Online fundraising platforms now intermediate a large and growing share of charitable giving. The sector reached \$7.5 billion in 2025 and is projected to nearly double to \$14.44 billion by 2030 (Research and Markets 2026), reflecting a broad shift in donation behavior: approximately 63% of donors now prefer to give online (National Philanthropic Trust 2026), and digital donations have continued to rise beyond their pandemic-era highs at a significantly higher rate than overall U.S. contributions (Blackbaud 2026). These developments have elevated digital intermediaries to a central position in nonprofit finance, making online fundraising essential to organizational survival.

Although their mission is to facilitate donations, many online fundraising platforms also seek to maximize the revenue they obtain from commissions and fees. GoFundMe, for example, has enabled over \$40 billion in giving since its inception in 2010, generating roughly \$150–200 million in annual revenue through transaction fees and donor tips (GoFundMe 2026).[1] This dual objective—facilitating

[1] Most of the largest online charitable fundraising platforms are for-profit entities, including GoFundMe, Fundly, and Classy (now GoFundMe Pro). However, nonprofit platforms may also be interested in maximizing commission revenue, potentially investing it in their operations or redistributing it among different causes.

contributions by matching donors and beneficiaries while also monetizing intermediation—naturally leads to viewing these platforms as two-sided marketplaces. Beneficiaries register on the platform and create campaigns describing their cause, while donors browse campaigns and contribute to those aligned with their preferences. To achieve their goals, platforms must therefore choose key design features that determine how effectively they match donors and beneficiaries.

One key decision is the commission charged on donations. Commissions reduce the share of each donation that reaches the beneficiary and, consequently, may be negatively viewed by potential donors. As a result, higher commissions increase revenue per dollar donated but may discourage participation and reduce contributions. When setting commission rates, platforms therefore face a fundamental trade-off between monetization and maintaining donor participation and contribution levels. Moreover, in addition to pricing, platforms must determine how many beneficiaries to onboard. Admitting more beneficiaries increases the chance that a well-aligned cause will be available for each potential donor, which should lead to more donations.[2] However, onboarding beneficiaries is costly: platforms must invest in verification, monitoring, and technological support to maintain trust and prevent misuse. As the number of beneficiaries grows, these administrative and screening costs increase. Moreover, onboarding too many beneficiaries could entail overlapping donor bases for different beneficiaries. Consequently, platforms must balance the benefits of expanding participation with the operational costs and potential congestion effects associated with a larger platform.

Importantly, these two design choices—commission and admission—are not independent, as they jointly determine market coverage, that is, what fraction of donors can find a sufficiently well-matched beneficiary to support. The optimal commission rate depends on the platform's admission decision, as a larger beneficiary base can affect both donor participation and platform costs. Conversely, the platform's ideal admission can differ depending on the commission levels because these affect the marginal value of onboarding each additional beneficiary. The solution to this joint platform design problem and its downstream effects are thus far from obvious.

In this paper, we address the following main research question: *How should a charitable fundraising platform jointly choose its commission rate and beneficiary admission?* Relatedly, we ask: *How do the platform's onboarding cost and beneficiaries' entry costs affect donations, beneficiary participation, and platform profit? And how do alternative objectives, such as profit maximization vs. donation maximization, shape these outcomes and the underlying trade-offs?*

To address these questions, we develop a game-theoretic model of a charitable fundraising platform, its beneficiaries, and its donors. The platform sets its commission and decides how many beneficiaries to admit, incurring an onboarding cost for doing so. Potential beneficiaries incur a

[2] Throughout the paper, we use "onboarding" and "admitting" interchangeably to refer to the platform accepting beneficiaries onto the platform.

fixed cost to apply and enter only when their expected payoff covers this cost. Donors contribute only to sufficiently well-aligned causes, with the commission affecting both their willingness to give and their contribution amount. We fully characterize the equilibrium and examine how onboarding and entry costs affect commissions, admissions, net donations, and platform profit. Finally, we consider a policymaker who caps the commission to maximize net donations and compare the resulting outcomes with those under profit maximization.

Contributions. Our main set of results shows that the effects of onboarding and entry costs are governed by market coverage. The platform either covers the market, so every donor finds a sufficiently well-aligned cause, or leaves it uncovered, so some donors do not donate. The two costs affect this choice through different channels. The platform's onboarding cost directly raises the cost of expanding its beneficiary base, whereas beneficiaries' entry cost creates an endogenous ceiling on the commission: to induce beneficiaries to apply, the platform must share at least enough donations with them to offset their entry cost. Because of the commission's crucial role—it finances onboarding but also determines both how much donors contribute and how many donors each beneficiary can reach—the ceiling imposed by the entry cost can have a significant effect on outcomes, different from that of the onboarding cost. Thus, although an increase in either cost weakly reduces platform profit, the effects on commissions, admissions, coverage, and net donations can differ across regimes.

Consider onboarding costs first. In a covered market, when entry costs are low enough that beneficiaries would participate at the platform's preferred commission, a higher onboarding cost leads the platform to lower its commission and admit fewer beneficiaries. The lower commission expands each beneficiary's donor reach, allowing the platform to preserve coverage with fewer costly admissions; donors also give more and beneficiaries retain a larger share, increasing the net donations sent to beneficiaries. When instead the entry cost is high enough to force the platform's commission below its unconstrained optimum, a marginal change in onboarding costs leaves the commission, admission, and net donations unchanged as long as the market remains covered. On the other hand, in an uncovered market, the platform responds to higher onboarding costs by holding its commission fixed and cutting admissions, which reduces donor participation and net donations. Entry costs, in turn, have no effect until they limit the commission the platform can charge; beyond that point, higher entry costs force the platform to lower its commission. At moderate onboarding costs, this response can move the market from uncovered to covered and back again: initial commission reductions expand donor reach enough to restore coverage—even with fewer beneficiaries—but further reductions leave the platform with too little revenue to justify sustaining admission. Net donations can therefore rise initially and then fall once the loss of admitted beneficiaries outweighs the gains from larger contributions, broader donor reach, and greater pass-through.

Our second set of results compares the profit-maximizing platform to a donation-maximizing benchmark, in which a policymaker imposes an upper bound on the commission the platform may charge in order to maximize net donations. We show that the policymaker's problem is equivalent to directly selecting the donation-maximizing commission while allowing the platform to retain its admission decision. Furthermore, we establish that the commission that maximizes net donations is weakly lower than the profit-maximizing commission. Importantly, a zero commission does not maximize net donations, as it would eliminate the platform's incentive to incur the costs of onboarding beneficiaries in the first place. We also show that the profit-maximizing platform admits at least as many beneficiaries as the donation-maximizing benchmark. Whenever the commissions differ, however, the additional beneficiaries admitted under profit maximization are insufficient to offset the effects of the higher commission, resulting in lower net donations. When the entry-cost constraint forces the two commissions to coincide, both gaps are zero. We refer to these differences in admissions and net donations as the admission and donation gaps, respectively. Higher beneficiary entry costs can close both gaps by exerting downward pressure on the commissions chosen under profit maximization and donation maximization. Closing the gaps, however, does not necessarily maximize net donations: the platform's equilibrium net donations rise as its commission approaches the donation-maximizing level but fall once the entry-cost constraint pushes both commissions below that level. Finally, our numerical analysis shows that higher onboarding costs can narrow the admission and donation gaps, although their effects depend more strongly on the underlying regime.

Our findings carry implications for the decisions of both platforms and policymakers. The commission cap is the direct policy lever in our analysis, and the link between commission and admission implies that an overly aggressive cap can reduce net donations in some regimes by leading the platform to admit fewer beneficiaries. Furthermore, shifts in onboarding and entry costs due to changes in the operating environment—for example, changes in onboarding technology, verification or compliance requirements, or beneficiaries' application burdens—should be carefully considered when setting the cap, as their effects on net donations vary across coverage regimes. Finally, closing the gap between the profit-maximizing and donation-maximizing cases is not, by itself, the right policy target: as entry costs increase, net donations may decline after the gap first closes. A policymaker seeking to maximize net donations should therefore evaluate commissions, admissions, donor coverage, and net donations jointly rather than focus on any single outcome.

Organization. The remainder of the paper is organized as follows. In Section 2, we provide a comprehensive review of the relevant literature. In Sections 3 and 4, we introduce our model and present our key theoretical findings. In Section 5, we present the results of our numerical study. Finally, in Section 6, we conclude.

2. Literature

Our paper lies at the intersection of three streams of literature. First, we contribute to the literature on the economics of charitable giving and online fundraising. Second, our work is related to the effects of commissions and fees on platform operations. Third, our paper contributes to the broader academic literature on nonprofit operations management. We now discuss each of these strands.

Charitable Giving and Online Fundraising. The literature on charitable giving asks two fundamental questions: why people donate and what makes them donate more. Early work by Andreoni (1989) and Andreoni (1990) shows that donors give because they care about the cause and enjoy the act of giving itself, which is known as the "warm glow." Reviewing more than 500 studies, Bekkers and Wiepking (2011) identify eight factors that drive donations, including awareness of need, reputation, and perceived impact. Related to the latter, previous work shows that donors care about how much of their gift actually reaches the cause, with contributions declining sharply when overhead or non-program costs increase (Gneezy et al. 2014, Hung et al. 2023).[3] Evidence from online fundraising platforms shows the same pattern. Using platform fee variation on DonorsChoose.org, Meer (2014) shows that higher fees reduce the likelihood of a project being funded. On Kiva, Meer and Rigbi (2013) find that even small transaction costs substantially decrease contributions. Our paper contributes to this literature by incorporating both donor aversion to platform fees and donor preferences for closely matched causes into a model of fundraising platform design. In contrast to prior work, we endogenize the platform's commission and beneficiary onboarding decisions and examine how these choices jointly shape contribution outcomes.

Commissions and Fees in Platform Operations. A related operations management literature studies how platform commissions—equivalently, the split of revenue between a platform and the parties transacting through it—shape outcomes. One strand analyzes revenue-sharing contracts and their coordinating properties (Cachon and Lariviere 2005), as well as how platforms use revenue sharing to motivate producers (Bhargava 2022, Bhargava et al. 2022). Another compares agency (marketplace) and reselling formats (Abhishek et al. 2016, Tian et al. 2018) and studies commissions on service and gig platforms, including their cross-channel externalities and coordination frictions (Chen et al. 2022, Feldman et al. 2023) and who should control pricing (Cachon et al. 2026). Our setting departs from this literature in the way money moves: rather than dividing the proceeds of a sale between two parties, the commission is deducted from a donation to a beneficiary. Hence, a higher commission directly reduces the funds reaching the cause, and donors are averse to the commission itself. In our setting, the platform sets the commission jointly with the number of beneficiaries it admits, and we compare the resulting net donations with those under the commission that maximizes donations to beneficiaries.

[3] Non-program costs are expenses not directly spent on a nonprofit's mission, such as administrative and fundraising costs. Donors often treat these costs as a signal that less of their contribution reaches the cause.

Not-for-profit Operations Management. A growing literature in operations management studies how nonprofit organizations can run their operations more effectively (Berenguer and Shen 2020). One stream of this work studies how design choices affect donor giving behavior. Examples include mechanisms such as pledging (Andreoni and Serra-Garcia 2021), earmarking (Özer et al. 2024, Aflaki and Pedraza-Martinez 2023), operational transparency in online fundraising (Mejia et al. 2019), identity-related factors (Charness and Holder 2019), default donation options (Castro and Rodilitz 2026), and incentive and solicitation mechanisms (Ghosh et al. 2025, 2026). A second stream studies how resource-constrained nonprofits should allocate funds and design services for beneficiaries, including fund allocation under ex-post funding (Devalkar et al. 2017), service portfolio design (Arora et al. 2022), partial completion (Zhang et al. 2022), payment-for-results funding (Sharma et al. 2021), and how NGOs should deploy limited resources to influence firms and regulators (Kraft et al. 2013). Related work also studies operational challenges associated with non-monetary resources, such as in-kind donations (Daniels and Valdés 2021) and time-sensitive food donations (Lee et al. 2026). Additionally, a growing stream of literature studies the operations of volunteering and how to improve the efficiency of volunteer engagement (Tipnis et al. 2025, 2026, Manshadi et al. 2026). Finally, a closely related body of work examines online crowdfunding platforms, addressing platform design, cause selection, beneficiary onboarding, and the role of signaling (Chang 2020, Chakraborty and Swinney 2021, Belavina et al. 2020). Most of this work, however, focuses on reward-based or entrepreneurial crowdfunding rather than charitable fundraising. The settings differ in a way that matters here: crowdfunding backers receive a product or equity stake, whereas charitable donors receive no material return and give based on cause alignment and on the share of their gift that reaches the beneficiary. To the best of our knowledge, ours is among the first papers to analyze the impact of commission and beneficiary admission as joint design choices, and to compare the platform's profit-maximizing design with the donation-maximizing benchmark.

3. Model

We develop a stylized model of an online charitable fundraising platform that acts as an intermediary between beneficiaries and donors.

Platform. The platform sets a commission f on donations and selects the number of beneficiaries n to admit from the pool of applicants. To facilitate clean insights, we allow n to take non-integer values and use the continuous extensions of the relevant functions of n. The platform's goal is to maximize its profit, given by the revenue from commissions minus the onboarding costs $c(n) := \gamma n^2/2$, where $\gamma > 0$ parametrizes the cost of admitting and managing beneficiaries.[4]

[4] Quadratic cost specifications of this form are common in the literature and capture an increasing marginal cost of onboarding. Each admitted beneficiary requires verification, ongoing monitoring, technical support, and compliance

Beneficiaries. We assume that the population of potential beneficiaries is sufficiently large and dense to accommodate the equilibrium applicant pool. Beneficiaries are risk-neutral and differ only in their location on a unit circle (Salop 1979), which captures how closely their causes align with donors' preferences. After observing the platform's commission, potential beneficiaries decide whether to apply to the platform. The platform then selects n beneficiaries from the applicant pool. Each beneficiary incurs a fixed entry cost $\phi > 0$ upon applying,[5] representing the time and resources needed to prepare its campaign, document its eligibility, and complete the platform's screening process. For tractability, we assume that admitted beneficiaries are evenly spaced around the unit circle.[6]

Donors. A unit mass of donors is uniformly distributed along the unit circumference, each with an ideal cause given by their position on the circle. The distance $k \geq 0$ between a donor and the nearest beneficiary captures the mismatch between their causes and, with n equally spaced beneficiaries, is bounded above by $1/(2n)$. Following the warm-glow theory of giving (Andreoni 1989, 1990), we assume that donors derive utility from their own contribution,[7] with two additional features: (i) donors dislike paying platform commissions and (ii) they also dislike giving to causes that do not closely match their preferences. Specifically, each donor chooses donation $g \in [0, w]$ to maximize

$$u(g,k,f) = \alpha(w-g) + \beta\sqrt{g(1-\theta f)^+} - \lambda k \cdot \mathbf{1}_{\{g>0\}}, \tag{1}$$

where $x^+ := \max\{x, 0\}$. The first term, $\alpha(w-g)$, represents the utility derived from private consumption, where $w > 0$ is the endowment or the disposable income. The parameter $\alpha > \frac{\beta}{2\sqrt{w}}$ captures the marginal utility of private consumption (the lower bound ensures that donors put at least enough weight on private consumption that they would not wish to donate all of their wealth). The second term, $\beta\sqrt{g(1-\theta f)^+}$, is the "warm-glow" or prosocial utility from giving, where β determines how strongly the donor values the act of contributing. The commission f reduces the impact of each dollar donated, and $\theta \geq 1$ captures how strongly donors react to this reduction.

oversight. As the beneficiary base expands, these activities become more complex and draw on increasingly scarce staff and technical capacity, making the marginal cost of admitting and managing another beneficiary rise with n. Changes in γ can therefore reflect shifts in verification or compliance requirements, onboarding technology, or subsidies for these activities.

[5] Our results are unchanged if the entry cost ϕ is incurred upon admission rather than upon application; see Appendix E.

[6] The even-spacing assumption is a stylized way to capture how curated platforms select beneficiaries from a large applicant pool. To maximize donor participation, platforms have incentives to admit causes that collectively cover a broad range of donor interests rather than concentrating on similar causes that may cannibalize.

[7] In Andreoni's framework (Andreoni 1989, 1990), a donor derives utility from their own private consumption, from the total contribution of all donors, and also directly from their own contribution—the "warm glow." Our model focuses on the pure "warm-glow" setting: on platforms with many anonymous donors, any individual contribution is negligible relative to a cause's total, so giving is driven more by the utility of contributing itself than by its effect on aggregate contributions.

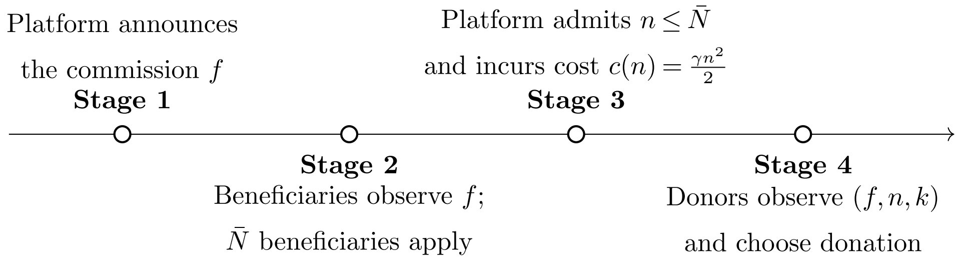

Figure 1 **Sequence of events.**

When $\theta = 1$, donors take the commission at face value; when $\theta > 1$, donors overweight the impact of commission and treat each dollar of commission as wiping out more than a dollar of impact, consistent with the overhead-aversion evidence (Gneezy et al. 2014, Gneezy 2023).[8] Observe that if $f \ge 1/\theta$, then the commission is high enough (and donors are commission-averse enough) that the second term in the utility is zero for all donors, which results in no donations. Thus, any $f \ge 1/\theta$ leads to the same outcome, and it is without loss of optimality for the platform to ignore $f > 1/\theta$ and optimize over $f \in [0, 1/\theta]$. The third term, $-\lambda k \cdot \mathbf{1}_{\{g>0\}}$, is the reduction in utility from cause mismatch, which is proportional to the distance k and is incurred only if the donor contributes.[9] The parameter $\lambda > 0$ captures how sensitive the donor is to the mismatch.

Timing. The sequence of events is as follows: First, the platform announces the commission f. Second, beneficiaries observe f and decide whether to apply to the platform, incurring the entry cost ϕ; let $\bar{N}$ denote the number of beneficiaries who want to apply. Third, the platform admits $n \le \bar{N}$ beneficiaries and incurs onboarding cost $c(n)$. Finally, if $n > 0$, each donor observes f, n, and their individual distance $k \in [0, 1/(2n)]$ from their nearest beneficiary, and chooses their contribution amount to maximize utility. If $n = 0$, no beneficiary can receive a contribution, so every donor contributes zero and aggregate donations are zero. Figure 1 summarizes this timeline and Table 1 in Appendix A summarizes the notation. We solve the game by backward induction.

3.1. Stage 4: Optimal Donation

In Proposition 1, we characterize donors' equilibrium contribution in response to the platform's commission and onboarding decisions. At the participation cutoff, a donor is indifferent between contributing and not contributing; we break this indifference in favor of contributing. This convention affects only a measure-zero set of donors and therefore does not affect any aggregate outcome.

[8] Gneezy et al. (2014) study aversion to overhead rather than platform commissions, but both reduce the share of a donor's contribution that actually reaches the cause, so the same behavioral logic applies in this setting. This concern is reinforced by charity watchdogs such as Charity Navigator, which rate nonprofits partly on overhead ratios and create constant pressure to keep these costs low.

[9] The indicator ensures that a donor does not experience mismatch disutility if she does not give. Moreover, the discrete nature of the disutility captures the fact that donors typically direct their support to a specific cause they feel connected to. Consistent with this, a nationally representative survey finds that 72% of donors report feeling strongly about a particular cause (CAF 2025).

Proposition 1 **(Donor Donation and Participation Cutoff)**. *For $f \in [0,1/\theta)$, a donor at distance k from the nearest beneficiary contributes $g^*(f) = \frac{\beta^2(1-\theta f)}{4\alpha^2}$ if $k \le \tilde{k}(f)$, and zero otherwise, where $\tilde{k}(f) = \frac{1}{\lambda}\left(\frac{\beta^2(1-\theta f)}{4\alpha}\right)$ is the cutoff distance beyond which donors do not contribute. At $f = 1/\theta$, every donor contributes zero; define $g^*(1/\theta) = \tilde{k}(1/\theta) = 0$. Both $g^*(f)$ and $\tilde{k}(f)$ are strictly decreasing in f on $[0,1/\theta]$.*

Proposition 1 shows that a higher commission hurts donations by (i) reducing each donor's contribution $g^*(f)$ and (ii) shrinking the cutoff $\tilde{k}(f)$, which can reduce the mass of donors who contribute. However, a higher commission helps the platform by increasing its take from each dollar contributed. As we will see later, the optimal commission must balance these competing forces.

The participation cutoff $\tilde{k}(f)$, together with the geometry of the circle, implies two possible regimes for the market. With n equally spaced beneficiaries, no donor is farther than $1/(2n)$ from their nearest beneficiary, so the radius of contributing donors on either side of each beneficiary is $\min\left\{\tilde{k}(f), \frac{1}{2n}\right\}$. If $\tilde{k}(f) \ge 1/(2n)$, every donor on the circle contributes $g^*(f)$ to the beneficiary nearest them; we refer to this as the *covered market*. On the other hand, if the commission is high enough to make $\tilde{k}(f) < 1/(2n)$, then only donors within $\tilde{k}(f)$ of a beneficiary contribute, leaving gaps around the circle; we refer to this as the *uncovered market*. We illustrate the market coverage in Figure 2. We will see later that the platform's trade-offs differ in each regime. Moreover, note that neither the equilibrium donation $g^*(f)$ nor the cutoff $\tilde{k}(f)$ depends on the number of beneficiaries n. Rather, it affects donations only through the geometry of the circle, by determining the maximum distance to the nearest beneficiary at $1/(2n)$; the commission determines each donor's gift amount and the participation cutoff, while n determines the mass of donors close enough to give.

3.2. Stage 3: Platform Admission

Given a commission $f \in [0,1/\theta]$, the platform chooses the number of beneficiaries $n \le \bar{N}$ to admit to maximize profit, anticipating donor behavior in Stage 4 (Section 3.1).[10] At either endpoint $f = 0$ or $f = 1/\theta$, commission revenue is zero, so the unique optimal admission is $n = 0$. The analysis below therefore focuses on $f \in (0,1/\theta)$, for which $\tilde{k}(f) > 0$. Here, $\bar{N}$ denotes the number of beneficiaries who have applied to join the platform in Stage 2; the platform's admission decision is therefore capped by the size of this applicant pool, affecting the regime in which the platform operates. In particular, if $\bar{N} < 1/(2\tilde{k}(f))$, then the platform cannot admit enough beneficiaries to cover the market, and the market is uncovered for any feasible n. Otherwise, if $\bar{N} \ge 1/(2\tilde{k}(f))$, then the platform can admit enough beneficiaries to cover the market. The feasible region then contains two

[10] We consider a platform that maximizes its profit through commission revenue. We address the issue of net donations received by beneficiaries (and a policymaker interested in maximizing them) in Section 4.2.

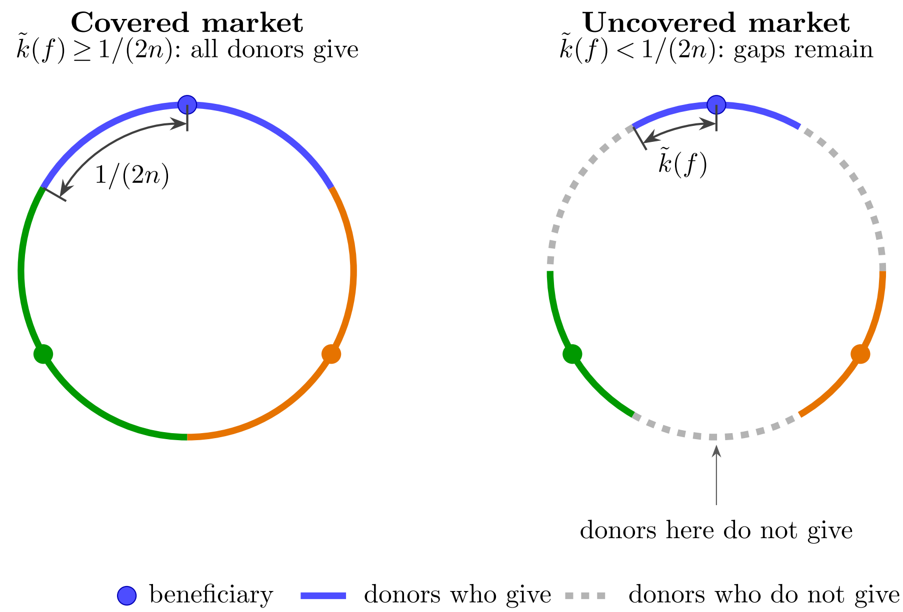

Figure 2 Covered vs. uncovered market on the unit circle. The circumference represents the continuum of donors, with $n = 3$ beneficiaries equally spaced.

intervals: the market is uncovered when $0 \leq n < 1/(2\tilde{k}(f))$ and covered when $1/(2\tilde{k}(f)) \leq n \leq \bar{N}$. We next characterize the platform's profit in each region.

In the uncovered region, only donors within distance $\tilde{k}(f)$ of a beneficiary contribute, so each additional beneficiary expands the donor base. The platform's profit in this case is

$$\Pi_U(f,n) = 2n \int_0^{\tilde{k}(f)} g^*(f) f \, dk - \frac{\gamma n^2}{2} = 2n g^*(f) f \tilde{k}(f) - \frac{\gamma n^2}{2},$$

which is easily shown to be strictly concave in n. Using the first-order condition, the unconstrained maximizer of $\Pi_U(f,n)$ in n is given by $n_U(f) := 2g^*(f) f \tilde{k}(f)/\gamma$.

In the covered region, every donor on the circle contributes, so the total donations are independent of n and the platform's profit is

$$\Pi_C(f,n) = 2n \int_0^{\frac{1}{2n}} g^*(f) f \, dk - \frac{\gamma n^2}{2} = g^*(f) f - \frac{\gamma n^2}{2}.$$

Since $\Pi_C(f,n)$ is strictly decreasing in n, the best covered solution is the minimum number of beneficiaries needed to cover the market, i.e., $\frac{1}{2\tilde{k}(f)}$. It is immediate that $\Pi_C(f, \frac{1}{2\tilde{k}(f)}) = \Pi_U(f, \frac{1}{2\tilde{k}(f)})$, and thus the platform's Stage 3 optimization problem is simply

$$\max_{0 \leq n \leq \min\{\frac{1}{2\tilde{k}(f)}, \bar{N}\}} \Pi_U(f,n). \tag{2}$$

Consequently, the platform covers the market if and only if the coverage threshold $1/(2\tilde{k}(f))$ is both optimal and feasible. Optimality requires $n_U(f) \geq \frac{1}{2\tilde{k}(f)}$, which is equivalent to $\gamma \leq \hat{\gamma}(f) := \frac{\beta^6(1-\theta f)^3 f}{16\alpha^4\lambda^2}$.

Feasibility additionally requires a sufficiently large applicant pool, i.e., $\bar{N} \geq \frac{1}{2\tilde{k}(f)}$. Thus, provided enough beneficiaries apply, admission remains at the coverage threshold until γ exceeds $\hat{\gamma}(f)$; beyond this point, full coverage becomes too costly and optimal admission declines with γ. If the applicant pool is insufficient, the market remains uncovered regardless of the onboarding cost. Proposition 2 formalizes these results.

PROPOSITION 2 **(Optimal Admission)**. *Given commission f and $\bar{N}$ beneficiaries who have applied, the platform's optimal admission decision is*

$$n^*(f,\bar{N}) = \begin{cases} 0 & \text{if } f \in \{0, 1/\theta\}, \\ \min\left\{\dfrac{1}{2\tilde{k}(f)}, \bar{N}\right\} & \text{if } f \in (0,1/\theta) \text{ and } \gamma \leq \hat{\gamma}(f), \\ \min\left\{n_U(f), \bar{N}\right\} & \text{if } f \in (0,1/\theta) \text{ and } \gamma > \hat{\gamma}(f), \end{cases}$$

where $\hat{\gamma}(f) = \frac{\beta^6(1-\theta f)^3 f}{16\alpha^4\lambda^2}$. Furthermore, the optimal solution $n^*(f,\bar{N})$ is weakly decreasing in γ.

3.3. Stage 2: Beneficiary Entry

Anticipating platform admission in Stage 3 (Section 3.2) and donor behavior in Stage 4 (Section 3.1), potential beneficiaries decide whether to apply to the platform. We first fix an interior commission $f \in (0,1/\theta)$; the endpoints are handled below. Let $\bar{N}$ denote the number of beneficiaries who want to apply, given commission f. The platform admits $n^* \leq \bar{N}$ beneficiaries: if $\bar{N} > n^*$, each beneficiary is admitted with probability $n^*/\bar{N}$. If $\gamma > \hat{\gamma}(f)$, we know from Proposition 2 that the platform sets $n^* = \min\{n_U(f), \bar{N}\}$ and the market is uncovered, so each admitted beneficiary therefore draws donations from all donors within $\tilde{k}(f)$ on either side and earns

$$2\,g^*(f)(1-f)\,\tilde{k}(f) \;=\; \frac{\beta^4(1-\theta f)^2(1-f)}{8\alpha^3\lambda} =: \hat{\phi}(f),$$

where the first equality follows from substituting $g^*(f)$ and $\tilde{k}(f)$ from Proposition 1. If instead $\gamma \leq \hat{\gamma}(f)$, then, again by Proposition 2, the platform will set $n = \min\{1/(2\tilde{k}(f)), \bar{N}\}$. If $\bar{N} < 1/(2\tilde{k}(f))$, then the market is uncovered, and the per-beneficiary payoff is $\hat{\phi}(f)$. If instead $\bar{N} \geq 1/(2\tilde{k}(f))$, then the platform admits exactly $1/(2\tilde{k}(f))$ beneficiaries—the minimum number that reaches all donors—so the market is covered; even so, each beneficiary still draws donations from within $\tilde{k}(f)$ on either side, so the per-beneficiary payoff will again be $\hat{\phi}(f)$.[11] Thus, regardless of the size of the applicant pool or the coverage regime, each admitted beneficiary receives $\hat{\phi}(f)$ in donations net of the platform's commission. This conditional payoff determines beneficiary entry. If $\phi > \hat{\phi}(f)$, no beneficiary applies because even certain admission would not cover the application cost; hence,

[11] This can also be seen by noting that if the platform sets $n = 1/(2\tilde{k}(f))$ and creates a covered market, then the total net donations of $g^*(f)(1-f)$ are evenly split among the $1/(2\tilde{k}(f))$ admitted beneficiaries, such that each one earns $g^*(f)(1-f)/(1/(2\tilde{k}(f))) = \hat{\phi}(f)$.

$\bar{N}=0$ and the market shuts down. If $\phi<\hat{\phi}(f)$, beneficiaries continue to apply until an additional applicant's expected payoff is zero, which determines the equilibrium applicant pool $\bar{N}(f)$ in each regime. Finally, when $\phi=\hat{\phi}(f)$, certain admission yields zero net utility. We assume that indifferent beneficiaries apply. Proposition 3 summarizes the resulting entry equilibrium.

PROPOSITION 3 **(Beneficiary Entry)**. *For $f\in(0,1/\theta)$, provided $\phi\leq\hat{\phi}(f)$, the equilibrium number of entrants $\bar{N}(f)$ and the number of beneficiaries admitted $n^*(f,\bar{N}(f))$ are provided in the table below.*

Condition	$\bar{N}(f)$	$n^*(f,\bar{N}(f))$	Regime
$\gamma\leq\hat{\gamma}(f)$	$\dfrac{g^*(f)(1-f)}{\phi}$	$\dfrac{1}{2\tilde{k}(f)}$	Covered
$\gamma>\hat{\gamma}(f)$	$\dfrac{4\left(g^*(f)\right)^2 f(1-f)\left(\tilde{k}(f)\right)^2}{\gamma\phi}$	$\dfrac{2g^*(f)f\,\tilde{k}(f)}{\gamma}$	Uncovered

In both regimes, the beneficiary supply constraint is non-binding, i.e., $\bar{N}(f)>n^*(f,\bar{N}(f))$, when $\phi<\hat{\phi}(f)$, and holds with equality when $\phi=\hat{\phi}(f)$; in the latter case, the platform's admission decision in Stage 3 is nevertheless the same as it would be without the beneficiary supply constraint. If instead $\phi>\hat{\phi}(f)$, then $\bar{N}(f)=n^*(f,\bar{N}(f))=0$, and the market does not operate. At either endpoint $f\in\{0,1/\theta\}$, Proposition 2 gives $n^*=0$. Applying would then yield payoff $-\phi<0$, so no beneficiary applies and the market does not operate.

Proposition 3 confirms that, at every interior commission for which the market operates, the equilibrium applicant pool supports the platform's unconstrained admission choice: the supply constraint is slack when $\phi<\hat{\phi}(f)$ and binds without distorting admission when $\phi=\hat{\phi}(f)$. Thus, for any such commission, we can ignore the minimum with $\bar{N}$ in Proposition 2, resulting in the equilibrium number of beneficiaries admitted given in the third column of the table in Proposition 3.

3.4. Stage 1: Platform Optimal Commission

Anticipating equilibrium beneficiary entry in Stage 2 (Section 3.3), optimal admission in Stage 3 (Section 3.2), and donor behavior in Stage 4 (Section 3.1), the platform chooses the commission $f\in[0,\frac{1}{\theta}]$ to maximize profit. With a slight abuse of notation, we write $\Pi_C(f)$ and $\Pi_U(f)$ for $\Pi_C\left(f,1/(2\tilde{k}(f))\right)$ and $\Pi_U\left(f,n_U(f)\right)$, respectively. Let $\bar{\gamma}:=\hat{\gamma}(1/(3\theta))=\beta^6/(162\alpha^4\lambda^2\theta)$. For $0<\gamma\leq\bar{\gamma}$, let $f_C^*(\gamma)\in[1/(3\theta),1/(2\theta))$ denote the unique maximizer of $\Pi_C(f)$.[12] We write $\Pi(f)$ for the

[12] Although $f_C^*(\gamma)$ does not have a closed form, we show in the proof of Proposition 4 in Appendix B that it is the unique root of a particular quartic equation on this interval for $0<\gamma\leq\bar{\gamma}$.

platform's profit as a function of the commission, obtained by substituting the optimal admission $n^*(f, \bar{N}(f))$ from Proposition 3 into the platform's profit (see Section 3.2), which yields

$$\Pi(f) = \begin{cases} \Pi_C(f) := g^*(f)\, f - \dfrac{\gamma}{8\,\tilde{k}(f)^2} & \text{if } \gamma \le \hat{\gamma}(f), \\ \Pi_U(f) := \dfrac{2\,(g^*(f))^2\, f^2 \left(\tilde{k}(f)\right)^2}{\gamma} & \text{if } \gamma > \hat{\gamma}(f). \end{cases} \tag{3}$$

For an interior commission, Stage 2 (Section 3.3) requires $\phi \le \hat{\phi}(f)$ for the market to operate. At $f = 0$, the platform also earns zero because it admits no beneficiaries. Thus, the platform's maximization problem can be written as

$$\max_{f \in [0, \frac{1}{\theta}]} \tilde{\Pi}(f), \qquad \text{where} \qquad \tilde{\Pi}(f) = \begin{cases} 0 & \text{if } \phi > \hat{\phi}(f), \\ \Pi(f) & \text{otherwise,} \end{cases}$$

where $\Pi(f)$ is given by (3).[13] Two structural features shape the platform's commission choice. First, the profit function $\Pi(f)$ switches between the covered and uncovered cases based on γ: both low and high commissions push the market into the uncovered regime, while intermediate commissions sustain coverage. The intuition is that at low commissions, the platform's share of each dollar donated is too small to justify paying the onboarding cost for full coverage; at high commissions, donors contribute less, and the reach of each beneficiary shrinks, making it both less lucrative and more costly to admit enough beneficiaries to cover the market.[14] By contrast, for intermediate commissions, the platform gets a healthy share of each dollar donated, but donors are not yet strongly dissuaded from giving, so it is worthwhile for the platform to admit enough beneficiaries to reach all donors. Let $\hat{\gamma}_{\max} := \hat{\gamma}(1/(4\theta)) = 27\beta^6/(4096\alpha^4\lambda^2\theta)$. For each $0 < \gamma < \hat{\gamma}_{\max}$, let $f_L(\gamma) < 1/(4\theta) < f_H(\gamma)$ denote the two solutions to $\hat{\gamma}(f) = \gamma$; the covered commissions are precisely $[f_L(\gamma), f_H(\gamma)]$. At $\gamma = \hat{\gamma}_{\max}$, only $f = 1/(4\theta)$ yields coverage, and for $\gamma > \hat{\gamma}_{\max}$, no commission does. Second, when $0 < \phi < \hat{\phi}(0)$, the interior commissions that induce entry are $(0, \bar{f}(\phi)]$, where $\bar{f}(\phi)$ is the unique solution to $\hat{\phi}(\bar{f}(\phi)) = \phi$; $f = 0$ remains a shutdown option yielding zero profit. We refer to $\bar{f}(\phi)$ as the *entry cost cap*. A higher entry cost ϕ forces the platform to charge a lower commission to retain beneficiaries. We formalize these properties in Lemmas 4 and 5 in Appendix D. The platform's optimal commission balances these two forces, as characterized next in Proposition 4.

PROPOSITION 4 **(Equilibrium Commission and Admission)**. *Recall that* $\bar{\gamma} = \hat{\gamma}\left(\frac{1}{3\theta}\right) = \frac{\beta^6}{162\alpha^4\lambda^2\theta}$. *Let* $f^\circ(\gamma) := f_C^*(\gamma)$ *for* $\gamma \le \bar{\gamma}$, *and* $f^\circ(\gamma) := \frac{1}{3\theta}$ *for* $\gamma > \bar{\gamma}$. *Then, if* $\phi < \hat{\phi}(0) = \frac{\beta^4}{8\alpha^3\lambda}$*:*

[13] Figure 9a in Appendix F illustrates the platform's profit function.

[14] Figure 9b in Appendix F illustrates $\hat{\gamma}(f)$ as a function of the commission f.

(i) There exists a threshold $\Phi(\gamma) := \hat{\phi}\big(f^\circ(\gamma)\big)$ *such that the entry cost cap does not distort the platform's choice, with* $f^* = f^\circ(\gamma)$, *if* $\phi \le \Phi(\gamma)$, *and binds and distorts the choice, with* $f^* = \bar{f}(\phi)$, *if* $\phi > \Phi(\gamma)$.

(ii) There exists a threshold $\Gamma(\phi) := \hat{\gamma}\big(\min\{\bar{f}(\phi), \frac{1}{3\theta}\}\big)$ *such that the market is covered if* $\gamma \le \Gamma(\phi)$ *and uncovered if* $\gamma > \Gamma(\phi)$.

(iii) The equilibrium commission f^* *and admission* n^* *are given in the table below.*

	Covered: $\gamma \le \Gamma(\phi)$	Uncovered: $\gamma > \Gamma(\phi)$
Entry Cap Non-Distorting: $\phi \le \Phi(\gamma)$	**Region I** $f^* = f_C^*(\gamma)$ $n^* = \dfrac{1}{2\tilde{k}(f^*)}$	**Region III** $f^* = \dfrac{1}{3\theta}$ $n^* = \dfrac{2g^*(f^*)f^*\tilde{k}(f^*)}{\gamma}$
Entry Cap Distorts: $\phi > \Phi(\gamma)$	**Region II** $f^* = \bar{f}(\phi)$ $n^* = \dfrac{1}{2\tilde{k}(f^*)}$	**Region IV** $f^* = \bar{f}(\phi)$ $n^* = \dfrac{2g^*(f^*)f^*\tilde{k}(f^*)}{\gamma}$

If $\phi \ge \hat{\phi}(0)$, *no beneficiaries apply, so the market does not operate. In this shutdown outcome, every commission announcement gives the platform zero profit; the commission is therefore not identified without an additional tie-breaking rule.*

Proposition 4 partitions the parameter space into four regions, determined by two costs: the platform's onboarding cost γ, and the beneficiaries' entry cost ϕ. Figure 3 visualizes this partition in (γ, ϕ)-space.[15] When onboarding is cheap ($\gamma \le \bar{\gamma}$) and the entry cost is sufficiently low, the platform prefers to cover the market and serve every donor (Regions I and II for $\gamma \le \bar{\gamma}$). It transitions to the uncovered market (Region IV for $\gamma \le \bar{\gamma}$) only when ϕ is large enough to force the commission below the level required to sustain coverage. When onboarding is moderately expensive ($\bar{\gamma} < \gamma < \hat{\gamma}_{\max}$) and entry is cheap, the platform no longer finds full coverage worthwhile and prefers to leave some donors unserved. However, a surprising phenomenon emerges here: an increase in the entry cost pulls the optimal commission down into a range where full coverage becomes optimal again (Region II in $\bar{\gamma} < \gamma < \hat{\gamma}_{\max}$), so more onerous beneficiary entry can paradoxically restore coverage. However, if the entry cost rises further, the commission falls, and the platform prefers to leave the market uncovered (Region IV in $\bar{\gamma} < \gamma < \hat{\gamma}_{\max}$). When onboarding is very expensive ($\gamma > \hat{\gamma}_{\max}$), the platform always operates in the uncovered regime. At $\gamma = \hat{\gamma}_{\max}$, the market is also uncovered except at the boundary value $\phi = \hat{\phi}(1/(4\theta))$, where $f^* = 1/(4\theta)$ and the market is covered. Across all four regions, the equilibrium commission equals either the platform's unconstrained optimum (which may be in the covered or the uncovered regime, depending on parameters) or the highest commission at

[15] Figure 9c in Appendix F presents a finer partition of the same γ, ϕ-space into nine regions, distinguishing the sub-cases within each of the four regions characterized here.

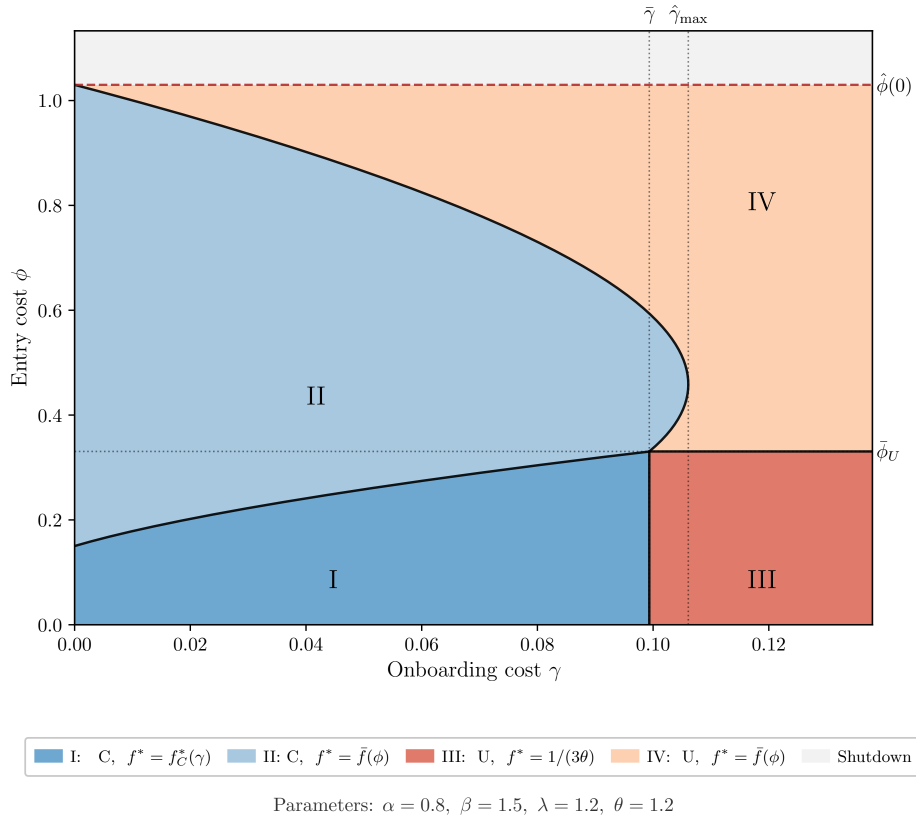

Figure 3 Equilibrium regions in γ, ϕ-space

which beneficiaries are still willing to apply, whichever is smaller. In what follows, we highlight some features of the equilibrium: (i) how the commission responds to the platform's onboarding cost γ, and (ii) how the coverage regime shifts non-monotonically with the beneficiary entry cost ϕ. We formalize these in Corollary 1.

Corollary 1 **(Comparative Statics of Commission, Coverage, and Admission)**. *The equilibrium commission f^* satisfies the following:*

(i) In the interior of Region I, where $0 < \gamma < \bar{\gamma}$ and $0 < \phi < \Phi(\gamma)$, the equilibrium commission $f^* = f_C^*(\gamma)$ is strictly decreasing in γ.

(ii) If $0 < \phi \le \bar{\phi}_U := \hat{\phi}\left(\frac{1}{3\theta}\right)$, the equilibrium commission and admission are both weakly decreasing in γ.

(iii) For any $\gamma \in (\bar{\gamma}, \hat{\gamma}_{\max})$, there exist thresholds $0 < \phi_H(\gamma) < \phi_L(\gamma) < \hat{\phi}(0)$ such that the market is uncovered for $0 < \phi < \phi_H(\gamma)$, covered for $\phi_H(\gamma) \le \phi \le \phi_L(\gamma)$, and uncovered for $\phi_L(\gamma) < \phi < \hat{\phi}(0)$.

The intuition behind each part of Corollary 1 is as follows. Part (i) captures the link between commission and admission as the onboarding cost increases in Region I, where the platform covers the market and sets its unconstrained optimal commission. As γ increases, admitting the same number of beneficiaries becomes more costly, so we might naturally expect the platform to increase its commission to offset this cost increase. However, doing so would shrink donor reach $\tilde{k}(f)$, in turn requiring the platform to admit even more beneficiaries if it wants to sustain coverage. On the other hand, *lowering* the commission increases donor reach, allowing the platform to cover the market with fewer beneficiaries than before. As the onboarding cost increases in a relatively low range, lowering the commission to cover the market is preferable to increasing the commission and admitting more, as well as to giving up full coverage. However, as γ continues to increase, eventually it is just too costly to cover the market, and we transition to the uncovered regime of Region III.

Part (ii) traces the commission and admission across the full range of onboarding costs when the entry cost is low. At low γ, the equilibrium starts in Region I if the entry cost cap does not distort the platform's preferred commission. If the cap initially distorts that choice, the equilibrium instead starts in Region II, where the commission and admission are flat in γ, before moving to Region I. At high γ, the platform leaves some donors unserved (Region III). One might expect that reaching every donor would force the platform to charge a lower commission, but in fact the opposite holds. The intuition follows from part (i). Coverage is only worthwhile when admitting is cheap, which is exactly when the platform prefers a high commission. As onboarding grows costlier within Region I, the platform reduces its commission to increase each beneficiary's reach and cover the market with fewer beneficiaries; once admitting is expensive, it abandons coverage entirely, settling on a commission that no longer depends on onboarding cost. The uncovered regime, therefore, has a lower commission, and admission follows the same pattern, as the commission governs the reach of beneficiaries. In short, while covering the market, the platform absorbs rising onboarding costs by reducing the commission, but once it stops covering the market, it holds the commission fixed and reduces admission instead.

Part (iii) of Corollary 1 captures the role of ϕ. The entry cost does not enter the platform's profit directly but restricts the feasible commission: a higher ϕ requires the platform to lower f to keep beneficiaries willing to apply (Lemma 5). When ϕ is low, the beneficiary entry cost cap does not distort the platform's choice, and at moderate onboarding costs, the platform leaves the market uncovered: the extra margin per donor beats the donations lost from those left out. One would expect that raising ϕ would admit fewer beneficiaries, which should push the market from covered to uncovered. However, the opposite holds as we transition from Region IV to II (when $\gamma \in (\bar{\gamma}, \hat{\gamma}_{\max})$). Coverage depends not just on how many beneficiaries join but on how far each one reaches—and this reach is set by the commission. A higher ϕ reduces the commission, increasing donors' incentive to give. Hence, each beneficiary draws donations from a wider stretch. The market becomes covered again not because more beneficiaries join, but because each reaches farther; in fact, coverage is achieved with strictly fewer beneficiaries than before. For an even higher ϕ, the commission falls so low that it is not profitable for the platform to admit enough beneficiaries and maintain coverage, and the market slips back to being uncovered.

4. Equilibrium Outcomes

The analysis so far has focused on characterizing agents' equilibrium decisions. In this section, we pivot to analyzing the equilibrium outcomes, with special focus on two key metrics of interest: (i) the platform's profit, and (ii) the total net donations. We first analyze these outcomes for the baseline model described above (see Sections 4.1.1 and 4.1.2), in which the platform sets its commission freely. We then turn to a setting where a policymaker caps the commission that the platform is allowed to charge with the aim of maximizing net donations (Section 4.2), and characterize the resulting gaps in net donations and admissions between the two settings (Section 4.3).

4.1. Outcomes without a Commission Cap

4.1.1. Platform Profit. We start by analyzing the platform's profit. Substituting the equilibrium pair (f^*, n^*) from Proposition 4 into equation (3) yields

$$\Pi^*(\gamma,\phi) = \begin{cases} \Pi_C(f_C^*(\gamma)) & \text{in Region I,} \\ \Pi_C(\bar{f}(\phi)) & \text{in Region II,} \\ \Pi_U(1/(3\theta)) & \text{in Region III,} \\ \Pi_U(\bar{f}(\phi)) & \text{in Region IV.} \end{cases}$$

The two costs affect profit through different channels. We first consider the effect of the onboarding-cost parameter γ, holding ϕ fixed. In Region I, the platform responds to a higher γ by lowering its commission and admitting fewer beneficiaries while maintaining full coverage. In Region II, the binding entry-cost constraint fixes the commission at $\bar{f}(\phi)$ (which is independent of γ), and the coverage requirement fixes admission; a higher γ therefore raises onboarding expenses without changing either decision. In the uncovered Regions III and IV, the commission is also independent of γ, and the platform responds by admitting fewer beneficiaries. In every operating region, these adjustments only partly offset the higher onboarding cost, so equilibrium profit decreases with γ.

We next consider the effect of the beneficiary entry cost ϕ, holding γ fixed. Because ϕ does not enter the platform's objective directly, it affects profit only through the entry-cost constraint $\phi \le \hat{\phi}(f)$. In Regions I and III, this constraint does not distort the platform's choice, so ϕ has no effect on the equilibrium commission, admission, or profit. In Regions II and IV, the constraint binds and distorts the choice, and the platform sets $f^* = \bar{f}(\phi)$. Because $\bar{f}(\phi)$ decreases with ϕ, a higher entry cost further restricts the platform's commission choice and lowers its maximized profit. These arguments yield the following formal result.

Corollary 2 **(Comparative Statics of Platform Profit)**. *Let $\Pi^*(\gamma,\phi)$ denote the platform's equilibrium profit, with $\Pi^*(\gamma,\phi) = 0$ when $\phi \ge \hat{\phi}(0)$. Equilibrium profit is jointly continuous in (γ,ϕ). It is strictly decreasing in γ whenever $\phi < \hat{\phi}(0)$, and weakly decreasing in ϕ. More precisely, for each $\gamma > 0$, profit is constant in ϕ for $0 < \phi \le \Phi(\gamma)$, strictly decreasing for $\Phi(\gamma) < \phi < \hat{\phi}(0)$, and equal to zero for $\phi \ge \hat{\phi}(0)$.*

4.1.2. Net Donations to Beneficiaries. We now examine the aggregate donations received by all admitted beneficiaries after the platform deducts its commission, which we refer to as *net donations*. In the covered regime, every donor on the circle contributes, and aggregate net donations are

$$D_C(f) = g^*(f)(1-f) = \frac{\beta^2(1-\theta f)(1-f)}{4\alpha^2}. \tag{4}$$

In the uncovered regime, each admitted beneficiary draws contributions from donors within $\tilde{k}(f)$ on either side. Each beneficiary thus earns $2\tilde{k}(f)g^*(f)(1-f)$, and with $n^*(f) = 2g^*(f)f\tilde{k}(f)/\gamma$ beneficiaries admitted,[16] the aggregate donation net of commission is then

$$D_U(f) = 2\tilde{k}(f)g^*(f)(1-f)n^*(f) = \frac{\beta^8(1-\theta f)^4 f(1-f)}{64\,\alpha^6\lambda^2\gamma}. \tag{5}$$

The expressions in (4) and (5) differ in two important ways that underlie the comparative statics. First, γ does not enter D_C directly, whereas it appears in the denominator of D_U. In the covered regime, the platform admits just enough beneficiaries to provide full donor coverage, implying that onboarding costs affect net donations only indirectly through their impact on the equilibrium commission. In contrast, in the uncovered regime, γ directly affects the platform's admission decision and therefore the number of beneficiaries admitted. Second, D_C is strictly decreasing in $f \in [0, 1/\theta]$, whereas D_U is generally non-monotonic in f. We now turn to the effects of the two cost parameters, γ and ϕ, on equilibrium net donations.

PROPOSITION 5 **(Comparative Statics of Net Donations)**. *Let $D^*(\gamma,\phi)$ denote the equilibrium net donations to beneficiaries, evaluated at the platform's profit-maximizing commission $f^*(\gamma,\phi)$ from Proposition 4. At parameter values in the interior of the respective regions, the partial derivatives satisfy:*

(i) (Covered Regions) In Region I, $\frac{\partial D^*}{\partial \gamma} > 0$ and $\frac{\partial D^*}{\partial \phi} = 0$. In Region II, $\frac{\partial D^*}{\partial \phi} > 0$ and $\frac{\partial D^*}{\partial \gamma} = 0$.

(ii) (Uncovered Regions) In Region III, $\frac{\partial D^*}{\partial \gamma} < 0$ and $\frac{\partial D^*}{\partial \phi} = 0$. In Region IV, $\frac{\partial D^*}{\partial \gamma} < 0$, and the sign of $\frac{\partial D^*}{\partial \phi}$ matches the sign of $-Q(\bar{f}(\phi))$, where $Q(f) := 1-(2+5\theta)f + 6\theta f^2$.

We first discuss the effect of γ. In the covered market of Region I, the platform admits $n^* = 1/(2\tilde{k}(f^*))$ to keep every donor within reach of some beneficiary. A high commission shrinks donor reach and requires more admissions to cover the market. When γ increases, the cost to admit the same number of beneficiaries becomes higher, and the platform responds by lowering f^* so that it can admit fewer beneficiaries while still reaching all donors (Corollary 1(i)). A lower commission has a two-pronged positive effect on net donations: it both increases each donor's contribution $g^*(f)$

[16] We henceforth drop the dependence on $\bar{N}(f)$ from our notation and simply write $n^*(f)$; this is for brevity and because, as discussed in Section 3.3, the platform's admission decision is not affected in equilibrium by the value of $\bar{N}(f)$.

and leaves a larger share for beneficiaries, so D^* increases. On the other hand, in the uncovered regime, the commission is $1/(3\theta)$ or $\bar{f}(\phi)$, neither of which depends on γ. Instead of adjusting the commission, now the platform absorbs a higher γ by admitting fewer beneficiaries. Since the commission is unchanged, this means fewer donors each donating the same amount as before (with the same percentage passing through to beneficiaries), so net donations fall. The same parameter that lowers the commission in Region I lowers the admission level in the uncovered regime, with opposite effects on aggregate net donations.

We now turn to the effect of the entry cost ϕ. In the covered regime, admissions are tied to the coverage constraint, and D_C is strictly decreasing in f. In Region I, ϕ does not affect the commission and therefore has no effect on D^*. In Region II, a higher ϕ lowers $f^* = \bar{f}(\phi)$, so net donations strictly increase. In the uncovered regime, when the beneficiary entry cost cap binds, the commission equals $\bar{f}(\phi)$, and a higher ϕ lowers it. A lower commission increases donor reach in the uncovered regime; moreover (and just like in the covered regime), it also increases the amount each donor gives and leaves a larger share for beneficiaries. However, admissions in the uncovered regime peak at $f = 1/(3\theta)$; pushing the commission below this peak reduces n^*, so fewer beneficiaries receive contributions. These effects pull in opposite directions, and which dominates depends on the commission charged—captured formally by the sign of $Q(\bar{f}(\phi))$.

In summary, Proposition 5 yields two important implications. First, the effect of γ reverses sign across regimes: it increases net donations in Region I (covered), has no effect in Region II (covered), and reduces them in both uncovered regions. Second, the effect of ϕ is ambiguous in Region IV (uncovered), with its sign determined by the quadratic $Q(\bar{f}(\phi))$. Thus, platforms should carefully adjust their decisions when facing changes in the operational environment, such as changes in onboarding technology, verification or compliance requirements, and beneficiaries' application burdens.

4.2. Outcomes under a Commission Cap

The analysis so far assumes that the platform freely chooses its commission. We now consider a setting in which a policymaker or nonprofit board caps the commission the platform is permitted to charge, with the aim of maximizing the donations that reach beneficiaries. We treat this as a donation-maximizing benchmark rather than a welfare benchmark. A commission cap is a natural instrument for a policymaker seeking to steer more of each donation to beneficiaries, and several platforms already operate at or near zero commission.[17]

[17] Several major platforms have moved toward minimal or zero commissions in recent years. GoFundMe eliminated its platform fee on personal fundraisers in 2017, relying instead on optional donor tips. Nonprofit platforms such as GlobalGiving publicly report operating at near-cost, returning roughly 93–96% of each donation to beneficiaries.

Concretely, suppose that a policymaker sets a commission cap $r \in [0, 1/\theta]$ in Stage 0, preceding the profit-maximizing platform's commission decision. In Stage 1, the platform sets $f \le r$ to maximize profit over the commissions it is permitted to charge, with profit $\Pi(f)$ when the market operates and zero when the entry cost exceeds the per-beneficiary payoff $\hat{\phi}(f)$. Stages 2–4 remain unchanged. Thus, under the commission cap, the platform's problem becomes

$$\max_{f \in [0,r]} \tilde{\Pi}(f), \qquad \text{where} \qquad \tilde{\Pi}(f) = \begin{cases} \Pi(f) & \text{if } \phi \le \hat{\phi}(f), \\ 0 & \text{if } \phi > \hat{\phi}(f). \end{cases} \tag{6}$$

Let $f^*(\gamma, \phi)$ denote the platform's profit-maximizing commission absent any cap, characterized in Proposition 4. As the platform's profit is single-peaked in the commission (Proposition 4) and the cap only removes commissions above r, the platform charges its unconstrained optimum whenever it is below the cap and charges the cap otherwise, i.e.,

$$f_r^*(\gamma, \phi) = \min\{f^*(\gamma, \phi), r\}. \tag{7}$$

The policymaker anticipates this response and chooses the commission cap to maximize net donations. As with the platform, net donations are $D(f)$ when the market operates and zero when it shuts down. The policymaker's Stage 0 problem is therefore

$$\max_{r \in [0, 1/\theta]} \tilde{D}\big(f_r^*(\gamma, \phi)\big), \qquad \text{where} \qquad \tilde{D}(f) = \begin{cases} D(f) & \text{if } \phi \le \hat{\phi}(f), \\ 0 & \text{if } \phi > \hat{\phi}(f), \end{cases} \tag{8}$$

where $f_r^*(\gamma, \phi)$ is given by (7). Although the policymaker only sets a commission cap, the following proposition shows that problem (8) is equivalent to the policymaker setting the commission directly to maximize net donations.

Proposition 6 **(Equivalence of the Commission Cap and Direct Regulation)**. *Fix (γ, ϕ) for which the market operates. Then*

$$\max_{r \in [0,1/\theta]} \tilde{D}\big(f_r^*(\gamma, \phi)\big) = \max_{f \in [0,1/\theta]} \tilde{D}(f), \tag{9}$$

and any commission $f^\dagger$ attaining the right-hand maximum is attained by the cap $r = f^\dagger$.

Proposition 6 implies that the commission cap problem (8) delivers the same net donations as the policymaker choosing the commission directly to maximize net donations. We therefore work with the equivalent direct formulation, in which the policymaker selects $f \in [0, 1/\theta]$ in Stage 1 and the platform retains control over admission. Stages 2–4 are unchanged: Proposition 1 gives the optimal donation $g^*(f)$ and the participation threshold $\tilde{k}(f)$, Proposition 2 gives the platform's admission decision, and equilibrium beneficiary entry follows Proposition 3. With a slight abuse of notation, we write $D_C(f)$ and $D_U(f)$ for $D_C(f,n^*(f))$ and $D_U(f,n^*(f))$, respectively. Substituting the equilibrium quantities into net donations yields the piecewise objective

$$D(f)=\begin{cases} D_C(f)=\frac{\beta^2(1-\theta f)(1-f)}{4\alpha^2} & \text{if } \gamma\le\hat{\gamma}(f),\\ D_U(f)=\frac{\beta^8(1-\theta f)^4 f(1-f)}{64\alpha^6\lambda^2\gamma} & \text{if } \gamma>\hat{\gamma}(f).\end{cases} \tag{10}$$

These expressions are the same as (4) and (5) for a given commission f, but unlike before, we now use them directly in the objective function. Specifically, the policymaker solves

$$\max_{f\in[0,1/\theta]} \tilde{D}(f), \qquad \text{where} \qquad \tilde{D}(f)=\begin{cases} 0 & \text{if } \phi>\hat{\phi}(f),\\ D(f) & \text{otherwise,}\end{cases}$$

where $D(f)$ is given by (10). The condition $\phi\le\hat{\phi}(f)$ caps the commission at $\bar{f}(\phi)$. The two branches of $D(f)$ coincide on the regime boundary $\gamma=\hat{\gamma}(f)$, and $D(f)$ is continuous in f.

Although $D(f)$ has the same piecewise structure as the platform's profit in (3), the trade-offs in the two objectives naturally differ. The platform keeps a share f of each donation, so the commission enters its profit-maximizing objective as direct revenue. Beneficiaries receive the complementary share $(1-f)$, so the same commission reduces the donations that ultimately reach them. Lemma 6 in Appendix D formalizes the shape of the net donations function $D(f)$: $D_C(f)$ is strictly decreasing on $[0,1/\theta]$, while $D_U(f)$ is single-peaked at an interior maximum $f_B^U\in(0,1/(4\theta))$ that depends only on θ. Combining these results with the cap $\bar{f}(\phi)$, we characterize the equilibrium in Proposition 7.

Proposition 7 **(Donation-Maximizing Solution)**. *Let* $\gamma_B:=\hat{\gamma}(f_B^U)=\frac{\beta^6(1-\theta f_B^U)^3 f_B^U}{16\alpha^4\lambda^2}$ *and* $\phi_B:=\hat{\phi}(f_B^U)=\frac{\beta^4(1-\theta f_B^U)^2(1-f_B^U)}{8\alpha^3\lambda}$, *and let* $f_B^\circ(\gamma):=f_L(\gamma)$ *for* $\gamma\le\gamma_B$ *and* $f_B^\circ(\gamma):=f_B^U$ *for* $\gamma>\gamma_B$. *Provided* $\phi<\frac{\beta^4}{8\alpha^3\lambda}$, *the donation-maximizing equilibrium is characterized below (otherwise, the market does not operate).*

	$\gamma\le\gamma_B$	$\gamma>\gamma_B$
Entry Cap Non-Distorting: $\phi\le\hat{\phi}(f_B^\circ(\gamma))$	**B-I** (Covered) $f_B^*=f_L(\gamma)$ $n_B^*=\dfrac{1}{2\tilde{k}(f_B^*)}$	**B-III** (Uncovered) $f_B^*=f_B^U$ $n_B^*=\dfrac{2g^*(f_B^*)f_B^*\tilde{k}(f_B^*)}{\gamma}$
Entry Cap Distorts: $\phi>\hat{\phi}(f_B^\circ(\gamma))$	**B-II** (Uncovered) $f_B^*=\bar{f}(\phi)$ $n_B^*=\dfrac{2g^*(f_B^*)f_B^*\tilde{k}(f_B^*)}{\gamma}$	**B-IV** (Uncovered) $f_B^*=\bar{f}(\phi)$ $n_B^*=\dfrac{2g^*(f_B^*)f_B^*\tilde{k}(f_B^*)}{\gamma}$

Proposition 7 partitions the parameter space into four regions along two costs: the onboarding cost γ and the beneficiary entry cost ϕ. Figure 4 visualizes this partition in (γ,ϕ)-space.

When onboarding is cheap ($\gamma\le\gamma_B$), the donation-maximizing commission lies in the covered regime. Since D_C is decreasing in f (Lemma 6 in Appendix D), the policymaker chooses the smallest commission consistent with coverage $f_L(\gamma)$. This commission increases with γ: a more expensive platform can only sustain coverage at a higher commission, since a higher per-donor revenue justifies

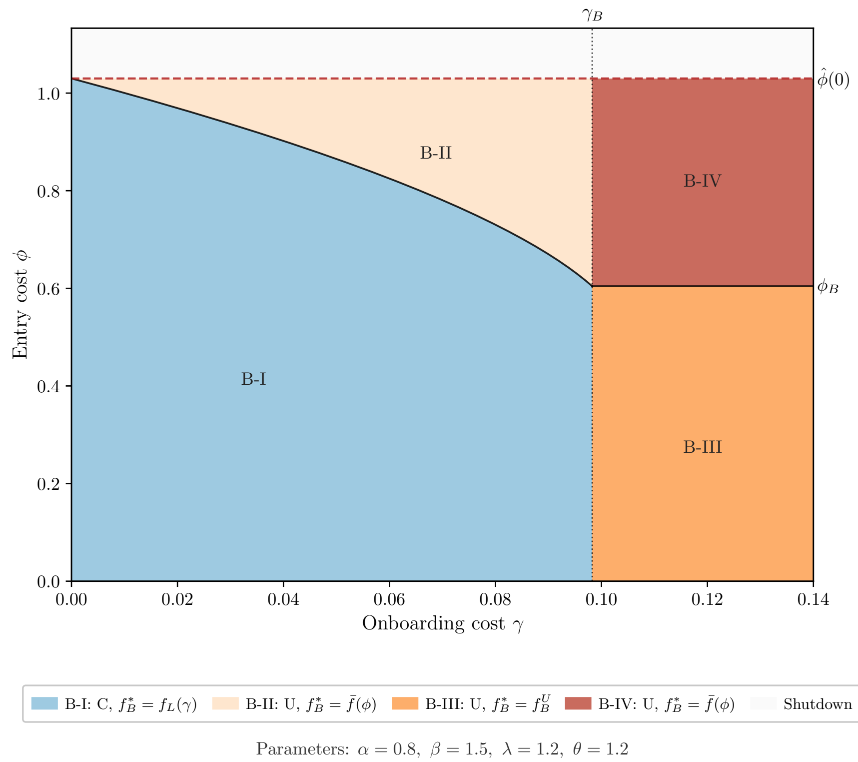

Figure 4 Equilibrium regions in γ, ϕ-space

the onboarding cost. When $\gamma_B < \gamma < \hat{\gamma}_{\max}$, the smallest commission consistent with coverage exists but exceeds f_B^U, and the covered optimum yields lower net donations than the uncovered peak. When $\gamma \geq \hat{\gamma}_{\max}$, no non-degenerate covered interval exists; at equality, coverage is possible only at $f = 1/(4\theta)$. In either case, the policymaker chooses the interior uncovered peak f_B^U (see the proof of Proposition 7). Since f_B^U is independent of γ, higher onboarding costs are absorbed entirely through admissions: n_B^* falls with γ in this regime, while the commission remains unchanged. When $\bar{f}(\phi)$ falls below this preferred commission, the condition $\phi \leq \hat{\phi}(f)$ pins f_B^* at $\bar{f}(\phi)$. We now turn to the comparative statics of n_B^*.

Lemma 1 **(Non-monotonic Admissions in γ).** *For $\phi \leq \phi_B$, the donation-maximizing admission level $n_B^*(\gamma)$ is strictly increasing in γ on $(0, \gamma_B]$ and strictly decreasing on (γ_B, ∞), attaining its unique maximum at $\gamma = \gamma_B$.*

From Lemma 1, we have that γ affects admissions in opposite directions across the two regimes, with the peak at the regime switch (from covered to uncovered). In Region B-I, the market is covered, so admissions are determined by how many beneficiaries it takes to reach every donor. A higher γ raises the smallest commission consistent with coverage, $f_L(\gamma)$, which shrinks each beneficiary's reach $\tilde{k}(f)$, and more beneficiaries are needed to cover the circle. In Region B-III, $f_L(\gamma)$ exceeds the uncovered optimum f_B^U, so coverage is no longer optimal, and the policymaker sets $f = f_B^U$. Since

f_B^U is independent of γ, the only remaining channel is the onboarding cost in the denominator of n_B^*: a higher γ means the platform can afford fewer admissions. The two regimes meet at γ_B, where the increasing floor in B-I just reaches f_B^U, and admissions peak at this switch point.

In Corollary 3, we compare the donation-maximizing commission with the profit-maximizing commission to find the ordering between the two, and we ask what happens when a regulator caps the commission below the donation-maximizing level.

COROLLARY 3 **(Ordering of Commissions and the Risk of Over-Regulation)**. *For every (γ, ϕ) at which the market operates, the donation-maximizing commission is weakly below the profit-maximizing one, i.e.,*

$$f_B^*(\gamma, \phi) \leq f^*(\gamma, \phi),$$

where $f^*(\gamma, \phi)$ and $f_B^*(\gamma, \phi)$ are the commissions from Propositions 4 and 7, respectively. Moreover, an exogenously imposed cap need not increase net donations: a cap set too aggressively can induce strictly lower net donations than the profit-maximizing outcome.

The ordering reflects the opposing objectives in the two cases. Holding total giving fixed, the platform receives more revenue from a higher commission, while beneficiaries retain the complementary share $1 - f$. This direct difference in incentives leads to a lower commission under the donation-maximizing case, after accounting for the effects of the commission on donations and admissions. The gap between the two measures how far profit maximization pushes the commission above the level that maximizes resources reaching beneficiaries.[18] This does not mean, however, that lowering the commission always increases net donations, since commission revenue also funds onboarding: as the commission approaches zero, so do admissions. A policymaker who caps the commission too far below f_B^* can therefore induce lower net donations than would arise without intervention.

4.3. Donation and Admission Gaps

The previous sections establish that the platform's profit-maximizing commission generally differs from the donation-maximizing one. To compare the two objectives, we focus on aggregate net donations—the policymaker's objective in our benchmark—and admissions, which capture how many causes gain access to donors.[19] We analyze each in turn.

[18] We illustrate how both commissions respond to the two cost parameters γ and ϕ numerically in Section 5.

[19] A policymaker concerned with the diversity of causes funded, or with ensuring that smaller and less-established nonprofits can reach donors, might weigh admissions alongside net donations.

4.3.1. Donation Gap. We define the donation gap as

$$\Delta_D(\gamma,\phi) := D_B^*(\gamma,\phi) - D^*(\gamma,\phi),$$

where D^* and D_B^* are the net donations from the profit-maximizing and donation-maximizing cases characterized in Propositions 4 and 7, respectively.[20] Hence, $D_B^*(\gamma,\phi) \geq D^*(\gamma,\phi)$, and $\Delta_D(\gamma,\phi) \geq 0$ measures the net donations forgone when the platform freely sets the commission to maximize its profit.

To analyze $\Delta_D(\gamma,\phi)$, we introduce notation for the unconstrained commissions. Let $f^\circ(\gamma)$ and $f_B^\circ(\gamma)$ denote the platform's and the policymaker's preferred commissions, respectively, before the condition $\phi \leq \hat{\phi}(f)$ is imposed. From Propositions 4 and 7, respectively, we have

$$f^\circ(\gamma) = \begin{cases} f_C^*(\gamma), & \gamma \leq \bar{\gamma}, \\ \frac{1}{3\theta}, & \gamma > \bar{\gamma}, \end{cases} \qquad f_B^\circ(\gamma) = \begin{cases} f_L(\gamma), & \gamma \leq \gamma_B, \\ f_B^U, & \gamma > \gamma_B. \end{cases}$$

Each case (profit-maximizing and donation-maximizing) charges its preferred commission $f^\circ(\gamma)$ or $f_B^\circ(\gamma)$ when the beneficiary entry cost cap $\bar{f}(\phi)$ does not distort its choice. Lemma 2 characterizes the donation gap and its comparative statics in ϕ.

Lemma 2 **(Structure and Comparative Statics of the Donation Gap)**. *Fix $\gamma > 0$. For $0 < \phi < \hat{\phi}(0)$, the donation gap satisfies*

$$\Delta_D(\gamma,\phi) = \begin{cases} D(f_B^\circ(\gamma);\gamma) - D(f^\circ(\gamma);\gamma) & \text{if } \bar{f}(\phi) \geq f^\circ(\gamma), \\ D(f_B^\circ(\gamma);\gamma) - D(\bar{f}(\phi);\gamma) & \text{if } f_B^\circ(\gamma) < \bar{f}(\phi) < f^\circ(\gamma), \\ 0 & \text{if } \bar{f}(\phi) \leq f_B^\circ(\gamma). \end{cases}$$

Moreover, $\Delta_D(\gamma,\phi)$ is independent of ϕ when $\bar{f}(\phi) \geq f^\circ(\gamma)$, strictly decreasing in ϕ when $f_B^\circ(\gamma) < \bar{f}(\phi) < f^\circ(\gamma)$, and zero when $\bar{f}(\phi) \leq f_B^\circ(\gamma)$. The gap first reaches zero at $\phi = \hat{\phi}(f_B^\circ(\gamma))$ and remains zero for larger ϕ in the operating range. If $\phi \geq \hat{\phi}(0)$, neither case operates; defining both net-donation outcomes as zero gives $\Delta_D(\gamma,\phi) = 0$.

Lemma 2 partitions the operating parameter space into three cases based on where $\bar{f}(\phi)$ is relative to the two preferred commissions. In the first case, the entry cost cap does not distort either choice (profit-maximizing or donation-maximizing), so each charges its preferred commission, and the gap depends only on γ; in the second case, the cap distorts the platform's choice but not the policymaker's; in the third case, the cap forces both choices to $\bar{f}(\phi)$. Since $\bar{f}(\phi)$ is strictly decreasing in ϕ (Lemma 5), increasing ϕ moves through the three cases of Lemma 2 in order. Figure 10a in Appendix F illustrates how the donation gap varies with ϕ.[21]

[20] For the rest of this section, "the platform" refers to the profit-maximizing case and "the policymaker" refers to the donation-maximizing benchmark.

[21] Lemma 2 considers a fixed γ; we examine the effect of γ numerically in Section 5.2.

A higher beneficiary entry cost therefore shrinks the donation gap, but only up to the threshold $\hat{\phi}(f_B^\circ(\gamma))$. A smaller gap, however, need not indicate an improvement in net donations. Beyond this threshold and while the market operates, the entry-cost-imposed ceiling makes both commissions equal to $\bar{f}(\phi)$, so the gap remains zero, but it also pushes the common commission below the donation-maximizing level and net donations fall. At $\phi = \hat{\phi}(0)$, the market shuts down and net donations reach zero; further increases in ϕ leave them at zero. Thus, the donation gap is a useful comparison between the two objectives but should not be viewed as a policy target in itself.

4.3.2. Admission Gap. We define the admission gap as

$$\Delta_n(\gamma,\phi) := n^*(\gamma,\phi) - n_B^*(\gamma,\phi),$$

where n^* and n_B^* are the admissions from the profit-maximizing and donation-maximizing cases characterized in Propositions 4 and 7, respectively. Unlike the donation gap, the sign of $n^* - n_B^*$ is not immediate from the two objectives. In fact, the admission gap runs in the opposite direction from the donation gap: the platform always admits at least as many beneficiaries as the policymaker. We formalize this result in Lemma 3.

LEMMA 3 **(Structure and Comparative Statics of the Admission Gap)**. *Fix $\gamma > 0$. For $0 < \phi < \hat{\phi}(0)$, the admission gap satisfies*

$$\Delta_n(\gamma,\phi) = \begin{cases} n^*(f^\circ(\gamma);\gamma) - n^*(f_B^\circ(\gamma);\gamma) & \text{if } \bar{f}(\phi) \geq f^\circ(\gamma), \\ n^*(\bar{f}(\phi);\gamma) - n^*(f_B^\circ(\gamma);\gamma) & \text{if } f_B^\circ(\gamma) < \bar{f}(\phi) < f^\circ(\gamma), \\ 0 & \text{if } \bar{f}(\phi) \leq f_B^\circ(\gamma), \end{cases}$$

where each expression above is non-negative. Moreover, $\Delta_n(\gamma,\phi)$ is independent of ϕ when $\bar{f}(\phi) \geq f^\circ(\gamma)$, strictly decreasing in ϕ when $f_B^\circ(\gamma) < \bar{f}(\phi) < f^\circ(\gamma)$, and zero when $\bar{f}(\phi) \leq f_B^\circ(\gamma)$. The gap first reaches zero at $\phi = \hat{\phi}(f_B^\circ(\gamma))$—the same threshold at which the donation gap closes. If $\phi \geq \hat{\phi}(0)$, neither case operates; defining both admission outcomes as zero gives $\Delta_n(\gamma,\phi) = 0$.

Lemma 3 mirrors the structure of Lemma 2: the three cases are determined by where the beneficiary entry cost cap $\bar{f}(\phi)$ sits relative to the profit-maximizing and donation-maximizing commissions. In the first case, the cap does not distort either choice; in the second case, it distorts the platform's choice but not the policymaker's; and in the third case, it pins both choices at the same commission. We illustrate it in Figure 10b in Appendix F.[22]

The platform admits at least as many beneficiaries as the policymaker for two distinct reasons across the two regimes. In the covered regime, admissions are determined by $n = 1/(2\tilde{k}(f))$: a

[22] Similar to the donation gap, we examine the effect of γ on Δ_n numerically in Section 5.2.

lower commission—which the policymaker prefers—widens each beneficiary's reach $\tilde{k}(f)$, reducing the number of beneficiaries required to cover the circle. The platform's higher commission, therefore, induces more admissions than the policymaker's. In the uncovered regime, admissions $n = 2g^*(f)f\tilde{k}(f)/\gamma$ are maximized at the platform's preferred commission $1/(3\theta)$. The policymaker chooses $f_B^U < 1/(3\theta)$, accepting fewer admissions in exchange for a larger share of each donation reaching beneficiaries. The platform's motive—in the uncovered case—aligns with maximizing admissions, while the policymaker trades admissions for per-beneficiary net donations to achieve the maximum net donations. As we show next, the same threshold of ϕ that closes the donation gap also closes the admission gap.

Lemma 3 also shows that the admission gap closes at the same threshold as the donation gap $\hat{\phi}(f_B^\circ(\gamma))$. As ϕ increases (for $f_B^\circ(\gamma) < \bar{f}(\phi) < f^\circ(\gamma)$), the beneficiary entry cost cap pulls the platform's commission toward $f_B^\circ(\gamma)$, and Δ_n shrinks. However, the admission gap closes in the opposite direction compared to the donation gap: the donation gap closes by pulling D^* up toward D_B^*, whereas the admission gap closes by pulling n^* down to n_B^*. This asymmetry has a practical implication for policymakers. Admission counts alone do not measure the resources reaching causes: whenever the two commissions differ, the profit-maximizing platform admits more beneficiaries than the donation-maximizing benchmark but generates lower aggregate net donations. Net donations under profit maximization attain their maximum when both gaps first close; beyond this threshold, the entry-cost constraint pushes both commissions below the donation-maximizing level, and net donations fall.

5. Numerical Study

Having characterized the equilibrium outcomes analytically, we now illustrate the comparative statics numerically, focusing on the effect of two cost parameters. Throughout this section, we set $w = 5000$, $\alpha = 0.001$, $\beta = 0.1$, $\lambda = 1000$, and $\theta = 1.2$. We examine how the profit-maximizing and donation-maximizing commissions respond to the cost parameters in Section 5.1, and then turn to the donation gap and the admission gap in Section 5.2.

5.1. Sensitivity of the Commission to Cost Parameters

We first study how the two commissions respond to each cost parameter in turn. The comparative statics and the ordering of the two commissions are both established analytically (Corollaries 1 and 3, Lemma 5), so the goal of this section is to illustrate them and to show how the two commissions move relative to each other.

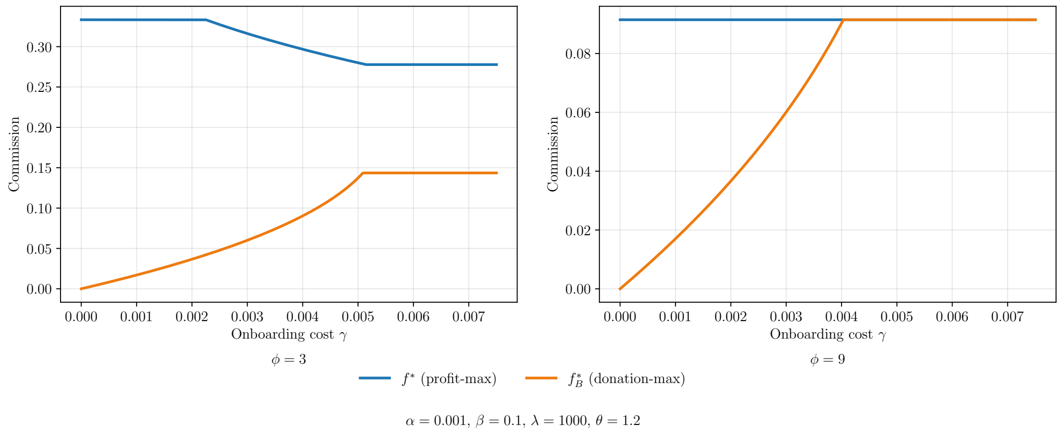

Figure 5 Commission vs. Onboarding cost γ

Sensitivity of commissions (w.r.t. γ). When the entry cost is low ($\phi = 3$ in Figure 5), the entry cost cap initially distorts the platform's choice, so the profit-maximizing commission $f^*(\gamma) = \bar{f}(\phi)$ is flat at the lowest values of γ. Once the cap no longer distorts the choice, $f^*(\gamma) = f_C^*(\gamma)$ decreases over the remainder of the covered region (Corollary 1), as the platform lowers its commission to widen each beneficiary's reach and cover the market with fewer beneficiaries. The donation-maximizing commission $f_L(\gamma)$ instead increases while its solution remains covered, since sustaining coverage at a higher onboarding cost requires a higher commission. When the entry cost is high ($\phi = 9$), the entry cost cap $\bar{f}(\phi)$ lies below the platform's preferred commission for every γ and holds it constant, so only the donation-maximizing commission increases—until it reaches the cap and the two coincide.

Sensitivity of commissions (w.r.t. ϕ). Figure 6 shows that both commissions are weakly decreasing in ϕ. The beneficiary entry cost affects the commission only through the entry cost cap $\bar{f}(\phi)$, which decreases in ϕ and constrains each commission from above. When the entry cost cap is at or above the preferred commission, it does not distort the choice; once the cap falls below it, the constraint binds and distorts the choice, and the commission equals the cap. Since the profit-maximizing commission is higher (Corollary 3), it is distorted first, and the two commissions coincide only once the cap falls below the donation-maximizing level. A higher entry cost therefore tightens the endogenous ceiling on both commissions; once that ceiling falls below the donation-maximizing level, it also distorts the benchmark.

5.2. Donation and Admission Gap

The donation and admission gaps are characterized analytically only with respect to the entry cost ϕ (Lemmas 2 and 3); we therefore use the numerical study to examine how each responds to the onboarding cost γ. We observe that a higher γ initially narrows the donation gap largely by lowering the donation-maximizing benchmark. Over the initial range in which the platform's

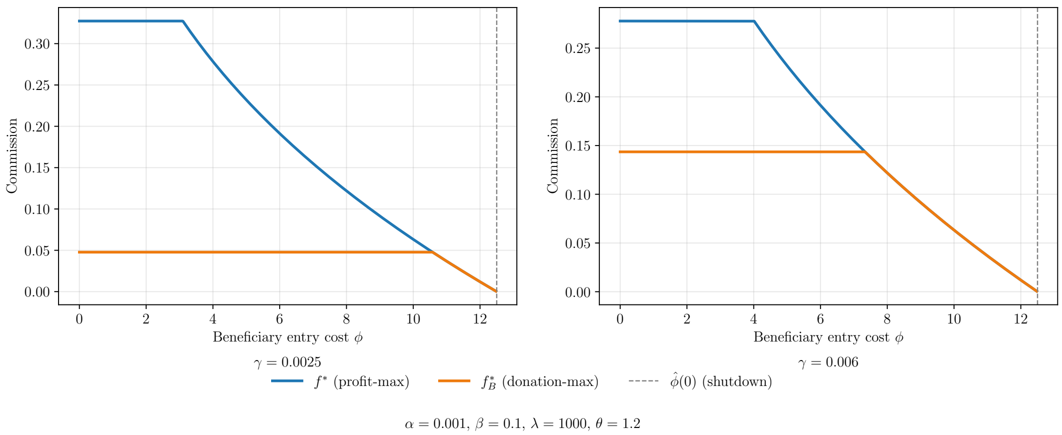

Figure 6 **Commission vs. Beneficiary entry cost ϕ**

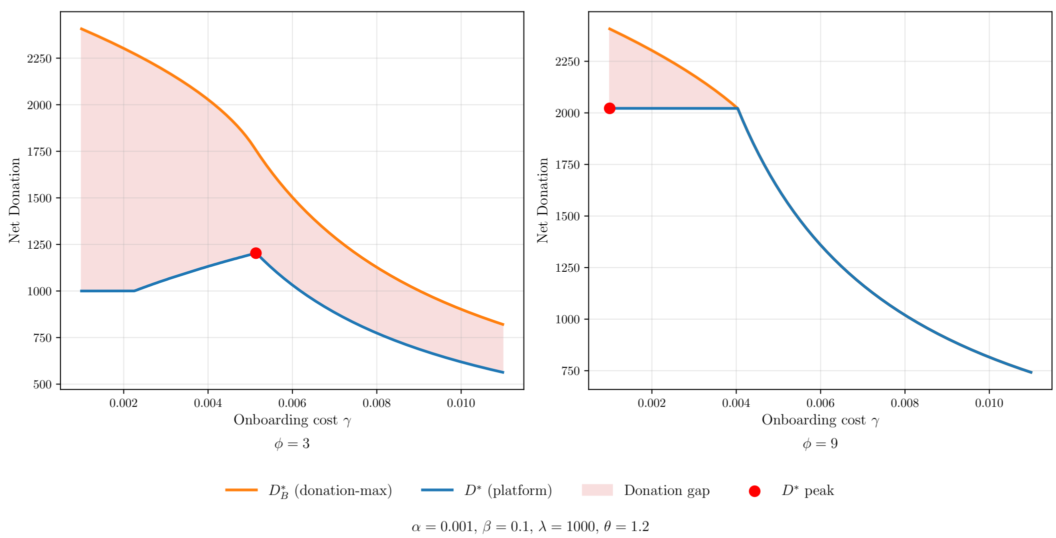

Figure 7 **Donation Gap**

admissions are flat and the policymaker's rise, it also narrows the admission gap, although whether that gap closes depends on ϕ.

Donation Gap (w.r.t. γ). We now examine the impact of the onboarding cost γ on the donation gap. Figure 7 plots net donations under both cases (profit-maximizing and donation-maximizing) against γ for a low ($\phi = 3$) and high ($\phi = 9$) entry cost. Net donations under profit maximization are non-monotonic in γ for $\phi = 3$. As γ increases toward the peak, D^* moves closer to D_B^* and the gap shrinks. But D_B^* also decreases with γ for both values of ϕ. The smaller gap at higher γ is therefore partly driven by a deterioration in the benchmark and need not imply higher net donations.

Admission Gap (w.r.t. γ). We now examine the impact of γ on the admission gap. Figure 8 plots admissions under both cases (profit-maximizing and donation-maximizing) against γ for a low ($\phi = 3$) and high ($\phi = 9$) entry cost. For both values of ϕ, the platform's profit-maximizing admissions n^* remain flat over an initial range of γ, whereas the policymaker's donation-maximizing

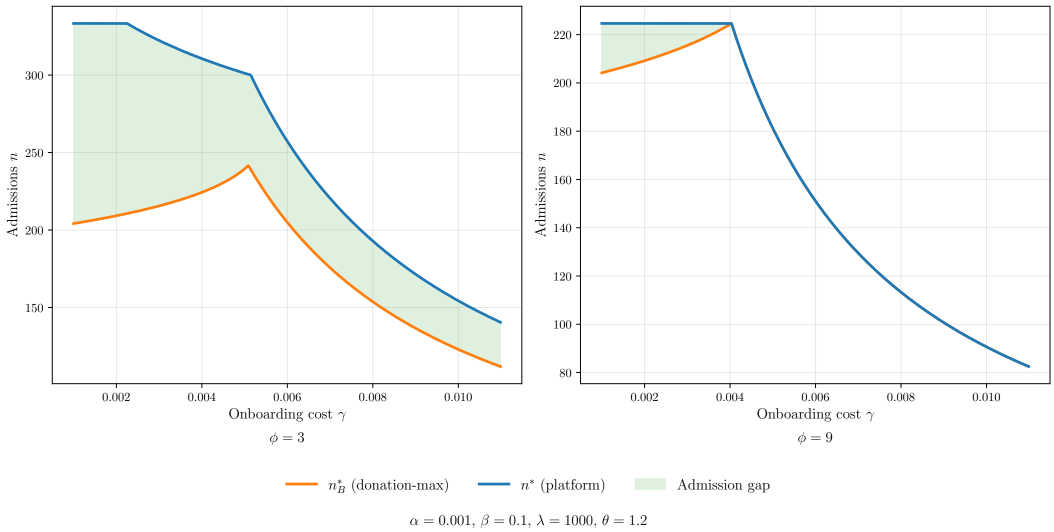

Figure 8 **Admission Gap**

admissions n_B^* increase over the same range. At $\phi = 9$, this increase closes the admission gap without lowering either curve. At $\phi = 3$, it narrows but does not close the gap at any finite γ; once both solutions are uncovered, the gap is a positive constant times $1/\gamma$ and therefore converges to zero as γ grows. Thus, the admission gap closes at higher γ in the high-entry-cost calibration, but this pattern does not hold uniformly across entry costs.

6. Conclusions

We introduce a stylized game-theoretic model of a profit-maximizing charitable fundraising platform that decides the commission on donations and the number of beneficiaries to onboard. We compare the resulting equilibrium with a donation-maximizing benchmark in which a policymaker caps the commission. Our results show that market coverage governs the effects of onboarding and entry costs. The two costs operate through different channels: the platform's onboarding cost directly raises the cost of expanding its beneficiary base, whereas the beneficiaries' entry cost creates an endogenous ceiling on the commission. The commission, in turn, plays three roles: it divides donations between the platform and beneficiaries, finances onboarding, and determines both how much donors contribute and how many donors each beneficiary can reach. As a result, the equilibrium outcomes respond to these costs in non-monotonic ways: the profit-maximizing commission can fall as onboarding costs increase; net donations can rise with the beneficiary entry cost before eventually falling; and the donation-maximizing benchmark yields weakly higher net donations but weakly fewer admissions than the profit-maximizing platform.

For platforms, our results show that the commission and the admission decision jointly determine market coverage: the commission sets each beneficiary's donor reach, and hence the number of admissions needed to reach all donors. The commission is thus not only a source of revenue but also a determinant of onboarding requirements. When covering the market is worthwhile and beneficiary entry cost is on the low end, a platform should resist the urge to raise its commission to recoup a higher onboarding cost: a higher commission shrinks each beneficiary's reach, so sustaining coverage would require admitting even more beneficiaries. The profitable response is instead to lower the commission and preserve coverage with a leaner beneficiary base. When onboarding is costly enough that full coverage is no longer worthwhile, the platform holds its commission and cuts admissions.

For policymakers seeking to maximize aggregate net donations, three implications follow. First, the appropriate commission cap depends on the operating environment. Changes in onboarding technology, verification or compliance requirements, or beneficiaries' application burdens can shift γ or ϕ, and their effects on net donations depend on the coverage regime. These cost comparative statics describe changes that a policymaker should anticipate; the direct policy lever in our analysis is the commission cap. Second, an overly restrictive commission cap can reduce the resources reaching causes. The commission finances onboarding, and a cap set too far below the donation-maximizing level can induce lower aggregate net donations than no cap. Third, closing the gaps between the profit-maximizing and donation-maximizing outcomes is not itself the policy objective. As entry costs rise, net donations under profit maximization attain their maximum when the gaps first close; further increases constrain both commissions below the donation-maximizing level and reduce net donations. Admission counts likewise do not measure the resources reaching causes: whenever the commissions differ, the profit-maximizing platform admits more beneficiaries while generating lower aggregate net donations.

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Appendix A: Table of Notation

Table 1 Summary of Notation

Model parameters	
$w > 0$	Disposable wealth or endowment of a donor
$\beta > 0$	Weight on charitable giving in a donor's utility
$\alpha > \frac{\beta}{2\sqrt{w}}$	Weight on private consumption in a donor's utility[a]
$\lambda > 0$	Disutility coefficient associated with distance
$\theta \geq 1$	Degree of donor aversion to platform commissions
$\gamma > 0$	Cost coefficient for onboarding and managing beneficiaries
$\phi > 0$	Fixed cost incurred by a potential beneficiary
Decision variables	
$f \in [0, \frac{1}{\theta}]$	Commission rate charged by the platform
$n \in [0, \bar{N}]$	Number of beneficiaries admitted to the platform
Equilibrium objects	
$\bar{N} \geq 0$:	Number of beneficiaries who apply to the platform
g^*:	Equilibrium donation
$\tilde{k}$:	Cutoff distance for contributions

[a] The lower bound $\alpha > \frac{\beta}{2\sqrt{w}}$ ensures that donors put at least enough weight on private consumption that they would not wish to donate all of their wealth.

Appendix B: Proofs of Propositions

Proof of Proposition 1. If no beneficiary is admitted, the convention in the timing of the game gives zero contributions. Suppose therefore that $n > 0$. First take $f \in [0, 1/\theta)$, and write $z := 1 - \theta f > 0$. For every $g > 0$, completing the square gives

$$u(g,k,f) - u(0,k,f) = -\alpha\left(\sqrt{g} - \frac{\beta\sqrt{z}}{2\alpha}\right)^2 + \frac{\beta^2 z}{4\alpha} - \lambda k.$$

Thus, the unique maximizer over positive contributions is

$$g^*(f) = \frac{\beta^2 z}{4\alpha^2} = \frac{\beta^2(1-\theta f)}{4\alpha^2}.$$

This is feasible and interior: the assumption $\alpha > \frac{\beta}{2\sqrt{w}}$ implies $\frac{\beta^2}{4\alpha^2} < w$, and hence $0 < g^*(f) < w$. The maximum utility gain from contributing is $\frac{\beta^2 z}{4\alpha} - \lambda k$. It is positive if $k < \tilde{k}(f)$, negative if $k > \tilde{k}(f)$, and zero if

$$k = \tilde{k}(f) := \frac{\beta^2(1-\theta f)}{4\alpha\lambda}.$$

At equality, both $g = 0$ and $g = g^*(f)$ are optimal; the tie-breaking convention in the text selects $g^*(f)$.

At $f = 1/\theta$, the warm-glow term vanishes. For every $g > 0$,

$$u(g,k,1/\theta) - u(0,k,1/\theta) = -\alpha g - \lambda k < 0,$$

so the unique optimum is $g = 0$. Finally, the displayed formulas for $g^*(f)$ and $\tilde{k}(f)$ are positive multiples of $1 - \theta f$, so both functions are strictly decreasing on $[0, 1/\theta]$. ■

Proof of Proposition 2. At $f=0$ and $f=1/\theta$, the platform earns no commission revenue. Any positive admission would then generate a strictly positive onboarding cost, so the unique optimum is $n^*=0$. Fix henceforth $f\in(0,1/\theta)$, so $\tilde{k}(f)>0$.

The optimization problem (2) is simply the maximization of a strictly concave function (i.e., $\Pi_U(f,n)$, where f is fixed) over an interval (i.e., $[0,\min\{\frac{1}{2\tilde{k}(f)},\bar{N}\}]$). As mentioned in the text, the unconstrained maximizer in n of $\Pi_U(f,n)$ is $n_U(f)=2g^*(f)f\tilde{k}(f)/\gamma$. If $n_U(f)$ falls in the relevant interval, then it is the optimal solution. Otherwise, $n_U(f)$ lies above the right endpoint, and $\Pi_U(f,n)$ is strictly increasing in n throughout the feasible interval, so the optimal solution is that endpoint. Thus, the optimal solution to (2) is

$$n^*(f,\bar{N}):=\min\{n_U(f),\frac{1}{2\tilde{k}(f)},\bar{N}\}.$$

After rearrangement, $n_U(f)\lesseqqgtr\frac{1}{2\tilde{k}(f)}$ is equivalent to $\gamma\gtreqqless 4g^*(f)f\tilde{k}(f)^2$, and after inputting the expressions for $g^*(f)$ and $\tilde{k}(f)$ from Proposition 1, we then have that

$$n_U(f)\lesseqqgtr\frac{1}{2\tilde{k}(f)}\iff\gamma\gtreqqless\hat{\gamma}(f):=\frac{\beta^6(1-\theta f)^3 f}{16\alpha^4\lambda^2}.$$

Thus, if $\gamma>\hat{\gamma}(f)$, then the optimal solution is $\min\{n_U(f),\bar{N}\}$, but if $\gamma\le\hat{\gamma}(f)$, then the optimal solution is $\min\{\frac{1}{2\tilde{k}(f)},\bar{N}\}$. This solution is covered if and only if $\gamma\le\hat{\gamma}(f)$ and $\frac{1}{2\tilde{k}(f)}\le\bar{N}$.

For monotonicity, the minimum representation above is especially useful. For fixed f and $\bar{N}$, both $1/(2\tilde{k}(f))$ and $\bar{N}$ are independent of γ, while $n_U(f)=2g^*(f)f\tilde{k}(f)/\gamma$ is strictly decreasing in γ. Their minimum is therefore weakly decreasing in γ. At $\gamma=\hat{\gamma}(f)$, the equality $n_U(f)=1/(2\tilde{k}(f))$ also shows that the solution is continuous. Hence $n^*(f,\bar{N})$ is weakly decreasing in γ. ■

Proof of Proposition 3. The expressions for $n^*(f)$ are the unconstrained admission optima from Proposition 2. We first verify that, regardless of $\bar{N}$ or whether $\gamma\le\hat{\gamma}(f)$, the payoff for each admitted beneficiary is equal to $\hat{\phi}(f)$. If $\gamma>\hat{\gamma}(f)$, the platform will set $n=\min\{n_U(f),\bar{N}\}$. Whichever quantity attains the minimum, the market will be uncovered, so each admitted beneficiary draws donations from all donors within $\tilde{k}(f)$ on either side and earns $2g^*(f)(1-f)\tilde{k}(f)=\hat{\phi}(f)$. If instead $\gamma\le\hat{\gamma}(f)$, the platform will set $n=\min\{1/(2\tilde{k}(f)),\bar{N}\}$. If $\bar{N}<1/(2\tilde{k}(f))$, then the market is uncovered, and the per-beneficiary payoff is $\hat{\phi}(f)$. If instead $1/(2\tilde{k}(f))$ attains the minimum, then since $1/(2\tilde{k}(f))$ is exactly the minimum number of beneficiaries that reaches all donors, it remains the case that each beneficiary draws donations from all donors within $\tilde{k}(f)$ on either side, so the per-beneficiary payoff will again be $\hat{\phi}(f)$. (This can also be seen by noting that if the platform sets $n=1/(2\tilde{k}(f))$ and creates a covered market, then the total net donations of $g^*(f)(1-f)$ are evenly split among the $1/(2\tilde{k}(f))$ admitted beneficiaries, such that each one earns $g^*(f)(1-f)/(1/(2\tilde{k}(f)))=\hat{\phi}(f)$.)

If $\phi>\hat{\phi}(f)$, even certain admission gives net payoff $\hat{\phi}(f)-\phi<0$, so $\bar{N}(f)=0$, $n^*(f)=0$, and the market does not operate. Suppose, then, that $\phi<\hat{\phi}(f)$ (we will later address the case of equality). First suppose $\gamma\le\hat{\gamma}(f)$. For any $\bar{N}(f)<n_C(f):=1/(2\tilde{k}(f))$, the platform admits all applying beneficiaries, and these beneficiaries have net utility $\hat{\phi}(f)-\phi>0$. So, more beneficiaries have an incentive to apply, and thus, we cannot have $\bar{N}(f) < n_C(f)$. If $\bar{N}(f) \geq n_C(f)$, the platform only admits a fraction $n_C(f)/\bar{N}(f)$ of beneficiaries, and thus a beneficiary's expected net utility from applying to the platform is

$$\frac{n_C(f)}{\bar{N}(f)} \cdot 2g^*(f)(1-f)\tilde{k}(f) - \phi = \frac{g^*(f)(1-f)}{\bar{N}(f)} - \phi.$$

This quantity is decreasing in $\bar{N}(f)$, and thus the equilibrium occurs where it reaches zero, i.e., at $\bar{N}_C(f) := \frac{g^*(f)(1-f)}{\phi}$. If instead $\gamma > \hat{\gamma}(f)$, then the platform prefers to admit fewer beneficiaries (Proposition 2), so the expected net utility for a given number of applicants is lower. Analogous reasoning to that above implies that we cannot have $\bar{N}(f) < n_U(f)$ in this case. For $\bar{N}(f) \geq n_U(f)$, the expected net utility for a beneficiary who applies is

$$\frac{n_U(f)}{\bar{N}(f)} \cdot 2g^*(f)(1-f)\tilde{k}(f) - \phi = \frac{4(g^*(f))^2 f(1-f)(\tilde{k}(f))^2}{\gamma \bar{N}(f)} - \phi.$$

Similar logic to the case with $\gamma \leq \hat{\gamma}(f)$ then allows us to determine $\bar{N}(f)$ by setting the above expected net utility to zero and isolating $\bar{N}(f)$, which yields $\bar{N}_U(f) := \frac{4(g^*(f))^2 f(1-f)(\tilde{k}(f))^2}{\gamma\phi}$. Using $\hat{\phi}(f) = 2g^*(f)(1-f)\tilde{k}(f)$, we obtain

$$\frac{\bar{N}_C(f)}{n_C(f)} = \frac{\bar{N}_U(f)}{n_U(f)} = \frac{\hat{\phi}(f)}{\phi} > 1,$$

so $\bar{N}(f) > n^*(f)$; the constraint is slack and the platform admits its unconstrained $n^*(f)$.

It remains to address $\phi = \hat{\phi}(f)$. In this case, admitted beneficiaries earn zero net utility and, by our tie-breaking convention, indifferent beneficiaries are willing to apply to the platform. If $\gamma \leq \hat{\gamma}(f)$, then for any $\bar{N}(f) \leq n_C(f)$, applying beneficiaries earn net utility zero, whereas for $\bar{N}(f) > n_C(f)$, applying beneficiaries earn strictly negative net utility (because their expected payoff will now be strictly less due to uncertain admission, but they still incur the entry cost). Thus, the equilibrium is $\bar{N}(f) = n_C(f)$. In this case, both quantities attain the minimum in Proposition 2, and although the constraint on the platform's stage-3 admission problem is binding in the sense that it holds at equality at the optimal solution, the platform's decision is nevertheless the same as it would be without the constraint. Analogous reasoning gives us that the equilibrium when $\gamma > \hat{\gamma}(f)$ is $\bar{N}(f) = n_U(f)$, with an analogous implication about the "binding" constraint. Hence, the number of admitted beneficiaries equals the unconstrained $n^*(f)$ for all $\phi \leq \hat{\phi}(f)$. ■

Proof of Proposition 4. We proceed in four steps. In Step 1, we solve the platform's Stage 1 problem over the whole interval $[0, 1/\theta]$, ignoring the entry-feasibility constraint; because the platform profit is piecewise, we solve the covered and uncovered branches separately, and then compare them globally as a function of γ. In Step 2, we impose the feasibility constraint $\phi \leq \hat{\phi}(f)$; since $\hat{\phi}$ is strictly decreasing, this constraint simply truncates the feasible commissions to an interval $[0, \bar{f}(\phi)]$, and part (i) follows. In Steps 3 and 4, we establish parts (ii) and (iii), respectively.

Step 1: the unconstrained problem. The Stage 1 objective of the platform is

$$\Pi(f) = \begin{cases} \Pi_C(f) = g^*(f)f - \dfrac{\gamma}{8\tilde{k}(f)^2} = \dfrac{\beta^2(1-\theta f)f}{4\alpha^2} - \dfrac{2\gamma\alpha^2\lambda^2}{\beta^4(1-\theta f)^2}, & \gamma \leq \hat{\gamma}(f), \\ \Pi_U(f) = \dfrac{2\left(g^*(f)\right)^2 f^2 \left(\tilde{k}(f)\right)^2}{\gamma} = \dfrac{\beta^8(1-\theta f)^4 f^2}{128\alpha^6\lambda^2\gamma}, & \gamma > \hat{\gamma}(f). \end{cases} \qquad f \in \left[0, \frac{1}{\theta}\right].$$

When $\gamma = \hat{\gamma}(f)$, the two formulas agree, since we have

$$\Pi_C(f) = g^*(f)f - \frac{\hat{\gamma}(f)}{8\tilde{k}(f)^2} = \frac{g^*(f)f}{2} = \frac{2\,(g^*(f))^2\, f^2 \left(\tilde{k}(f)\right)^2}{\hat{\gamma}(f)} = \Pi_U(f),$$

so the objective is continuous. It is straightforward to show that $f = 0$ and $f = 1/\theta$ both lead to zero profit for the platform and that it is possible to achieve strictly positive profit for some $f \in (0, 1/\theta)$; thus, the maximizer must fall in the open interval $(0, 1/\theta)$.

We first study the uncovered case. Differentiating gives

$$\Pi'_U(f) = \frac{\beta^8}{64\alpha^6\lambda^2\gamma}\, f(1-\theta f)^3(1-3\theta f).$$

The fractional term is strictly positive, and for $f \in (0, 1/\theta)$, we have $1 - \theta f > 0$; thus, on this interval, the sign of $\Pi'_U(f)$ matches that of the last term, namely $1 - 3\theta f$. Accordingly, we have

$$\Pi'_U(f) \gtreqqless 0 \iff f \lesseqqgtr \frac{1}{3\theta}.$$

Hence, Π_U is strictly increasing on $(0, 1/(3\theta))$ and strictly decreasing on $(1/(3\theta), 1/\theta)$, so its unique maximizer is

$$f_U^* = \frac{1}{3\theta}.$$

We next consider the covered case. For later use, define

$$\bar{\gamma} := \hat{\gamma}\left(\frac{1}{3\theta}\right) = \frac{\beta^6}{162\alpha^4\lambda^2\theta}.$$

It is useful to express the derivative of Π_C using an auxiliary function; specifically, we have

$$\Pi'_C(f) = \frac{\beta^2}{4\alpha^2}(1-2\theta f) - \frac{4\gamma\alpha^2\lambda^2\theta}{\beta^4(1-\theta f)^3} = \frac{4\alpha^2\lambda^2\theta}{\beta^4(1-\theta f)^3}\big(\Psi(f) - \gamma\big),$$

where $\Psi(f) := \frac{\beta^6(1-\theta f)^3(1-2\theta f)}{16\alpha^4\lambda^2\theta}$. Note that $\frac{4\alpha^2\lambda^2\theta}{\beta^4(1-\theta f)^3} > 0$, so the sign of $\Pi'_C(f)$ is exactly the sign of $\Psi(f) - \gamma$. Differentiating Ψ, we obtain

$$\Psi'(f) = -\frac{\beta^6}{16\alpha^4\lambda^2}(1-\theta f)^2(5-8\theta f),$$

On $[0, 1/(2\theta)]$, both $(1-\theta f)^2$ and $(5-8\theta f)$ are strictly positive, so $\Psi'(f) < 0$, and hence Ψ is strictly decreasing on $[0, 1/(2\theta)]$. Moreover, $\Psi\left(\frac{1}{3\theta}\right) = \frac{\beta^6}{162\alpha^4\lambda^2\theta} = \bar{\gamma}$, and $\Psi\left(\frac{1}{2\theta}\right) = 0$. Therefore, for every $\gamma \in (0, \bar{\gamma}]$, there is a unique

$$f_C^*(\gamma) \in \left[\frac{1}{3\theta}, \frac{1}{2\theta}\right],$$

such that $\Psi(f_C^*(\gamma)) = \gamma$, equivalently $\Pi'_C(f_C^*(\gamma)) = 0$.

Because Ψ is strictly decreasing on the whole interval $[0, 1/(2\theta)]$ and $\Psi(f_C^*(\gamma)) = \gamma$, it follows that $\Psi(f) > \gamma$ for every $f \in [0, f_C^*(\gamma))$ and that $\Psi(f) < \gamma$ for every $f \in (f_C^*(\gamma), 1/(2\theta)]$. As previously argued, the sign of $\Pi'_C(f)$ matches the sign of $\Psi(f) - \gamma$, so we conclude that $\Pi'_C(f) > 0$ on $[0, f_C^*(\gamma))$ and $\Pi'_C(f) < 0$ on $(f_C^*(\gamma), 1/(2\theta)]$.

For $f \in [1/(2\theta), 1/\theta)$, both terms in the formula for $\Pi'_C(f)$ are non-positive and the second is strictly negative, so $\Pi'_C(f) < 0$. Hence, $\Pi_C(f)$ is strictly increasing up to $f_C^*(\gamma)$ and strictly decreasing afterward, with a unique maximizer at $f_C^*(\gamma)$. The same monotonicity of Ψ implies that $f_C^*(\gamma)$ is strictly decreasing in γ. In particular,

$$f_C^*(0) = \frac{1}{2\theta}, \qquad f_C^*(\bar{\gamma}) = \frac{1}{3\theta}.$$

We now compare the covered and uncovered regimes globally, for three intervals of γ.

Case 1: $0 < \gamma \le \bar{\gamma}$. In this case, Lemma 4 implies that the covered set is $[f_L(\gamma), f_H(\gamma)]$, where $f_L(\gamma) \in (0, 1/(4\theta))$ and $f_H(\gamma) \in (1/(4\theta), 1/\theta)$ are defined in the lemma. The same lemma also establishes that $\hat{\gamma}$ is strictly decreasing on $[1/(4\theta), 1/\theta]$, and $f_H(\gamma)$ is defined by $\hat{\gamma}(f_H(\gamma)) = \gamma$. So, the inequality $\gamma \le \bar{\gamma} = \hat{\gamma}\big(\frac{1}{3\theta}\big)$ implies $f_H(\gamma) \ge \frac{1}{3\theta}$. We next verify that the covered maximizer $f_C^*(\gamma)$ actually lies in the covered set. For $f \in [1/(3\theta), 1/(2\theta)]$, we have

$$\Psi(f) = \hat{\gamma}(f)\frac{1 - 2\theta f}{\theta f},$$

and the factor $(1 - 2\theta f)/(\theta f)$ is at most one on that interval, and, thus, $\Psi(f) \le \hat{\gamma}(f)$. Evaluating at $f_C^*(\gamma)$ gives

$$\gamma = \Psi(f_C^*(\gamma)) \le \hat{\gamma}(f_C^*(\gamma)),$$

so $f_C^*(\gamma)$ satisfies the covered condition and thus belongs to $[f_L(\gamma), f_H(\gamma)]$.

Hence, the global shape of Π is clear. On $[0, f_L(\gamma))$, the objective equals Π_U, which is increasing because this interval lies to the left of $1/(3\theta)$. On $[f_L(\gamma), f_C^*(\gamma)]$, it equals Π_C, which is increasing by construction of $f_C^*(\gamma)$. On $[f_C^*(\gamma), f_H(\gamma)]$, it still equals Π_C, but now decreasing. Finally, on $(f_H(\gamma), 1/\theta]$, it equals Π_U, and decreasing because $f_H(\gamma) \ge 1/(3\theta)$ as argued above. Therefore, the platform's profit $\Pi(f)$ is strictly increasing up to $f_C^*(\gamma)$ and strictly decreasing afterward, so the unique unconstrained maximizer is

$$f^\circ(\gamma) = f_C^*(\gamma).$$

Case 2: $\bar{\gamma} < \gamma < \hat{\gamma}_{\max}$. In this case, the covered set is again $[f_L(\gamma), f_H(\gamma)]$, but now the same monotonicity argument gives

$$f_H(\gamma) < \frac{1}{3\theta},$$

because now $\gamma > \hat{\gamma}(1/(3\theta))$. We claim that the covered branch is increasing on the entire covered interval. Indeed, if $f < 1/(3\theta)$, then $(1 - 2\theta f)/(\theta f) > 1$, so

$$\Psi(f) = \hat{\gamma}(f)\frac{1 - 2\theta f}{\theta f} > \hat{\gamma}(f) \ge \gamma \qquad \text{for every } f \in [f_L(\gamma), f_H(\gamma)].$$

Since $\Pi_C'(f)$ has the same sign as $\Psi(f) - \gamma$, this implies $\Pi_C'(f) > 0$ throughout the covered interval. The global objective is therefore increasing on $[0, 1/(3\theta)]$: it equals Π_U and is increasing on $[0, f_L(\gamma))$, equals Π_C and is increasing on $[f_L(\gamma), f_H(\gamma)]$, and then equals Π_U and is still increasing on $[f_H(\gamma), 1/(3\theta)]$. On $[1/(3\theta), 1/\theta]$, it equals Π_U and is decreasing. Hence the unique unconstrained maximizer is

$$f^\circ(\gamma) = \frac{1}{3\theta}.$$

Case 3: $\gamma \ge \hat{\gamma}_{\max}$**.** In this case, the covered set is a singleton $\{1/(4\theta)\}$ when $\gamma = \hat{\gamma}_{\max}$ and is empty when $\gamma > \hat{\gamma}_{\max}$. At the singleton, $\Pi_C = \Pi_U$, so the global objective has the same values as the uncovered branch throughout the domain. Since that branch has the unique maximizer $1/(3\theta)$, we can write

$$f^\circ(\gamma) = \frac{1}{3\theta}.$$

In all three cases above, on the interval $(0, 1/\theta)$, the platform's profit $\Pi(f)$ is strictly increasing to a unique maximum, then strictly decreasing thereafter. Combining the three cases, we obtain

$$f^\circ(\gamma) = \begin{cases} f_C^*(\gamma), & 0 < \gamma \le \bar{\gamma}, \\ \frac{1}{3\theta}, & \gamma > \bar{\gamma}. \end{cases}$$

In particular, $f^\circ(\gamma) \in \left[\frac{1}{3\theta}, \frac{1}{2\theta}\right]$ for every $\gamma > 0$.

Step 2: impose the feasibility constraint (part (i)). By Lemma 5, $\hat{\phi}$ is strictly decreasing on $[0, 1/\theta]$, with $\hat{\phi}(0) = \beta^4/(8\alpha^3\lambda)$ and $\hat{\phi}(1/\theta) = 0$. Hence, if $\phi > \hat{\phi}(0)$, no commission can induce entry and the market does not operate. If $\phi = \hat{\phi}(0)$, $f = 0$ is the only commission satisfying the beneficiary participation condition, but it yields $n^* = 0$; every other commission also induces shutdown and the same zero platform profit. Thus the shutdown outcome is unique but the commission announcement is not identified. If $0 < \phi < \hat{\phi}(0)$, there is a unique $\bar{f}(\phi) \in (0, 1/\theta)$ such that

$$\hat{\phi}(\bar{f}(\phi)) = \phi,$$

and the feasible set is exactly $[0, \bar{f}(\phi)]$.

Since the unconstrained objective is single-peaked with maximizer $f^\circ(\gamma)$, the constrained maximizer is simply

$$f^* = \min\{f^\circ(\gamma), \bar{f}(\phi)\}.$$

Because $\hat{\phi}$ is strictly decreasing with inverse $\bar{f}$, for any $f \in [0, 1/\theta]$ we have $\phi \le \hat{\phi}(f)$ if and only if $\bar{f}(\phi) \ge f$. Taking $f = f^\circ(\gamma)$ yields

$$\phi \le \Phi(\gamma) = \hat{\phi}\big(f^\circ(\gamma)\big) \iff \bar{f}(\phi) \ge f^\circ(\gamma) \iff f^* = \min\{f^\circ(\gamma), \bar{f}(\phi)\} = f^\circ(\gamma),$$

and, symmetrically, $\phi > \Phi(\gamma)$ if and only if $f^* = \bar{f}(\phi) < f^\circ(\gamma)$. This proves part (i).

Step 3: the coverage threshold (part (ii)). Recall from Proposition 2 that, given the commission, the platform's optimal admission results in a covered market if and only if $\gamma \le \hat{\gamma}(f)$ and covering the market is feasible. Proposition 3 further shows that, at any operating commission, the equilibrium applicant pool does not distort the platform's admission decision: the supply constraint is slack when $\phi < \hat{\phi}(f)$ and binds without changing admission when equality holds. Since $f^* \le \bar{f}(\phi)$, i.e., $\phi \le \hat{\phi}(f^*)$, the market operates at f^*, and the equilibrium market is covered if and only if $\gamma \le \hat{\gamma}(f^*)$. It therefore suffices to show that

$$\gamma \le \hat{\gamma}(f^*) \iff \gamma \le \Gamma(\phi) = \hat{\gamma}\left(\min\left\{\bar{f}(\phi), \frac{1}{3\theta}\right\}\right).$$

We distinguish two cases, based on the position of $\bar{f}(\phi)$ relative to $\frac{1}{3\theta}$.

Suppose first that $\bar{f}(\phi) \le \frac{1}{3\theta}$, i.e., $\phi \ge \hat{\phi}\big(\frac{1}{3\theta}\big) =: \bar{\phi}_U = \frac{\beta^4(3\theta-1)}{54\alpha^3\lambda\theta}$. Since $f^\circ(\gamma) \ge \frac{1}{3\theta}$ for every $\gamma > 0$ (Step 1), we have

$$f^* = \min\{f^\circ(\gamma), \bar{f}(\phi)\} = \bar{f}(\phi) = \min\left\{\bar{f}(\phi), \frac{1}{3\theta}\right\},$$

so $\hat{\gamma}(f^*) = \Gamma(\phi)$, and the equivalence holds identically.

Suppose instead that $\bar{f}(\phi) > \frac{1}{3\theta}$, i.e., $\phi < \bar{\phi}_U$, so that $\Gamma(\phi) = \hat{\gamma}\big(\frac{1}{3\theta}\big) = \bar{\gamma}$. If $\gamma > \bar{\gamma}$, then $f^\circ(\gamma) = \frac{1}{3\theta} < \bar{f}(\phi)$, so $f^* = \frac{1}{3\theta}$ and $\hat{\gamma}(f^*) = \bar{\gamma} < \gamma$: the market is uncovered, consistent with $\gamma > \Gamma(\phi)$. If instead $\gamma \le \bar{\gamma}$, then $f^\circ(\gamma) = f_C^*(\gamma)$, and thus

$$f^* = \min\{f_C^*(\gamma), \bar{f}(\phi)\} \in \left[\frac{1}{3\theta}, f_C^*(\gamma)\right] \subseteq \left[\frac{1}{3\theta}, \frac{1}{2\theta}\right].$$

By Lemma 4, $\hat{\gamma}$ is strictly decreasing on $[1/(4\theta), 1/\theta]$, and hence on $[1/(3\theta), 1/(2\theta)]$, so $\hat{\gamma}(f^*) \geq \hat{\gamma}(f_C^*(\gamma))$. Moreover, as shown in Case 1 of Step 1, $\gamma = \Psi(f_C^*(\gamma)) \leq \hat{\gamma}(f_C^*(\gamma))$. Combining the two inequalities gives $\hat{\gamma}(f^*) \geq \gamma$: the market is covered, consistent with $\gamma \leq \bar{\gamma} = \Gamma(\phi)$. This proves part (ii).

Step 4: the equilibrium table (part (iii)). Part (i) determines the equilibrium commission in each row of the table, and part (ii) determines the coverage regime in each column. Given the commission and the regime, Proposition 2 and Proposition 3 yield the equilibrium admission reported in the table:

$$n^* = \begin{cases} n_C(f^*) = \dfrac{1}{2\tilde{k}(f^*)}, & \gamma \leq \Gamma(\phi), \\[2ex] n_U(f^*) = \dfrac{2g^*(f^*)f^*\tilde{k}(f^*)}{\gamma}, & \gamma > \Gamma(\phi). \end{cases}$$

For the distorting row, part (i) gives $f^* = \bar{f}(\phi)$ in both cells, as stated in the table. For the non-distorting row, part (i) gives $f^* = f^\circ(\gamma)$, and it remains to identify which branch of f° applies in each cell. When the cap is non-distorting, $f^\circ(\gamma) \geq \frac{1}{3\theta}$ (Step 1) and the monotonicity of $\hat{\phi}$ give

$$\phi \leq \Phi(\gamma) = \hat{\phi}\big(f^\circ(\gamma)\big) \leq \hat{\phi}\left(\frac{1}{3\theta}\right) = \bar{\phi}_U,$$

so $\bar{f}(\phi) \geq \frac{1}{3\theta}$ and hence $\Gamma(\phi) = \hat{\gamma}\big(\frac{1}{3\theta}\big) = \bar{\gamma}$ (Step 3). In the non-distorting row, the coverage condition $\gamma \leq \Gamma(\phi)$ therefore reduces to $\gamma \leq \bar{\gamma}$, under which $f^* = f^\circ(\gamma) = f_C^*(\gamma)$, as stated in the non-distorting–covered cell; for $\gamma > \bar{\gamma}$, we have $f^* = f^\circ(\gamma) = \frac{1}{3\theta}$, as stated in the non-distorting–uncovered cell. This concludes the proof. ■

Proof of Proposition 5. Fix a parameter pair in the interior of the relevant region. *Region I.* Here $f^* = f_C^*(\gamma)$ and $D^*(\gamma,\phi) = D_C(f_C^*(\gamma))$, with $\frac{\partial f_C^*}{\partial \gamma} < 0$ by Corollary 1(i). Since D_C (see (4)) depends on γ and ϕ only through f^*, the chain rule gives

$$\frac{\partial D^*}{\partial \gamma} = \frac{\partial D_C}{\partial f}\bigg|_{f=f_C^*(\gamma)} \cdot \frac{\partial f_C^*}{\partial \gamma}.$$

Differentiating $D_C(f) = \frac{\beta^2(1-\theta f)(1-f)}{4\alpha^2}$ yields $\frac{\partial D_C}{\partial f} = -\frac{\beta^2}{4\alpha^2}[1+\theta-2\theta f]$, which is strictly negative on $f \in [0, 1/\theta)$ since $1+\theta-2\theta f \geq \theta - 1 \geq 0$ at $f = 1/\theta$. Recalling from the proof of Proposition 4 that $f_C^*(\gamma) \in [1/(3\theta), 1/(2\theta)] \subset [0, 1/\theta)$, we conclude that $\frac{\partial D_C}{\partial f}|_{f=f_C^*(\gamma)} < 0$. Thus, both factors in the chain rule are negative, so $\frac{\partial D^*}{\partial \gamma} > 0$. Since ϕ does not enter f_C^* or D_C, $\frac{\partial D^*}{\partial \phi} = 0$.

Region II. Here $f^* = \bar{f}(\phi)$, which is strictly decreasing in ϕ by Lemma 5 and independent of γ, and $D^*(\gamma,\phi) = D_C(\bar{f}(\phi))$. Since D_C does not contain γ, $\frac{\partial D^*}{\partial \gamma} = 0$. By the chain rule,

$$\frac{\partial D^*}{\partial \phi} = \frac{\partial D_C}{\partial f}\bigg|_{f=\bar{f}(\phi)} \cdot \frac{\partial \bar{f}}{\partial \phi},$$

and both factors are negative, so the product is positive.

Region III. Here $f^* = \frac{1}{3\theta}$ is independent of both γ and ϕ, and $D^*(\gamma,\phi) = D_U(1/(3\theta))$. Substituting into $D_U(f)$ in (5) yields $D^*(\gamma,\phi) = \frac{\beta^8(3\theta-1)}{2916\alpha^6\lambda^2\theta^2\gamma}$, which is strictly decreasing in γ and independent of ϕ.

Region IV. Here $f^* = \bar{f}(\phi)$, which is strictly decreasing in ϕ and independent of γ, and $D^*(\gamma,\phi) = D_U(\bar{f}(\phi))$. Since $D_U(f;\gamma) = \frac{\beta^8(1-\theta f)^4 f(1-f)}{64\alpha^6\lambda^2\gamma}$ contains γ only in the denominator and $\bar{f}(\phi)$ does not depend on γ, $\frac{\partial D^*}{\partial \gamma} = -D^*/\gamma < 0$. For the effect of ϕ, direct differentiation gives

$$\frac{\partial D_U}{\partial f} = \frac{\beta^8(1-\theta f)^3}{64\alpha^6\lambda^2\gamma} \underbrace{\left[1-(2+5\theta)f+6\theta f^2\right]}_{:=Q(f)}.$$

The prefactor $\frac{\beta^8(1-\theta f)^3}{64\alpha^6\lambda^2\gamma}$ is strictly positive on $f \in [0, 1/\theta)$, so the sign of $\frac{\partial D_U}{\partial f}$ matches the sign of $Q(f)$. By the chain rule,

$$\frac{\partial D^*}{\partial \phi} = \frac{\partial D_U}{\partial f}\bigg|_{f=\bar{f}(\phi)} \cdot \frac{\partial \bar{f}}{\partial \phi},$$

and since $\frac{\partial \bar{f}}{\partial \phi} < 0$, the sign of $\frac{\partial D^*}{\partial \phi}$ matches the sign of $-Q(\bar{f}(\phi))$. ■

Proof of Proposition 6. Since the market operates, $\phi < \hat{\phi}(0) = \beta^4/(8\alpha^3\lambda)$, so by Lemma 5 the feasibility threshold $\bar{f}(\phi)$, the unique solution of $\hat{\phi}(f) = \phi$, satisfies $0 < \bar{f}(\phi) < 1/\theta$, and $f \le \bar{f}(\phi)$ if and only if $\phi \le \hat{\phi}(f)$. We write $f^* := f^*(\gamma, \phi) = \min\{f^\circ(\gamma), \bar{f}(\phi)\}$ for the platform's profit-maximizing commission absent the cap, where, by Proposition 4, $f^\circ(\gamma) = f_C^*(\gamma)$ for $0 < \gamma \le \bar{\gamma}$ and $f^\circ(\gamma) = 1/(3\theta)$ for $\gamma > \bar{\gamma}$, with $\bar{\gamma} = \hat{\gamma}(1/(3\theta))$. Specifically, we can write $f^* \le \bar{f}(\phi) < 1/\theta$.

Step 1: Shape of Donation Function D. Differentiating the covered net donations $D_C(f) = \frac{\beta^2(1-\theta f)(1-f)}{4\alpha^2}$ gives $D_C'(f) = \frac{\beta^2}{4\alpha^2}[2\theta f - (1+\theta)]$, which is strictly negative on $[0, 1/\theta)$ because $2\theta f < 2 \le 1 + \theta$ there; hence D_C is strictly decreasing on $[0, 1/\theta)$. Differentiating the uncovered net donations $D_U(f) = \frac{\beta^8(1-\theta f)^4 f(1-f)}{64\alpha^6\lambda^2\gamma}$ gives $D_U'(f) = \frac{\beta^8(1-\theta f)^3}{64\alpha^6\lambda^2\gamma} Q(f)$, where $Q(f) := 6\theta f^2 - (2 + 5\theta)f + 1$.

The prefactor is strictly positive on $[0, 1/\theta)$, so the sign of $D_U'(f)$ matches that of $Q(f)$. The quadratic Q has positive leading coefficient, and $Q(0) = 1 > 0$, $Q(1/(4\theta)) = -(1 + 2\theta)/(8\theta) < 0$, and $Q(1/\theta) = 4(1-\theta)/\theta \le 0$; hence its smaller root lies in $(0, 1/(4\theta))$ and its larger root at or above $1/\theta$, so $Q(f) < 0$ on $[1/(4\theta), 1/\theta)$. Thus D_U is strictly decreasing on $[1/(4\theta), 1/\theta)$. Finally, D is continuous, since D_C and D_U agree wherever $\gamma = \hat{\gamma}(f)$: substituting $\hat{\gamma}(f) = \frac{\beta^6(1-\theta f)^3 f}{16\alpha^4\lambda^2}$ into D_U yields $\frac{\beta^2(1-\theta f)(1-f)}{4\alpha^2} = D_C(f)$.

Step 2: The commission cap problem. By the platform's best response (7), a cap r induces the commission $\min\{f^*, r\} \in [0, f^*] \subseteq [0, \bar{f}(\phi)]$. At every positive induced commission, Lemma 5 implies that the participation condition holds and $\tilde{D}(\min\{f^*, r\}) = D(\min\{f^*, r\})$; at an induced commission of zero, both the platform outcome and net donations are zero, so the same equality holds. As r ranges over $[0, 1/\theta]$ the induced commission takes every value in $[0, f^*]$, and since D is continuous on $[0, f^*]$,

$$\max_{r \in [0, 1/\theta]} \tilde{D}\big(f_r^*(\gamma, \phi)\big) = \max_{f \in [0, f^*]} D(f).$$

Step 3: The donation-maximizing problem. In this case, $\tilde{D}(f) = D(f)$ on $[0, \bar{f}(\phi)]$ and $\tilde{D}(f) = 0$ on $(\bar{f}(\phi), 1/\theta]$ by Lemma 5. Each factor of D_C and of D_U is positive on $(0, 1/\theta)$, so $D > 0$ on the nonempty interval $(0, \bar{f}(\phi))$ and $\max_{[0, \bar{f}(\phi)]} D > 0$; hence

$$\max_{f \in [0, 1/\theta]} \tilde{D}(f) = \max_{f \in [0, \bar{f}(\phi)]} D(f).$$

Step 4: Monotonicity of D on $[f^*, \bar{f}(\phi)]$. If $f^* = \bar{f}(\phi)$ this is immediate, so suppose $f^* = f^\circ(\gamma) < \bar{f}(\phi)$; since $\bar{f}(\phi) < 1/\theta$, it suffices to show that D is strictly decreasing on $[f^\circ(\gamma), 1/\theta)$. By Proposition 2 the market is covered at commission f if and only if $\gamma \le \hat{\gamma}(f)$.

Case 1: $0 < \gamma \le \bar{\gamma}$. Since $\hat{\gamma}$ attains its unique maximum at $1/(4\theta)$ (Lemma 4) and $1/(3\theta) \ge 1/(4\theta)$, we have $\bar{\gamma} = \hat{\gamma}(1/(3\theta)) < \hat{\gamma}_{\max}$, so by Lemma 4 the covered set is $[f_L(\gamma), f_H(\gamma)]$ with $f_L(\gamma) < 1/(4\theta) < f_H(\gamma) < 1/\theta$ (the last inequality because $\hat{\gamma}(1/\theta) = 0 < \gamma$), and $\hat{\gamma}$ is strictly decreasing on $[1/(4\theta), 1/\theta]$. By Proposition 4, $f^\circ(\gamma) = f_C^*(\gamma) \in [1/(3\theta), 1/(2\theta))$ and belongs to $[f_L(\gamma), f_H(\gamma)]$. Because $1/(3\theta)$ and $f_H(\gamma)$ both lie in $[1/(4\theta), 1/\theta]$, where $\hat{\gamma}$ is decreasing, and $\hat{\gamma}(1/(3\theta)) = \bar{\gamma} \geq \gamma = \hat{\gamma}(f_H(\gamma))$, it follows that $f_H(\gamma) \geq 1/(3\theta)$. On $[f_C^*(\gamma), f_H(\gamma)]$ the market is covered, so $D = D_C$ is strictly decreasing; on $[f_H(\gamma), 1/\theta)$ it is uncovered, so $D = D_U$ is strictly decreasing because $f_H(\gamma) \geq 1/(3\theta) > 1/(4\theta)$. As D is continuous at $f_H(\gamma)$ and strictly decreasing on each adjoining interval, it is strictly decreasing on $[f_C^*(\gamma), 1/\theta)$.

Case 2: $\gamma > \bar{\gamma}$. Here $f^\circ(\gamma) = 1/(3\theta)$, and for every $f \in [1/(3\theta), 1/\theta]$ the monotonicity of $\hat{\gamma}$ on $[1/(4\theta), 1/\theta]$ (Lemma 4) gives $\hat{\gamma}(f) \leq \hat{\gamma}(1/(3\theta)) = \bar{\gamma} < \gamma$, so the market is uncovered and $D = D_U$ is strictly decreasing on $[1/(3\theta), 1/\theta)$.

In either case D is strictly decreasing on $[f^*, \bar{f}(\phi)]$.

Step 5: Conclusion. By Step 4, $D(f) < D(f^*)$ for every $f \in (f^*, \bar{f}(\phi)]$, so the maximum of D over $[0, \bar{f}(\phi)]$ is not attained in $(f^*, \bar{f}(\phi)]$; thus $\max_{[0,\bar{f}(\phi)]} D = \max_{[0,f^*]} D$ and every maximizer of D over $[0, \bar{f}(\phi)]$ lies in $[0, f^*]$. Combining this with Steps 2 and 3,

$$\max_{r\in[0,1/\theta]} \tilde{D}\big(f_r^*(\gamma,\phi)\big) = \max_{[0,f^*]} D = \max_{[0,\bar{f}(\phi)]} D = \max_{f\in[0,1/\theta]} \tilde{D}(f).$$

Finally, let $f^\dagger$ attain $\max_{[0,1/\theta]} \tilde{D}$. This maximum equals $\max_{[0,\bar{f}(\phi)]} D > 0$, while $\tilde{D} = 0$ on $(\bar{f}(\phi), 1/\theta]$, so $f^\dagger \in [0, \bar{f}(\phi)]$ and $\tilde{D}(f^\dagger) = D(f^\dagger) = \max_{[0,\bar{f}(\phi)]} D$; hence $f^\dagger$ maximizes D over $[0, \bar{f}(\phi)]$ and so $f^\dagger \leq f^*$. The cap $r = f^\dagger$ then induces $f^*(\gamma, \phi, f^\dagger) = \min\{f^*, f^\dagger\} = f^\dagger$ and attains the maximum. ■

Proof of Proposition 7. Suppose $\phi < \hat{\phi}(0) = \frac{\beta^4}{8\alpha^3\lambda}$. First, we solve the unconstrained problem and then impose feasibility.

Step 1: Unconstrained problem. By Lemma 6, D_C is strictly decreasing on $[0, 1/\theta]$ and D_U is strictly increasing on $[0, f_B^U]$ and strictly decreasing on $[f_B^U, 1/\theta]$, with $f_B^U \in (0, 1/(4\theta))$. Additionally, by Lemma 4, $\hat{\gamma}$ is strictly increasing on $[0, 1/(4\theta)]$, so for $0 < \gamma < \hat{\gamma}_{\max}$, the position of $f_L(\gamma)$ relative to f_B^U is governed by $\gamma_B := \hat{\gamma}(f_B^U)$: $f_L(\gamma) \leq f_B^U$ if and only if $\gamma \leq \gamma_B$. Since $f_B^U < 1/(4\theta)$, we have $\gamma_B < \hat{\gamma}_{\max}$, so the case $\gamma \leq \gamma_B$ always lies within the range where the covered interval $[f_L(\gamma), f_H(\gamma)]$ exists.

Case 1: $\gamma \leq \gamma_B$. Since $\gamma_B < \hat{\gamma}_{\max}$, the covered interval exists and $f_L(\gamma) \leq f_B^U$; furthermore, $f_B^U < 1/(4\theta) \leq f_H(\gamma)$, so $f_B^U \in [f_L(\gamma), f_H(\gamma)]$. We evaluate D on each of the three pieces $[0, f_L(\gamma))$, $[f_L(\gamma), f_H(\gamma)]$, and $(f_H(\gamma), 1/\theta]$. On $[0, f_L(\gamma))$, $D = D_U$ is strictly increasing because $[0, f_L(\gamma)) \subseteq [0, f_B^U]$, and its supremum on this piece is $D_U(f_L(\gamma)) = D_C(f_L(\gamma))$ by continuity. On $[f_L(\gamma), f_H(\gamma)]$, $D = D_C$ is strictly decreasing, so its maximum is $D_C(f_L(\gamma))$. On $(f_H(\gamma), 1/\theta]$, $D = D_U$ is strictly decreasing because $f_H(\gamma) \geq 1/(4\theta) > f_B^U$ places this piece in the decreasing region of D_U; hence values are strictly below $D_U(f_H(\gamma)) = D_C(f_H(\gamma)) < D_C(f_L(\gamma))$. The unconstrained maximum is therefore $D_C(f_L(\gamma))$, attained at $f_B^\circ(\gamma) = f_L(\gamma)$.

Case 2: $\gamma > \gamma_B$. If $\gamma_B < \gamma < \hat{\gamma}_{\max}$, the covered interval exists with $f_L(\gamma) > f_B^U$, so f_B^U lies in the left uncovered piece $[0, f_L(\gamma))$. On this piece, $D = D_U$ peaks at f_B^U. On $[f_L(\gamma), f_H(\gamma)]$, $D = D_C$ is strictly decreasing with maximum $D_C(f_L(\gamma)) = D_U(f_L(\gamma)) < D_U(f_B^U)$, where the inequality uses that D_U is strictly decreasing on $[f_B^U, 1/\theta]$ and $f_L(\gamma) > f_B^U$. On $(f_H(\gamma), 1/\theta]$, $D = D_U$ is strictly decreasing, so values are below $D_U(f_H(\gamma)) < D_U(f_B^U)$. If instead $\gamma > \hat{\gamma}_{\max}$, the entire domain is uncovered by Lemma 4 and $D = D_U$ throughout. At $\gamma = \hat{\gamma}_{\max}$, the market is covered only at $f = 1/(4\theta)$, where continuity gives $D_C = D_U$, so maximizing D_U over the domain remains valid. Thus, the unique maximizer is f_B^U. In both subcases, $f_B^\circ(\gamma) = f_B^U$.

Step 2: Feasibility. By Lemma 5, the feasibility set is $[0, \bar{f}(\phi)]$, which is non-empty iff $\phi \leq \hat{\phi}(0) = \frac{\beta^4}{8\alpha^3\lambda}$. We claim D is strictly increasing on $[0, f_B^\circ(\gamma)]$ in both cases. In Case 1, $[0, f_B^\circ(\gamma)] = [0, f_L(\gamma)]$ lies in the uncovered region except at the right endpoint, where continuity gives $D_U(f_L(\gamma)) = D_C(f_L(\gamma))$; on the interior $D = D_U$ is strictly increasing because $f \leq f_L(\gamma) \leq f_B^U$. In Case 2, $[0, f_B^\circ(\gamma)] = [0, f_B^U]$ is uncovered. When $\gamma < \hat{\gamma}_{\max}$, Case 2 gives $\gamma > \gamma_B$, and strict monotonicity of $\hat{\gamma}$ on $[0, 1/(4\theta)]$ yields $f_B^U < f_L(\gamma)$; when $\gamma \geq \hat{\gamma}_{\max}$, every point in $[0, f_B^U]$ is uncovered directly from Lemma 4. On this interval, $D = D_U$ is strictly increasing by Lemma 6. Hence D is single-peaked at $f_B^\circ(\gamma)$ with strictly increasing left branch, and the constrained maximizer is

$$f_B^*(\gamma, \phi) = \min\{f_B^\circ(\gamma), \bar{f}(\phi)\}.$$

The four regions follow by comparing $\bar{f}(\phi)$ to $f_B^\circ(\gamma)$. Since $\hat{\phi}$ is strictly decreasing, $\bar{f}(\phi) \geq f_B^\circ(\gamma)$ iff $\phi \leq \hat{\phi}(f_B^\circ(\gamma))$. In Case 1, this gives Region B-I when $\phi \leq \hat{\phi}(f_L(\gamma))$ and Region B-II when $\phi > \hat{\phi}(f_L(\gamma))$. In Region B-II, the market is uncovered because $f_B^* = \bar{f}(\phi) < f_L(\gamma)$ implies $\gamma > \hat{\gamma}(f_B^*)$ by Lemma 4. In Case 2, this gives Region B-III when $\phi \leq \phi_B$ and Region B-IV when $\phi > \phi_B$. When $\gamma < \hat{\gamma}_{\max}$, both are uncovered because $f_B^* = \min\{f_B^U, \bar{f}(\phi)\} \leq f_B^U < f_L(\gamma)$; when $\gamma \geq \hat{\gamma}_{\max}$, the same conclusion follows from Lemma 4 and $f_B^* \leq f_B^U < 1/(4\theta)$.

Finally, as in the profit-maximizing setting, if $\phi = \hat{\phi}(0) = \frac{\beta^4}{8\alpha^3\lambda}$, $f = 0$ is the only commission satisfying the beneficiary participation condition, and it yields $n = 0$. Every other commission also induces shutdown and the same zero platform profit, so the commission announcement is not identified. If $\phi > \hat{\phi}(0)$, no commission induces entry and the market does not operate. ■

Appendix C: Proofs of Lemmas and Corollaries

Proof of Corollary 1. Part (i): From the proof of Proposition 4, $f_C^*(\gamma)$ is defined implicitly by $\Psi(f_C^*(\gamma)) = \gamma$ for $0 < \gamma \leq \bar{\gamma}$, where Ψ is strictly decreasing on $[0, 1/(2\theta)]$. Hence its inverse $f_C^*(\gamma)$ is strictly decreasing in γ. In the interior of Region I the entry cost cap does not distort this choice, so $f^* = f_C^*(\gamma)$, proving part (i).

Part (ii): Fix $0 < \phi \leq \bar{\phi}_U$. By the monotonicity of $\hat{\phi}$, $\bar{f}(\phi) \geq 1/(3\theta)$. For $0 < \gamma \leq \bar{\gamma}$, Proposition 4 gives

$$f^*(\gamma, \phi) = \min\{f_C^*(\gamma), \bar{f}(\phi)\}.$$

Because $f_C^*(\gamma)$ is strictly decreasing, this minimum is weakly decreasing in γ. The market is covered throughout this range, and $n^* = 1/(2\tilde{k}(f^*))$. Since $\tilde{k}$ is strictly decreasing in f, admission is increasing in f^* and therefore weakly decreasing in γ. For $\gamma > \bar{\gamma}$, the equilibrium is uncovered, $f^* = 1/(3\theta)$, and

$$n^* = n_U(1/(3\theta)) = \frac{2g^*(1/(3\theta))(1/(3\theta))\tilde{k}(1/(3\theta))}{\gamma},$$

which is strictly decreasing in γ. At $\gamma = \bar{\gamma} = \hat{\gamma}(1/(3\theta))$, $f_C^*(\bar{\gamma}) = 1/(3\theta)$ and $n_U(1/(3\theta)) = 1/(2\tilde{k}(1/(3\theta)))$, so both choices are continuous at the regime boundary. This proves the global weak monotonicity in part (ii); depending on ϕ, the equilibrium may start in either Region I or Region II before entering Region III.

For part (iii), fix $\gamma \in (\bar{\gamma}, \hat{\gamma}_{\max})$. Lemma 4 gives two roots $f_L(\gamma) < 1/(4\theta) < f_H(\gamma)$. Moreover, $\gamma > \hat{\gamma}(1/(3\theta))$ and the strict decrease of $\hat{\gamma}$ to the right of $1/(4\theta)$ imply $f_H(\gamma) < 1/(3\theta)$. Define

$$\phi_H(\gamma) := \hat{\phi}(f_H(\gamma)), \qquad \phi_L(\gamma) := \hat{\phi}(f_L(\gamma)).$$

Because $\hat{\phi}$ is strictly decreasing, $0 < \phi_H(\gamma) < \phi_L(\gamma) < \hat{\phi}(0)$. Proposition 4 gives $f^* = \min\{1/(3\theta), \bar{f}(\phi)\}$. Thus $f^* > f_H(\gamma)$ when $0 < \phi < \phi_H(\gamma)$, $f^* \in [f_L(\gamma), f_H(\gamma)]$ when $\phi_H(\gamma) \le \phi \le \phi_L(\gamma)$, and $f^* < f_L(\gamma)$ when $\phi_L(\gamma) < \phi < \hat{\phi}(0)$. Lemma 4 then yields the three coverage claims. ■

Proof of Corollary 2. Write the platform's profit after optimizing admission as $\Pi(f;\gamma)$, making its dependence on γ explicit. For $f \in (0, 1/\theta)$,

$$\Pi(f;\gamma) = \begin{cases} g^*(f)f - \dfrac{\gamma}{8\tilde{k}(f)^2}, & \gamma \le \hat{\gamma}(f), \\[2ex] \dfrac{2(g^*(f))^2 f^2 \tilde{k}(f)^2}{\gamma}, & \gamma > \hat{\gamma}(f). \end{cases}$$

At $\gamma = \hat{\gamma}(f) = 4g^*(f)f\tilde{k}(f)^2$, both expressions equal $g^*(f)f/2$. Both branches are strictly decreasing in γ on their respective domains, and the function extends continuously to the commission endpoints with value zero. Thus, $\Pi(f;\gamma)$ is jointly continuous in (f,γ) and, for each $f \in (0,1/\theta)$, strictly decreasing in γ.

Define

$$b(\phi) := \begin{cases} \bar{f}(\phi), & 0 < \phi \le \hat{\phi}(0), \\ 0, & \phi > \hat{\phi}(0). \end{cases}$$

Lemma 5 implies that $b(\phi)$ is continuous and weakly decreasing. The equilibrium value can be written as

$$\Pi^*(\gamma,\phi) = \max_{0 \le f \le b(\phi)} \Pi(f;\gamma) = \max_{t \in [0,1]} \Pi\big(tb(\phi);\gamma\big).$$

The maximum theorem therefore implies that Π^* is jointly continuous.

Because $b(\phi)$ is weakly decreasing, a higher ϕ shrinks the platform's feasible set of commissions, so Π^* is weakly decreasing in ϕ. Proposition 4 further shows that the unconstrained profit function is strictly increasing up to $f^\circ(\gamma)$. Hence, profit is constant while $b(\phi) \ge f^\circ(\gamma)$, or equivalently while $\phi \le \Phi(\gamma)$, and strictly decreases as $b(\phi)$ falls below $f^\circ(\gamma)$. At $\phi = \hat{\phi}(0)$, $b(\phi) = 0$ and profit is zero; it remains zero thereafter.

Finally, fix $\phi < \hat{\phi}(0)$ and $0 < \gamma_1 < \gamma_2$. Let f_2 maximize profit at γ_2. The market operates, so equilibrium profit is positive and $f_2 \in (0,1/\theta)$. Therefore,

$$\Pi^*(\gamma_1,\phi) \ge \Pi(f_2;\gamma_1) > \Pi(f_2;\gamma_2) = \Pi^*(\gamma_2,\phi),$$

which establishes strict monotonicity in γ before shutdown. ■

Proof of Lemma 1. Fix $\phi \le \phi_B$. Since $\hat{\phi}$ is strictly decreasing (Lemma 5) and $f_L(\gamma) \le f_B^U$ for $\gamma \le \gamma_B$, we have $\hat{\phi}(f_L(\gamma)) \ge \hat{\phi}(f_B^U) = \phi_B \ge \phi$. Thus, for $\phi \le \phi_B$, Proposition 7 implies that we are in Region B-I for $\gamma \le \gamma_B$ and in Region B-III for $\gamma > \gamma_B$.

Region B-I ($\gamma \le \gamma_B$). Proposition 7 gives $n_B^*(\gamma) = \frac{1}{2\tilde{k}(f_L(\gamma))}$. By Lemma 4, $\hat{\gamma}(f)$ is increasing on $(0, 1/(4\theta))$, which implies that $f_L(\gamma)$ is strictly increasing in γ. Also, by Proposition 1, $\tilde{k}(f)$ is strictly decreasing in f. Hence $n_B^*(\gamma)$ is strictly increasing in γ.

Region B-III ($\gamma > \gamma_B$). Proposition 7 gives $n_B^*(\gamma) = \frac{2g^*(f_B^U)f_B^U\tilde{k}(f_B^U)}{\gamma}$. The numerator is independent of γ, so $n_B^*(\gamma)$ is strictly decreasing in γ.

Continuity at γ_B. Since $\gamma_B = \hat{\gamma}(f_B^U)$ and $f_B^U \in (0, 1/(4\theta))$, we have $f_L(\gamma_B) = f_B^U$. Substituting $g^*(f_B^U)$, $\tilde{k}(f_B^U)$, and γ_B into the B-III expression yields $\frac{1}{2\tilde{k}(f_B^U)}$, matching the B-I expression at γ_B.

Combining the three steps, n_B^* is continuous, strictly increasing on $(0, \gamma_B]$, and strictly decreasing on (γ_B, ∞), so γ_B is the unique maximizer. ■

Proof of Corollary 3 The ordering of commissions is immediate from Step 5 of the proof of Proposition 6. For the second claim, by Step 2 of the proof of Proposition 6 a cap $r < f^*$ induces the commission r and yields net donations $D(r)$. By Steps 1 and 2 of the proof of Proposition 7, D is continuous and strictly increasing on $[0, f_B^\circ(\gamma)]$; moreover $D(0) = 0$, since $\hat{\gamma}(0) = 0 < \gamma$ makes $f = 0$ uncovered and (5) gives $D_U(0) = 0$. If $f_B^* = f^*$, then $f^* \le f_B^\circ(\gamma)$, so $D(r) < D(f^*) = D^*$ for every $r < f^*$. If instead $f_B^* < f^*$, then $f_B^* = f_B^\circ(\gamma)$—otherwise $f_B^* = \bar{f}(\phi)$ would give $f^* \le \bar{f}(\phi) = f_B^*$—and since $D(0) = 0 < D^*$, continuity gives $D(r) < D^*$ for all sufficiently small r. ■

Proof of Lemma 2. If $\phi \ge \hat{\phi}(0)$, neither setting operates, so both net-donation outcomes and their gap are zero. Suppose henceforth that $0 < \phi < \hat{\phi}(0)$. By the proofs of Propositions 4 and 7, the equilibrium commissions are $f^*(\gamma, \phi) = \min\{f^\circ(\gamma), \bar{f}(\phi)\}$ and $f_B^*(\gamma, \phi) = \min\{f_B^\circ(\gamma), \bar{f}(\phi)\}$. If $\gamma \le \gamma_B$, then $f_B^\circ(\gamma) = f_L(\gamma) < 1/(4\theta)$; if $\gamma > \gamma_B$, then $f_B^\circ(\gamma) = f_B^U < 1/(4\theta)$. Since $f^\circ(\gamma) \ge 1/(3\theta)$, we obtain $f_B^\circ(\gamma) < f^\circ(\gamma)$ for every $\gamma > 0$. The three cases follow.

Case 1. If $\bar{f}(\phi) \ge f^\circ(\gamma)$, the cap does not distort the commission in either setting, so $f^* = f^\circ(\gamma)$ and $f_B^* = f_B^\circ(\gamma)$, giving $\Delta_D = D(f_B^\circ(\gamma); \gamma) - D(f^\circ(\gamma); \gamma)$.

Case 2. If $f_B^\circ(\gamma) < \bar{f}(\phi) < f^\circ(\gamma)$, the cap binds only in the profit-maximizing case, so $f^* = \bar{f}(\phi)$ and $f_B^* = f_B^\circ(\gamma)$, giving $\Delta_D = D(f_B^\circ(\gamma); \gamma) - D(\bar{f}(\phi); \gamma)$.

Case 3. If $\bar{f}(\phi) \le f_B^\circ(\gamma)$, we have $f^* = f_B^* = \bar{f}(\phi)$ and $\Delta_D = 0$.

We now establish the comparative statics in ϕ. The first claim is immediate, as the Case 1 expression for Δ_D does not involve ϕ. For the second, in Case 2, $D_B^* = D(f_B^\circ(\gamma); \gamma)$ is independent of ϕ, while $D^* = D(\bar{f}(\phi); \gamma)$. The proof of Proposition 7 shows that the donation function $D(\cdot; \gamma)$ is single-peaked at $f_B^\circ(\gamma)$ with strictly decreasing right branch on $[f_B^\circ(\gamma), 1/\theta]$. By Lemma 5, $\bar{f}(\phi)$ is strictly decreasing in ϕ, so as ϕ rises, $\bar{f}(\phi)$ falls toward $f_B^\circ(\gamma)$ on this branch and D^* strictly increases. Hence $\Delta_D = D_B^* - D^*$ strictly decreases in ϕ. The third claim follows directly from Case 3. The threshold solves $\bar{f}(\phi) = f_B^\circ(\gamma)$, giving $\phi = \hat{\phi}(f_B^\circ(\gamma))$. ■

Proof of Lemma 3. If $\phi \ge \hat{\phi}(0)$, neither setting operates, so both admission outcomes and their gap are zero. Suppose henceforth that $0 < \phi < \hat{\phi}(0)$. Recall from the proof of Lemma 2 that $f^* = \min\{f^\circ(\gamma), \bar{f}(\phi)\}$ and $f_B^* = \min\{f_B^\circ(\gamma), \bar{f}(\phi)\}$, with $f_B^\circ(\gamma) < f^\circ(\gamma)$ for all $\gamma > 0$. The three cases follow. If $\bar{f}(\phi) \ge f^\circ(\gamma)$, the cap distorts neither choice, so $\Delta_n = n^*(f^\circ(\gamma); \gamma) - n^*(f_B^\circ(\gamma); \gamma)$. If $f_B^\circ(\gamma) < \bar{f}(\phi) < f^\circ(\gamma)$, the cap distorts only the profit-maximizing choice, so $\Delta_n = n^*(\bar{f}(\phi); \gamma) - n^*(f_B^\circ(\gamma); \gamma)$. If $\bar{f}(\phi) \le f_B^\circ(\gamma)$, we have $f^* = f_B^* = \bar{f}(\phi)$, so $\Delta_n = 0$.

It remains to show that the expressions in the first two cases are non-negative; equivalently, that $n^*(f; \gamma) \ge n^*(f_B^\circ(\gamma); \gamma)$ for all $f \in [f_B^\circ(\gamma), f^\circ(\gamma)]$. Define $n_C(f) := 1/(2\tilde{k}(f))$. For $f \in (0, 1/\theta)$, the unconstrained admission can be written as

$$n^*(f; \gamma) = \min\{n_C(f), n_U(f)\}.$$

The covered formula $n_C(f)$ is strictly increasing because $\tilde{k}(f)$ is strictly decreasing, and $n_U(f)$ is strictly increasing on $[0, 1/(3\theta)]$ by Lemma 7.

If $\gamma > \bar{\gamma}$, then $f^\circ(\gamma) = 1/(3\theta)$. Both n_C and n_U are strictly increasing on $[f_B^\circ(\gamma), f^\circ(\gamma)]$, so their minimum $n^*(\cdot;\gamma)$ is also strictly increasing on this interval.

Now suppose $\gamma \le \bar{\gamma}$. Then $f^\circ(\gamma) = f_C^*(\gamma)$, and $0 < \gamma < \hat{\gamma}_{\max}$, so $f_L(\gamma)$ is well-defined. Moreover, $f_B^\circ(\gamma) \le f_L(\gamma)$: equality holds when $\gamma \le \gamma_B$, while $f_B^\circ(\gamma) = f_B^U < f_L(\gamma)$ when $\gamma > \gamma_B$. For $f \in [f_B^\circ(\gamma), f_L(\gamma))$, the market is uncovered and $n^* = n_U(f)$, which is strictly increasing because $f_L(\gamma) < 1/(4\theta) < 1/(3\theta)$. For $f \in [f_L(\gamma), f_C^*(\gamma)]$, the market is covered and $n^* = n_C(f)$, which is strictly increasing. The two expressions agree at $f_L(\gamma)$, so $n^*(\cdot;\gamma)$ is strictly increasing on the full interval $[f_B^\circ(\gamma), f^\circ(\gamma)]$. Thus the first two gap expressions are non-negative.

We now establish the comparative statics in ϕ. The first claim is immediate, as the expression for Δ_n for $\bar{f}(\phi) \ge f^\circ(\gamma)$ does not involve ϕ. For $f_B^\circ(\gamma) < \bar{f}(\phi) < f^\circ(\gamma)$, we have $\Delta_n = n^*(\bar{f}(\phi);\gamma) - n^*(f_B^\circ(\gamma);\gamma)$, where the second term is independent of ϕ. The proof above shows that $n^*(\cdot;\gamma)$ is strictly increasing on $[f_B^\circ(\gamma), f^\circ(\gamma)]$. By Lemma 5, $\bar{f}(\phi)$ is strictly decreasing in ϕ, so as ϕ rises, $\bar{f}(\phi)$ falls toward $f_B^\circ(\gamma)$ and $n^*(\bar{f}(\phi);\gamma)$ strictly decreases, while $n^*(f_B^\circ(\gamma);\gamma)$ remains constant. Hence Δ_n strictly decreases in ϕ. For $\bar{f}(\phi) \le f_B^\circ(\gamma)$, the gap is zero. The threshold solves $\bar{f}(\phi) = f_B^\circ(\gamma)$, giving $\phi = \hat{\phi}(f_B^\circ(\gamma))$. ■

Appendix D: Technical Lemmas and Proofs

LEMMA 4 **(Shape of the $\hat{\gamma}(f)$ Function)**. *The function $\hat{\gamma}(f) = \frac{\beta^6(1-\theta f)^3 f}{16\alpha^4\lambda^2}$ is strictly increasing on $[0, \frac{1}{4\theta}]$ and strictly decreasing on $[\frac{1}{4\theta}, \frac{1}{\theta}]$, with $\hat{\gamma}(0) = \hat{\gamma}(\frac{1}{\theta}) = 0$ and a unique maximum of $\hat{\gamma}_{\max} = \frac{27\beta^6}{4096\alpha^4\lambda^2\theta}$ at $f = \frac{1}{4\theta}$. Consequently, for any $0 < \gamma < \hat{\gamma}_{\max}$, the equation $\hat{\gamma}(f) = \gamma$ has exactly two roots $f_L(\gamma) < \frac{1}{4\theta} < f_H(\gamma)$, and the market is covered (i.e., $\gamma \le \hat{\gamma}(f)$) for $f \in [f_L(\gamma), f_H(\gamma)]$ and uncovered (i.e., $\gamma > \hat{\gamma}(f)$) for $f \in [0, f_L(\gamma)) \cup (f_H(\gamma), \frac{1}{\theta}]$. For $\gamma = \hat{\gamma}_{\max}$, the market is covered at $f = 1/(4\theta)$ and uncovered for all other f. For $\gamma > \hat{\gamma}_{\max}$, the market is uncovered for every f.*

Proof of Lemma 4. Note that $\hat{\gamma}(0) = \hat{\gamma}(\frac{1}{\theta}) = 0$ follows directly by substitution. Differentiating, we obtain

$$\frac{\partial\hat{\gamma}}{\partial f} = \frac{\beta^6}{16\alpha^4\lambda^2}(1-\theta f)^2(1-4\theta f).$$

Since $(1-\theta f)^2 > 0$ for all $f \in (0, \frac{1}{\theta})$, the sign of $\frac{\partial\hat{\gamma}}{\partial f}$ is determined entirely by $(1-4\theta f)$, which is positive for $f < \frac{1}{4\theta}$ and negative for $f > \frac{1}{4\theta}$. Hence $\hat{\gamma}(f)$ is strictly increasing on $[0, \frac{1}{4\theta}]$ and strictly decreasing on $[\frac{1}{4\theta}, \frac{1}{\theta}]$, attaining its unique maximum at $f = \frac{1}{4\theta}$. Substituting gives $\hat{\gamma}_{\max} = \frac{27\beta^6}{4096\alpha^4\lambda^2\theta}$. The claims about the roots and market coverage follow from the inverted-U shape and the boundary values. ■

LEMMA 5 **(Monotonicity of $\hat{\phi}(f)$ and the Feasibility Threshold)**. *The function $\hat{\phi}(f) = \frac{\beta^4(1-\theta f)^2(1-f)}{8\alpha^3\lambda}$ is strictly decreasing on $[0, \frac{1}{\theta}]$, with $\hat{\phi}(0) = \frac{\beta^4}{8\alpha^3\lambda}$ and $\hat{\phi}(\frac{1}{\theta}) = 0$. Hence, if $\phi \le \hat{\phi}(0)$, then the feasibility constraint $\phi \le \hat{\phi}(f)$ is satisfied if and only if $f \in [0, \bar{f}(\phi)]$, where $\bar{f}(\phi)$ is the unique solution in $f \in [0, 1/\theta]$ to $\hat{\phi}(f) = \phi$ and is strictly decreasing in ϕ. If instead $\phi > \hat{\phi}(0)$, then the market does not operate.*

Proof of Lemma 5. Both $(1-\theta f)^2$ and $(1-f)$ are strictly positive and strictly decreasing in f on $[0,\frac{1}{\theta})$, so their product—and hence $\hat{\phi}(f)$—is strictly decreasing on $[0,\frac{1}{\theta}]$, where $\hat{\phi}(0)=\frac{\beta^4}{8\alpha^3\lambda}$ and $\hat{\phi}(\frac{1}{\theta})=0$ follow from direct substitution. Strict monotonicity makes $\hat{\phi}$ invertible on its range $[0,\frac{\beta^4}{8\alpha^3\lambda}]$, so if $\phi\le\hat{\phi}(0)$, then the equation $\hat{\phi}(f)=\phi$ has a unique solution in $f\in[0,1/\theta]$, which we denote by $\bar{f}(\phi)$, and moreover, we have $f\le\bar{f}(\phi)\iff\phi\le\hat{\phi}(f)$. Since $\hat{\phi}$ is strictly decreasing, $\bar{f}(\phi)$ is strictly decreasing in ϕ.

Finally, if $\phi>\hat{\phi}(0)$, then since $\hat{\phi}$ is decreasing in f, we have $\phi>\hat{\phi}(0)\ge\hat{\phi}(f)$ for all $f\in[0,1/\theta]$. Proposition 3 then implies that no beneficiaries will apply to the platform for any $f\in[0,1/\theta]$, so the market does not operate. ■

LEMMA 6 **(Shape of the Net Donations Function)**. *The net donations function $D(f)$ satisfies the following:*

(i) $D_C(f)$ is strictly decreasing on $[0,1/\theta]$, with $D_C(0)=\frac{\beta^2}{4\alpha^2}$ and $D_C(1/\theta)=0$.

(ii) $D_U(f)$ is strictly increasing on $[0,f_B^U]$ and strictly decreasing on $[f_B^U,1/\theta]$, with its unique interior maximum at $f_B^U=\frac{(2+5\theta)-\sqrt{25\theta^2-4\theta+4}}{12\theta}\in(0,1/(4\theta))$, which does not depend on γ.

Proof of Lemma 6. For Part (i), $D_C'(f)=\frac{\beta^2}{4\alpha^2}[2\theta f-(1+\theta)]$. Since $\theta\ge1$, $2\theta f\le2\le1+\theta$ on $[0,1/\theta]$, with equality only at $f=1/\theta$ when $\theta=1$. Hence $D_C'(f)<0$ on the interior, so D_C is strictly decreasing on $[0,1/\theta]$.

For Part (ii), we differentiate $D_U(f;\gamma)$ to obtain

$$D_U'(f;\gamma)=\frac{\beta^8(1-\theta f)^3}{64\alpha^6\lambda^2\gamma}\cdot Q(f),\qquad Q(f):=1-(2+5\theta)f+6\theta f^2.$$

Note that $\frac{\beta^8(1-\theta f)^3}{64\alpha^6\lambda^2\gamma}$ is strictly positive on $[0,1/\theta)$, so the sign of D_U' matches the sign of $Q(f)$. The quadratic Q has discriminant $(2+5\theta)^2-24\theta=25\theta^2-4\theta+4>0$ for $\theta\ge1$, so Q has two distinct real roots, given by $f=\frac{(2+5\theta)\pm\sqrt{25\theta^2-4\theta+4}}{12\theta}$. To locate the roots, we evaluate Q at three points: $Q(0)=1>0$, $Q(1/(4\theta))=-\frac{2\theta+1}{8\theta}<0$, and $Q(1/\theta)=\frac{4(1-\theta)}{\theta}\le0$. Hence the smaller root f_B^U lies in $(0,1/(4\theta))$, and the larger root lies at or above $1/\theta$. Hence, $Q(f)>0$ on $[0,f_B^U)$ and $Q(f)<0$ on $(f_B^U,1/\theta)$, so D_U is strictly increasing on $[0,f_B^U]$ and strictly decreasing on $[f_B^U,1/\theta]$. ■

LEMMA 7 **(Shape of Uncovered Unconstrained Maximizer $n_U(f)$)**. *The function*

$$n_U(f)=\frac{2g^*(f)f\tilde{k}(f)}{\gamma}=\frac{\beta^4(1-\theta f)^2f}{8\alpha^3\lambda\gamma}$$

is strictly increasing on $[0,1/(3\theta)]$ and strictly decreasing on $[1/(3\theta),1/\theta]$, thus attaining its unique maximizer on $[0,1/\theta]$ at $f=1/(3\theta)$.

Proof of Lemma 7. The second equality in the result statement follows from substitution of the expressions for $g^*(f)$ and $\tilde{k}(f)$ from Proposition 1. Differentiating the expanded form of $n_U(f)$ then gives

$$\frac{\partial n_U}{\partial f}=\frac{\beta^4(1-\theta f)(1-3\theta f)}{8\alpha^3\lambda\gamma}.$$

Since $\frac{\beta^4}{8\alpha^3\lambda\gamma}>0$ and $1-\theta f>0$ on $[0,1/\theta)$, the sign of $\frac{\partial n_U}{\partial f}$ matches the sign of $1-3\theta f$, which is positive for $f<1/(3\theta)$ and negative for $f>1/(3\theta)$. The result follows. ■

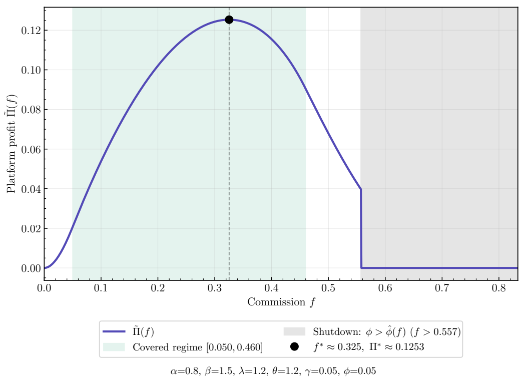
Appendix E: Beneficiary Entry Cost Incurred upon Admission

In the baseline model, each beneficiary incurs ϕ upon applying. Consider now the alternative timing in which applying is costless and ϕ is incurred only upon admission. All equilibrium outcomes characterized in the paper are unchanged. The reason is that the entry cost affects equilibrium outcomes only by determining whether beneficiaries are willing to be on the platform at commission f—that is, only through the participation condition $\phi \leq \hat{\phi}(f)$, which imposes the ceiling on the commission—and this condition compares the same two quantities, the per-beneficiary payoff $\hat{\phi}(f)$ and the cost ϕ, no matter when the entry cost is paid. The timing changes how many beneficiaries apply, but not whether they do.

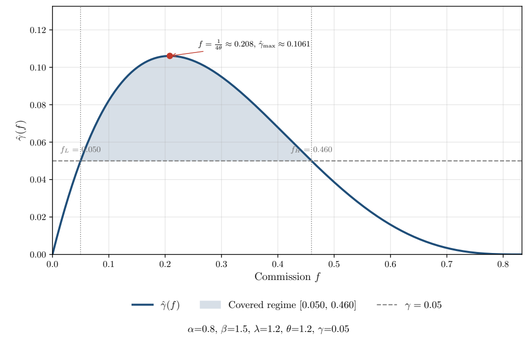
To see that the participation condition is identical, note first that at $f \in \{0, 1/\theta\}$, the platform admits no beneficiaries (Proposition 2) and all outcomes are zero under either timing, so fix $f \in (0, 1/\theta)$, for which $n^*(f) > 0$ whenever the applicant pool is positive. By the proof of Proposition 3, each admitted beneficiary earns $\hat{\phi}(f) = 2\, g^*(f)(1-f)\tilde{k}(f)$ regardless of the applicant pool or the coverage regime. Under admission-contingent timing, the entry cost is paid in the same event in which the payoff is received, so the expected net utility from applying is $(n/\bar{N})\big(\hat{\phi}(f) - \phi\big)$ rather than the baseline $(n/\bar{N})\,\hat{\phi}(f) - \phi$: in the admission-contingent case, the admission probability scales the entire net payoff without changing its sign, so applying is worthwhile if and only if $\phi \leq \hat{\phi}(f)$ (indifferent beneficiaries apply, by our tie-breaking convention). In the baseline, the cost is sunk before admission is known, so the willingness to apply depends on the odds. But the applicant pool adjusts until it does not matter: beneficiaries apply precisely when certain admission yields $\hat{\phi}(f) - \phi \geq 0$, and they keep applying until the expected net utility of an additional entrant reaches zero at the finite $\bar{N}(f)$ (Proposition 3). Hence, at every interior commission, the market operates under either timing if and only if $\phi \leq \hat{\phi}(f)$. By Lemma 5, the positive commissions that induce operation are therefore $(0, \bar{f}(\phi)]$ under both timings, while $f = 0$ is a common shutdown option and $\hat{\phi}(0)$ is the common shutdown threshold: the entry cost forces the platform to leave beneficiaries the same share of donations whether ϕ is handed over upon applying or upon admission.

It remains to be verified that the applicant pool—the only equilibrium decision that differs between the two timings—is payoff-irrelevant. By Proposition 3, beneficiary supply does not distort the platform's admission decision: it is slack under a strict participation inequality and binds without changing admission at equality. A larger pool only relaxes this constraint further. Hence, the platform admits the unconstrained $n^*(f)$ of Proposition 2 in both cases. Since ϕ enters neither the platform's profit nor donor utility, Stages 3 and 4 are unchanged given (f, n), the platform's Stage 1 problem is identical, and Proposition 4 applies verbatim, as do the donation-maximizing benchmark (Propositions 6 and 7) and the gap analysis (Lemmas 2 and 3).

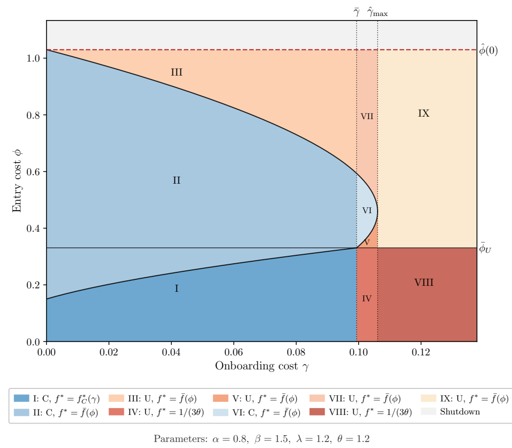
Appendix F: Plots and Diagrams



(a) Platform Profit

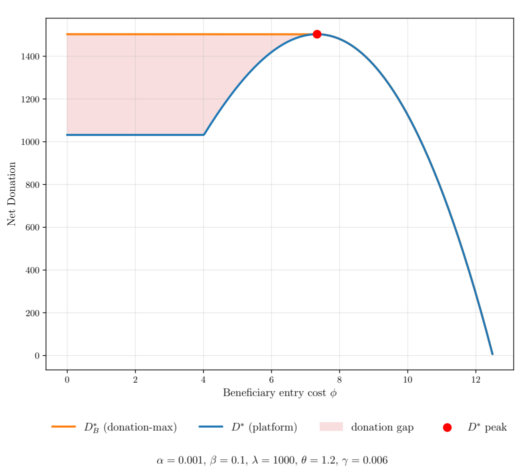


(b) $\hat{\gamma}(f)$ Function

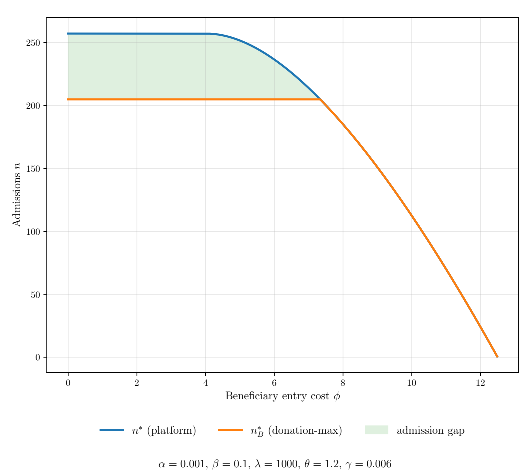


(c) Equilibrium regions in (γ, ϕ)-space

Figure 9 Equilibrium Outcomes



(a) Donation



(b) Admission

Figure 10 Donation and Admission Gaps