Increased Transparency in Procurement: The Role of Peer Effects

(Authors' names blinded for peer review)

Motivated by recent initiatives to increase transparency in procurement, we study the effects of disclosing information about previous purchases in a setting where an organization delegates its purchasing decisions to its employees. When employees can use their own discretion—which may be influenced by personal preferences—to select a supplier, the incentives of the employees and the organization may be misaligned. Disclosing information about previous purchasing decisions made by other employees can reduce or exacerbate this misalignment, as peer effects may come into play. To understand the effects of transparency, we introduce a theoretical model that compares employees' actions in two settings: one where employees cannot observe each other's choices and one where they can observe the decision previously made by a peer before making their own. Two behavioral considerations are central to our model: that employees are heterogeneous in their reciprocity towards their employer, and that they experience peer effects in the form of income inequality aversion towards their peer. As a result, our model predicts the existence of *negative spillovers* as a reciprocal employee is more likely to choose the expensive supplier (which gives him a personal reward) when he observes that a peer did so. A laboratory experiment confirms the existence of negative spillovers and the main behavioral mechanisms described in our model. A surprising result not predicted by our theory, is that employees whose decisions are observed by their peers are less likely to choose the expensive supplier than the employees in the no transparency case. We show that observed employees' preferences for compliance with the social norm of "appropriate purchasing behavior" explain our data well.

Key words: behavioral operations, delegation, transparency, procurement, peer effects, laboratory experiments.

1. Introduction

As part of recent initiatives to increase operational transparency, several organizations have launched online platforms to make information related to procurement transactions visible to various parties within the organization. For example, the government of Buenos Aires, Argentina, launched in 2017 a new platform, BuenosAiresCompra, that allows government employees to search and buy products and services from a set of preselected suppliers and to observe the past purchases of other government employees, among other features. ProZorro, an open public procurement platform launched by the government of Ukraine in 2014, goes even further and, under the motto "*Everyone can see everything*," discloses public procurement data, including the list of all potential suppliers with their bids, the decisions of the evaluation committee, contracts, and all qualification/certification documents to the general public. Similar initiatives have been adopted by the governments of Chile (ChileCompra), the UK (ContractsFinder), the state of California (CaleProcure), and other governments and private companies through third-party platforms such as OpenGov and Procurify. As opposed to ProZorro, most of these platforms provide only aggregate information to the public and, as in BuenosAiresCompra, individual decisions are observable only to employees within the organization.

In general, the transactions recorded by these platforms can be broadly classified into two types. The first type comprises the purchases of products and services that are central to the operation of the organization (e.g., the purchase of meals by a school district). As such, these purchases are the responsibility of procurement teams that must follow well-specified purchasing protocols. The second type comprises those purchases that, instead of being carried out by specialized procurement teams, are *delegated* to individual employees who will ultimately be the ones using these products and services. Examples of the latter include employees booking their own air tickets and hotels for business travel purposes, or choosing which work computer to buy. While employees still need to follow some purchasing guidelines, they are typically allowed much more freedom in selecting products and suppliers. This freedom can be somewhat problematic as the incentives of employees and the organization are not always fully aligned: while organizations typically care about price and quality, employees' preferences may be influenced by personal considerations. For example, employees might prefer to choose airlines for which they obtain reward points for personal use, ignoring lower-priced alternatives. This can be concerning for organizations because, even though individually delegated purchases generally involve small expenses, they can add up to large amounts. For instance, in 2014 the US government spent $17.1 billion through the Government Purchase Card Program, which allows public agencies to pay for small purchases.[1]

Disclosing information about others' purchasing decisions can reduce or exacerbate this misalignment, as social comparisons and peer effects may come into play. Moreover, mechanisms to mitigate the misalignment are typically unavailable—individuals' needs are idiosyncratic and purchases are usually small, making punishment and monitoring unfeasible—and therefore understanding the impact of increased transparency on purchasing decisions is particularly important in this setting.

The goal of this paper is to study, both theoretically and experimentally, the impact of increased transparency on delegated purchasing behavior. Our contribution is threefold. First, we introduce a theoretical model incorporating employees' social preferences in the presence of transparency into

[1] Source: SBTDC, September 2, 2015. http://www.sbtdc.org/2015/09/what-is-the-government-purchase-card/.

the employees' utility, and derive testable hypotheses. Second, we design a procurement game to test the predictions of our model and to analyze the main drivers leading to changes in behavior. Finally, based on our results, we provide concrete managerial insights for organizations seeking to understand the potential consequences of increased transparency on their procurement costs.

More precisely, we introduce a stylized model of an organization consisting of a director and two employees, whose wages are identical and determined by the director. The organization needs to purchase two identical items and the director delegates the supplier choice to the employees: each of the employees must choose one of two suppliers to provide an item, which will be paid by the organization, and their decisions can be neither overruled nor punished by the director. Purchasing from the expensive supplier provides an extra personal benefit for the employees (such as purchasing a flight from a preferred airline) but also results in a higher procurement cost for the organization. This model captures the main features of the real-world settings described above and, at the same time, it is simple enough to allow us to squarely focus on studying the impact of transparency on employees' purchasing decisions, both theoretically and experimentally.

In particular, to understand the effect of transparency, we compare the employees' actions in two settings: one where employees make their decisions simultaneously and cannot observe each other's choices (baseline) and one where they make their decisions sequentially and the second employee can observe the first employee's supplier choice before making his own (peer). We assume that, besides the motivation to maximize monetary payoffs, two behavioral factors drive employees' decisions. First, we assume that (some) employees have reciprocal preferences towards the employer such that the employees are willing to forgo the personal benefit and choose the cheaper supplier if they perceive their employer is treating them kindly. Second, employees are subject to peer effects, which we model as income inequality aversion towards their peer's payoff. In line with previous literature, we assume that the latter is only present among employees who can observe their peer's decisions.

We show that, in both the baseline and peer settings, the probability that an employee chooses the expensive supplier is decreasing in the wage offered by the director, in the price difference between suppliers, and in how much the employee cares about reciprocity. Our main theoretical contribution is to show the existence of a *negative spillover* price-difference region for reciprocal employees, i.e., if the price difference between the suppliers falls in that region, a reciprocal employee is more likely to choose the expensive supplier (relative to the baseline) if he observes that his peer did so. This effect, results from the interaction of two behavioral considerations: the heterogeneity in employees' reciprocity towards the employer, and the aversion to disadvantageous income inequality relative to the peer. Moreover, our model predicts the absence of positive spillovers as observing that a peer chose the cheapest option does not affect employees' behavior, regardless of their reciprocity type.

To test these predictions we introduce a new game, the *procurement game*, that replicates the setting in our theoretical model. Our experimental design consists of a baseline treatment, where both employees choose a supplier simultaneously, and a peer treatment, in which employees make their decisions sequentially. The main experimental finding is that increasing transparency has a heterogeneous effect on buyers. In line with our theoretical results, we find evidence of negative spillover effects on reciprocal employees and no evidence of positive spillover effects. These results suggest that increasing transparency negatively affects reciprocal employees who observe their peers' decisions. Besides confirming the existence of negative spillover effects, which have previously been uncovered experimentally in related settings (see Section 2 for a detailed discussion), our theoretical model and experimental design allow us to identify the heterogeneity in reciprocity towards the employer as a key behavioral mechanism leading to this effect.

Moreover, we analyze how a buyer's behavior changes when he is being observed by a peer. The effects on observed employees have been mostly overlooked by the previous literature on peer effects, which mainly focuses on the effects on observers. We find evidence that employees who are observed are less likely to choose the expensive option. This result is particularly significant among non-reciprocal employees, suggesting that reciprocity is not the main mechanism driving their behavior. We propose an alternative explanation based on preferences for compliance with the *social norm*, i.e., the collective agreement about the appropriateness of choosing the expensive supplier. To test it, we conduct two social norm elicitation treatments to measure the appropriateness of choosing the expensive supplier in the baseline and peer settings respectively. We find that it is less appropriate to choose the expensive supplier when employees are observed by others, and that the differences between the elicited social norms are consistent with the differences in purchasing behavior. These results suggest that a model where observed employees seek to comply with social norms better explains their purchasing behavior.

We conduct two additional treatments to examine the effects of transparency when the two conditions we isolated in the peer treatment overlap, such that an employee observes a peer's decision before making his own, *and* is also himself observed by a peer. These treatments confirm that the two main effects that we identified—negative spillovers associated with observing that a peer chose the expensive supplier and positive effects associated with being observed by a peer—are still present when an employee both observes and is observed.

Our results provide useful managerial insights that can be applied when designing procurement platforms that increase transparency. In particular, firms' internal communication policies should emphasize that employees' decisions will be observed by their peers: this would help reduce overspending by non-reciprocal employees which in turn would mitigate the negative spillovers on reciprocal employees, leading to lower procurement costs. In addition, our results show that overspending is perceived to be less appropriate when an employee's decision will be observed by other employees. This should also be exploited by the organization to reduce procurement costs: as employees comply, to a certain level, with what is perceived as socially appropriate, organizations should make communication efforts that reinforce what is perceived as appropriate spending behavior in an attempt to increase compliance with the social norm.

Finally, while our experiment captures decision making in a procurement setting, we believe our findings and the behavioral mechanisms we identify (and, consequently, the managerial implications we derive) can more broadly explain the effects of increased transparency in related settings.

The remainder of this paper is organized as follows. In Section 2 we discuss the related literature. In Section 3 we present the theoretical model. Section 4 describes the experimental design and the hypotheses derived from our model, and Section 5 presents the main results. Section 6 provides a discussion. Finally, Section 7 describes the managerial implications of this research and concludes.

2. Related Literature

Our paper lies at the intersection of several streams of literature. First, it contributes to a growing literature studying the effects of transparency in operations management. So far, this literature has focused mostly on the effects of transparency on consumers' valuations for a product or service. For example, Buell and Norton (2011) show that operational transparency signaling that a service provider has exerted effort leads to a higher customer value perception. Buell et al. (2017) find similar positive effects of transparency when customers observe operational processes and employees can observe customers. Kraft et al. (2018) show that increased transparency through greater supply chain visibility increases consumers' valuations for a firm's social responsibility practices. Our paper shifts the focus towards the effects of transparency regarding employees' procurement decisions on the behavior of those same employees and their peers, and provides insights into how transparency can be most effectively implemented in this setting.

Our paper is also related to the literature studying the impact of human behavior on the design of procurement policies; see Elmaghraby and Katok (2017) for a comprehensive overview. Several papers focus on comparing the outcomes of alternative mechanisms (Engelbrecht-Wiggans and Katok 2006, Katok and Kwasnica 2008, Wan and Beil 2009, Wan et al. 2012, Tunca et al. 2014, Chaturvedi et al. 2016, among others) and on analyzing the behavioral factors affecting bidders' decisions (Kwasnica and Katok 2007, Davis et al. 2011). In particular, Elmaghraby et al. (2012) and Haruvy and Katok (2013) identify adverse effects of increased information transparency in suppliers' bidding behavior. Elmaghraby et al. (2012) find that rank-based feedback leads to lower prices than full price feedback. Haruvy and Katok (2013) find that bidders act more aggressively under a sealed-bid first-price format than under open-bid dynamic auctions. While suppliers' monetary payoffs directly depend on other suppliers' bids, our focus is on the increased visibility of employees' decisions, where their choices do not affect each other's monetary payoffs.

Our paper studies a procurement setting where an organization delegates the supplier choice to its employees. This resembles the traditional delegation setting introduced in the seminal paper by Aghion and Tirole (1997). Recent papers have studied behavioral aspects of delegation in different settings. Charness et al. (2012) conduct an experiment where an employer can decide either to choose an employee's wage or to delegate this choice to the employee and show that both the employer's and the employee's earnings are larger under delegation. Hamman et al. (2010) find that delegation may also be used to avoid taking direct responsibility for selfish or unethical behavior. Unlike these papers, we do not seek to study whether a decision should be delegated or not. Rather, we contribute to this literature by studying a setting where the choice of delegating has already been made (and, as under A-formal authority in Aghion and Tirole 1997, employees' decisions cannot be overruled or punished by the director), and focus on understanding how increasing transparency affects the procurement outcome.

Evidence of peer effects has been found in various related settings. The closest papers study peer effects in a three-person gift-exchange game, where an employer first chooses a wage for each of two workers, who then individually choose—either simultaneously or sequentially—a costly effort level that benefits the employer and has no monetary effect on the coworker.[2] Gächter and Thöni (2010) and Gächter et al. (2012) find that reciprocal preferences towards the employer play an important role in explaining employees' behavior, and that higher wages are associated with higher effort. These papers focus on employees' responses to unequal treatment from the employer, and find that effort comparisons are present when the employer pays equal and generous wages to both employees. Instead, we focus squarely on the peer effects resulting from increased transparency of the procurement decision and only allow for equal wages. Gächter et al. (2013) and Thöni and Gächter (2015) identify spillover effects when one worker can observe the other worker's effort before choosing his own. The former find a positive correlation between the decisions of employees making their choices sequentially, and shows that the second worker's behavior is better explained by income inequality aversion towards the peer (Fehr and Schmidt 1999) than by preferences for compliance with social norms. The latter find that agents follow a low-performing but not a high-performing peer. This asymmetry is also identified by Dimant (2019), who shows that unethical behavior is more contagious than ethical behavior in a two-stage dictator game where subjects can

[2] The original gift-exchange game, introduced by (Fehr et al. 1993), consists of one employer and one employee.

donate to or take away money from charity. In the operations management literature, Ho et al. (2014) study peer effects when two retailers interact with the same supplier and find that, due to the retailers' peer-induced and distributional fairness (Ho and Su 2009), the second retailer has a higher wholesale price, makes a lower profit, and has a lower share of the total supply chain profit than the first retailer. We make three important contributions to this literature. First, we show that the negative spillover effects are a robust result, which also arises in our procurement game. Second, our experimental design allows us to test the mechanisms leading to this result by identifying that the negative spillovers affect primarily reciprocal employees. Third, we show the existence of "positive effects" on employees who are observed by their peers, a result that has been mostly overlooked in previous literature.[3] and analyze the behavioral drivers leading to it.

Finally, our paper is also broadly related to the behavioral operations management literature studying the effects of social preferences in supply chain management and procurement (Croson et al. 2013). This stream of literature has established that factors such as fairness (Haitao Cui et al. 2007, Loch and Wu 2008, Katok and Pavlov 2013), trust and trustworthiness (Özer et al. 2011, Özer et al. 2014, Özer and Zheng 2017, Spiliotopoulou et al. 2016, Beer et al. 2018), and long-term relational concerns (Davis and Hyndman 2017) are important for understanding how firms make decisions in procurement and how they relate with their suppliers. While most of these papers focus on firm-level decisions, our focus is on employee-level decisions and in particular on how employees affect each other's decisions when transparency is introduced.

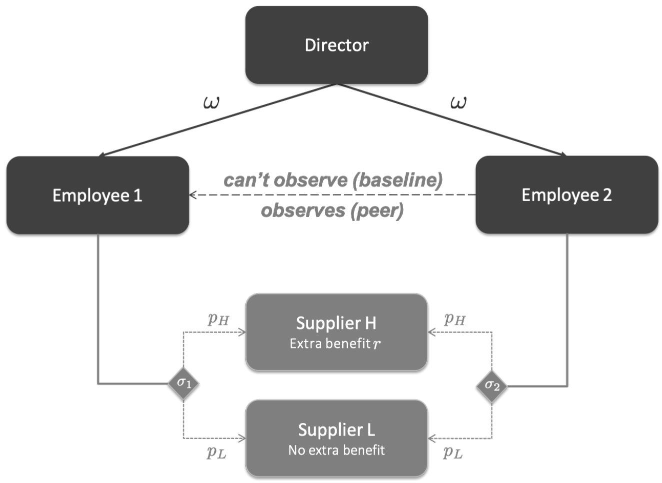
3. Model

We consider an organization comprised of three agents: a director (D) and two employees (E_1 and E_2). The organization needs to purchase two items and the director delegates this task to the employees, such that each employee is in charge of buying one item. The employees can purchase the item from one of two suppliers, supplier H and supplier L (S_H and S_L), who offer identical items at prices p_H and p_L, respectively. For simplicity, p_L is fixed and $p_H = p_L + \Delta$, where Δ is a random variable uniformly distributed in $\left[0, \bar{\delta}\right]$. That is, supplier H is at least as expensive as supplier L, and the realized price difference δ is randomly determined.[4] These prices are exogenously given, i.e., suppliers are non-strategic and/or employees have negligible market power, as in the motivating examples. Finally, employees obtain a personal reward $r > 0$ when purchasing from S_H that is commonly known. This reward can represent the mileage obtained from choosing the preferred airline, the extra utility of choosing a preferred brand, and so on.

[3] Mittone and Ploner (2011) study a five-person trust game, with one sender and four receivers, who are paired and make their choices sequentially. They find evidence of higher returns in receivers who move first, but only when the investment is high.

[4] When clear from the context we abuse notation and use δ for both the random price difference Δ and its realization.

Figure 1 The Procurement Game



Note: In Stage 1 the director chooses a wage that is equal for both employees. In Stage 2 the employees observe the wage chosen by the director and the realized price difference between suppliers, and make their decision. In the baseline model employees make their decisions simultaneously. In the peer model employees make their decisions sequentially, with E_1 choosing first and E_2 observing E_1's choice before making his own.

The interactions between the director and the employees are described in terms of a two-stage *procurement game* as follows. In the first stage, the director chooses a wage $\omega \in [\underline{\omega}, \infty)$ that is the same for both employees. The director chooses the wage knowing only the distribution of the price difference but not its realization. Her goal is to minimize the total procurement cost, given by the sum of the employees' wages and the prices of the suppliers selected by the employees. In the second stage, and after observing the wage chosen by the director and the realized prices p_H and p_L, each employee chooses a supplier. Employees' decisions can be neither overruled nor punished by the director. The game is illustrated in Figure 1.

In the absence of social preferences (i.e., when all agents maximize their own monetary payoff), this game has a unique equilibrium: both employees always choose supplier H, and the director chooses a wage $\omega = \underline{\omega}$. However, previous work has found evidence that agents not only care about their monetary payoff, but also incorporate social considerations in their utility function (e.g. Rabin 1993, Charness and Rabin 2002, Fehr and Schmidt 1999, Bolton and Ockenfels 2000). We focus on the interplay of two such considerations: (1) reciprocity towards the director, which reflects the desire to reward kind actions (high wage) and punish hostile ones (low wage); and (2) distributional preferences towards the peer, which we model as income inequality aversion.[5] In

[5] One could also consider including distributional fairness towards the director. However, since employees' actions follow the director's wage decision, we expect the employees' social preferences to be primarily driven by their perception of how kind the employer's action was. Indeed Ho and Su 2009 show that, in ultimatum games played sequentially by a leader and two followers, peer-induced fairness between the followers is significantly stronger than the followers' distributional fairness towards the leader. Consistent with this result, we focus on employees' reciprocity rather than employees' distributional fairness towards their employer.

the next subsections we consider two variants of the model. We start with a *baseline model* where employees cannot observe each other's decisions. Later, we consider a *peer model*, where employees make their decisions sequentially, starting with E_1 and followed by E_2, and E_2 can observe E_1's supplier choice before making his own decision.

3.1. Baseline Model

In the baseline model, both employees choose a supplier simultaneously. As employees have no information about each other's payoff, we assume that they have no distributional preferences towards their peer and that they have only reciprocity towards the director in their utility function.

We consider employees who are heterogeneous in their sensitivity to reciprocity, and denote employee i's sensitivity to reciprocity by γ_i. We assume that employees can be classified into two types, i.e., $\gamma_i \in \{\gamma_L, \gamma_H\}$ with $0 \leq \gamma_L \leq \gamma_H$. In addition, we let $\gamma_i = \gamma_H$ with probability q, and let $\gamma_i = \gamma_L$ with probability $1-q$, and assume that this distribution is commonly known. We focus on the special case where $\gamma_L = 0$ and $\gamma_H = \gamma > 0$, and say that employee i is *reciprocal* if $\gamma_i = \gamma$ and is *non-reciprocal* otherwise. This is consistent with previous work (see Englmaier and Leider 2012 and Beer et al. 2018).[6]

The strategy for employee $i \in \{1,2\}$ can be described by a function $\sigma_i : [\underline{\omega}, \infty) \times \left[0, \bar{\delta}\right] \times \{0, \gamma\} \to \{S_H, S_L\}$ where $\sigma_i(\omega, \delta, \gamma_i)$ represents the supplier chosen by employee i given wage ω, price difference δ, and its reciprocity coefficient[7] γ_i. When there is no risk of confusion, we sometimes omit the arguments and simply denote by σ_i the decision made by employee i.

For a given wage, price difference, and strategy, we model the utility of employee i as the sum of three terms as follows:

$$u_i(\omega, \delta, \gamma_i, \sigma_i) = \omega + r \cdot \mathbb{1}_{\{\sigma_i = S_H\}} + \gamma_i \cdot R_\rho(\omega, \delta, \sigma_i),$$

where the first two terms represent the monetary payoff (the wage and the reward if the expensive supplier is chosen), and the last term captures the additional utility from reciprocity. We model the latter as the product of the employee's sensitivity to reciprocity, γ_i, and a function R_ρ that depends on the employee's belief about how kind the director is (how the received wage compares to a reference wage) and the employee's kindness towards the director. For simplicity we assume that all employees have the same reference wage, which we denote by ρ. Formally, the function $R_\rho : [\underline{\omega}, \infty) \times \left[0, \bar{\delta}\right] \times \{S_H, S_L\} \to \mathbb{R}$ is defined as

$$R_\rho(\omega, \delta, \sigma_i) = \underbrace{(\omega - \rho)}_{\substack{:=\lambda_\rho(\omega) \\ director's\ kindness}} \cdot \underbrace{\frac{\delta}{2}\left(\mathbb{1}_{\{\sigma_i = S_L\}} - \mathbb{1}_{\{\sigma_i = S_H\}}\right)}_{\substack{:=\kappa_i(\delta, \sigma_i) \\ employee's\ reciprocation}}. \tag{1}$$

[6] As we shall see in Section 5, the assumption $\gamma_L = 0$ is consistent with our own experimental results.

[7] We restrict our attention to pure symmetric strategies, and in the case of indifference we assume that the employees choose supplier H and the director chooses the lowest wage.

The first term, $\lambda_\rho(\omega)$, captures the employee's belief about how generous the wage offered by the director is. We extend Dufwenberg and Kirchsteiger (2000) and assume that E_i's assessment of the director's kindness (or unkindness) is proportional to the difference between the wage received and the reference wage ρ, i.e. $\lambda_\rho(\omega) = \omega - \rho$. That is, the wage offered by the director is perceived as (un)kind if it is (below) above the reference wage ρ. The second term, $\kappa_i(\delta, \sigma_i)$, captures the employee's kindness towards the director. We again follow Dufwenberg and Kirchsteiger (2000) and assume that $\kappa_i(\delta, \sigma_i) = \frac{\delta}{2} \cdot (\mathbb{1}_{\{\sigma_i = S_L\}} - \mathbb{1}_{\{\sigma_i = S_H\}})$; i.e., employee i is kind if $\sigma_i = S_L$ (unkind if $\sigma_i = S_H$), and the magnitude of E_i's (un)kindness is equal to the average impact of his decision on the director's payoff, which is equal to $\delta/2$ (that is, $-\delta$ if $\sigma_i = S_H$ and 0 if $\sigma_i = S_L$).

Notice that, when $\omega > \rho$, the probability of choosing S_H is non-increasing in ω and δ as

$$\begin{aligned} \mathrm{P}\left(\sigma_i = S_H \mid \omega, \delta\right) &= \mathrm{P}\left(u_i(\omega, \delta, \gamma_i, S_H) > u_i(\omega, \delta, \gamma_i, S_L)\right) \\ &= \mathrm{P}\left(r + \gamma_i \cdot \left[R_\rho(\omega, \delta, \gamma_i, S_H) - R_\rho(\omega, \delta, \gamma_i, S_L)\right] > 0\right), \end{aligned} \tag{2}$$

is non-increasing in both ω and δ. This captures the fact that it is more costly to be unkind when the director offers a high wage or when the employee's decision has a higher impact on the director's payoff. By contrast, if $\omega < \rho$ then $R_\rho(\omega, \delta, S_H) - R_\rho(\omega, \delta, S_L)$ is non-negative and non-decreasing in both ω and δ, so the employees will always choose the expensive supplier.

The goal of the director is to choose a wage ω that minimizes her expected cost, defined as

$$c_D(\omega) = 2\omega + \mathbb{E}_{\delta, \gamma_1, \gamma_2}\left[p_{\sigma_1(\omega, \delta, \gamma_1)} + p_{\sigma_2(\omega, \delta, \gamma_2)}\right], \tag{3}$$

the expected sum of the prices of the suppliers chosen by the employees, choices that depend on the realized price difference and the reciprocity coefficients, both of which are unknown to the director at the time she chooses a wage.

Proposition 1 characterizes the equilibrium in the baseline model.

Proposition 1. *For a given wage ω and price difference δ, employee i's optimal strategy function can be characterized depending on his reciprocity type as follows:*

$$\sigma_i(\omega, \delta, 0) = S_H, \quad \text{and} \quad \sigma_i(\omega, \delta, \gamma) = \begin{cases} S_H & \text{if } \lambda_\rho(\omega) \le 0 \\ S_H & \text{if } \lambda_\rho(\omega) > 0 \text{ and } \delta \le \frac{r}{\gamma \lambda_\rho(\omega)} \\ S_L & \text{if } \lambda_\rho(\omega) > 0 \text{ and } \delta > \frac{r}{\gamma \lambda_\rho(\omega)}. \end{cases} \tag{4}$$

The director's optimal wage ω_B^* is given by:

$$\omega_B^* = \begin{cases} \rho + \psi & \text{if } r < q\gamma\bar{\delta}^2,\ 2(\rho - \underline{\omega}) < q\bar{\delta} - 3\psi \\ \underline{\omega} & \text{otherwise.} \end{cases}, \quad \text{where} \quad \psi = (q/\delta)^{\frac{1}{3}} \times (r/\gamma)^{\frac{2}{3}}. \tag{5}$$

The proof can be found in Appendix A. Intuitively, if an employee is non-reciprocal (i.e., $\gamma_i = 0$), he always chooses supplier H regardless of the wage and the price difference. By contrast, reciprocal employees (i.e. those for which $\gamma_i > 0$) always choose supplier H if the wage is below the reference wage; if the wage is above the reference wage, they employ a threshold strategy: they select supplier H for low price differences ($\delta \le \frac{r}{\gamma \lambda_\rho(\omega)}$) and supplier L otherwise. Based on the model primitives and on the employees' responses, the director will either choose to pay the minimum wage or she will incentivize pro-social behavior to achieve a lower procurement cost by offering $\omega_B^* = \rho + \psi$.

3.2. Peer Model

Consider now the case where E_2 observes E_1's decision before making his own. In this context we refer to E_1 and E_2 as the *observed* and the *observer* employees, respectively. As E_2 can perfectly observe E_1's monetary payoff, we assume that he incorporates distributional preferences in his utility, which becomes

$$u_2(\omega,\delta,\gamma_i,\sigma_2)=\pi_2+\underbrace{\gamma_2\cdot R_\rho(\omega,\delta,\sigma_2)}_{\text{reciprocity}}+\underbrace{(\pi_1-\pi_2)\cdot\left(\alpha\cdot\mathbb{1}_{\{\pi_2>\pi_1\}}-\beta\cdot\mathbb{1}_{\{\pi_2<\pi_1\}}\right)}_{\text{peer-effects}}, \tag{6}$$

where α and β are non-negative constants, $R_\rho(\omega,\delta,\sigma_2)$ is defined as in Equation (1), and π_i is a shorthand for the function $\pi_i(\omega,\sigma_i)=\omega+r\cdot\mathbb{1}_{\{\sigma_i=S_H\}}$, representing the monetary payoff. As in Ho and Su (2009) and Ho et al. (2014), we assume that only employee 2 incorporates distributional concerns as he is the only one who can perfectly infer monetary payoffs. Thus, the utility functions of the director and employee 1 remain unchanged. Moreover, following Fehr and Schmidt (1999) we restrict our attention to distributional preferences for "difference aversion"; i.e., α and β reflect the strength of E_2's aversion to advantageous and disadvantageous income inequality, respectively. Previous related work on trilateral gift-exchange games finds estimates for α and β in $[0,1]$ (Gächter et al. 2013, Thöni and Gächter 2015), and so we focus our analysis on α and β in this range.

Proposition 2 characterizes the equilibrium outcome of the peer model. As employee 2 now conditions his actions on the decision of employee 1, we solve for the equilibrium using backward induction in three steps: we first solve for the strategy of employee 1, then for the strategy of employee 2 given the strategy of employee 1, and, finally, for the director's decision.

Proposition 2. *Employee 1's optimal strategy σ_1 is as characterized in Proposition 1. Suppose that $\alpha,\beta\in[0,1]$. Then, given wage ω, price difference δ, and employee 1's optimal strategy σ_1, employee 2's optimal strategy can be characterized based on his reciprocity coefficient as follows:*

$$\sigma_2(\omega,\delta,0,\sigma_1)=S_H,\quad \text{and}\quad \sigma_2(\omega,\delta,\gamma,\sigma_1)=\begin{cases} S_H \text{ if } \lambda_\rho(\omega)\le 0 \text{ or } \lambda_\rho(\omega)>0 \text{ and } \delta<\frac{r}{\gamma\lambda_\rho(\omega)},\\ S_H \text{ if } \lambda_\rho(\omega)>0,\delta\in\left[\frac{r}{\gamma\lambda_\rho(\omega)},\frac{r(1+\beta)}{\gamma\lambda_\rho(\omega)}\right] \text{ and } \sigma_1=S_H,\\ S_L \text{ if } \lambda_\rho(\omega)>0,\delta\in\left[\frac{r}{\gamma\lambda_\rho(\omega)},\frac{r(1+\beta)}{\gamma\lambda_\rho(\omega)}\right] \text{ and } \sigma_1=S_L,\\ S_L \text{ if } \lambda_\rho(\omega)>0,\delta>\frac{r\cdot(1+\beta)}{\gamma\lambda_\rho(\omega)}.\end{cases} \tag{7}$$

Finally, let ψ be defined as in Equation (5) and define

$$\xi=\left[\frac{(1+q)}{2}\right]^{\frac{1}{3}},\quad \zeta=\left[\frac{(1+q)+(1+\beta)^2\cdot(1-q)}{2}\right]^{\frac{1}{3}}.$$

Then, the optimal wage ω_P^* is given by:

$$\omega_P^*=\begin{cases} \rho+\psi\zeta \text{ if } \xi\in\left(\frac{r}{\gamma\bar{\delta}\psi},\frac{r(1+\beta)}{\gamma\bar{\delta}\psi}\right),\ \zeta\in\left(\frac{r(1+\beta)}{\gamma\bar{\delta}\psi},\infty\right),\ 2(\rho-\underline{\omega})<q\bar{\delta}-3\psi\zeta,\ 3\psi(\zeta-\xi)<\frac{q(1-q)\bar{\delta}}{2}\\ \rho+\psi\zeta \text{ if } \xi\notin\left(\frac{r}{\gamma\bar{\delta}\psi},\frac{r(1+\beta)}{\gamma\bar{\delta}\psi}\right),\ \zeta\in\left(\frac{r(1+\beta)}{\gamma\bar{\delta}\psi},\infty\right),\ 2(\rho-\underline{\omega})<q\bar{\delta}-3\psi\zeta\\ \rho+\psi\xi \text{ if } \xi\in\left(\frac{r}{\gamma\bar{\delta}\psi},\frac{r(1+\beta)}{\gamma\bar{\delta}\psi}\right),\ \zeta\in\left(\frac{r(1+\beta)}{\gamma\bar{\delta}\psi},\infty\right),\ 2(\rho-\underline{\omega})<\frac{q(1+q)\bar{\delta}}{2}-3\psi\xi,\ 3\psi(\zeta-\xi)\ge\frac{q(1-q)\bar{\delta}}{2}\\ \rho+\psi\xi \text{ if } \xi\in\left(\frac{r}{\gamma\bar{\delta}\psi},\frac{r(1+\beta)}{\gamma\bar{\delta}\psi}\right),\ \zeta\le\frac{r(1+\beta)}{\gamma\bar{\delta}\psi},\ 2(\rho-\underline{\omega})<\frac{q(1+q)\bar{\delta}}{2}-3\psi\xi,\\ \underline{\omega} \quad \text{otherwise.}\end{cases} \tag{8}$$

The proof can be found in Appendix A. The equilibrium strategy for employee 1 is equal to that in the baseline model, but now employee 2 uses a different strategy. If employee 2 is non-reciprocal ($\gamma_i = 0$), he always chooses supplier H. If employee 2 is reciprocal ($\gamma_i > 0$) he always chooses supplier H if the wage is below the reference wage, and uses a threshold strategy when the wage is above the reference wage: if the price difference is below $\frac{r}{\gamma\lambda_\rho(\omega)}$, then he chooses supplier H; if the price difference is above $\frac{r\cdot(1+\beta)}{\gamma\lambda_\rho(\omega)}$, then he chooses supplier L; finally, if the price difference is in the intermediate region (i.e., $\delta \in \left[\frac{r}{\gamma\lambda_\rho(\omega)}, \frac{r(1+\beta)}{\gamma\lambda_\rho(\omega)}\right]$), then employee 2 mimics employee 1's choice. We refer to the latter region as the *negative spillover* region as, in this region, reciprocal employees choose supplier H when they observe their peer did so, whereas in the absence of transparency reciprocal employees always choose supplier L. Finally, and similarly to the baseline case, the director will either choose to pay the minimum wage or she will incentivize pro-social behavior to achieve a lower procurement cost by offering either $\rho + \psi\xi$ or $\rho + \psi\zeta$, depending on the model primitives.

Note that a direct consequence of $\alpha \leq 1$ is the absence of positive spillover effects when E_1 is reciprocal and E_2 is non-reciprocal. The reason is that the benefit from choosing the expensive supplier, r, outweighs the harm from the aversion to advantageous income inequality ($r\alpha$).

In Figure 2 we summarize the equilibria described in Propositions 1 and 2 in the case where both ω_B^* and ω_P^* are above ρ and $\omega_B^* \leq \omega_P^*$. Since the director knows neither the price difference δ nor the reciprocity type of each employee, the figure shows what she expects for each possible combination of γ_1 and γ_2 and each price difference. We observe that the most relevant difference between the two columns is the decision made by E_2 in the region $\left[\frac{r}{\gamma\lambda(\omega_B^*)}, \frac{r\cdot(1+\beta)}{\gamma\lambda(\omega_P^*)}\right]$ when $(\gamma_1, \gamma_2) = (0, \gamma)$ and[8] $\beta > 0$. In the baseline model E_2 chooses S_L, while in the peer model he chooses S_H. We refer to this as the *negative spillover region*, since a non-reciprocal E_1 can negatively influence a reciprocal E_2.

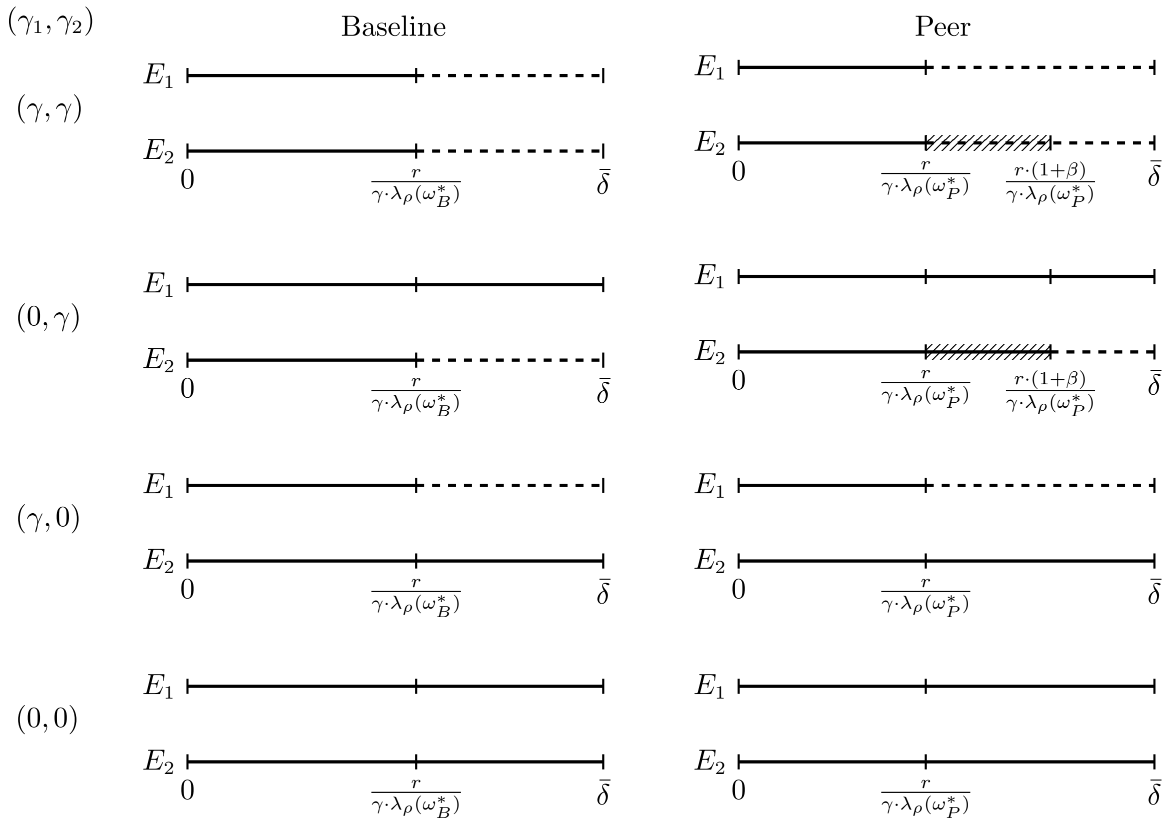
Remark 1. (The role of heterogeneity in reciprocity towards the director) The aforementioned differences disappear if $\gamma_L = \gamma_H = \gamma$, as $\omega_B^* = \omega_P^*$ and all the cases reduce to $(\gamma_1, \gamma_2) = (\gamma, \gamma)$.

4. Experimental Design

To test the predictions derived from our theoretical model, we designed a computerized laboratory experiment consisting of a *procurement game* that reproduces the game presented in the theory section. At the beginning of each session, subjects are randomly assigned to the role of director or employee, and they keep their role for the entire session. Subjects then play six rounds of the procurement game, where the sequence of events is as follows. At the beginning of each round, subjects are randomly and anonymously matched into an organization consisting of one director

[8] Considering $\beta \geq 0$ ensures that $\frac{r}{\gamma\lambda(\omega_B^*)} < \frac{r\cdot(1+\beta)}{\gamma\lambda(\omega_P^*)}$. Otherwise, if $\beta = 0$ then $\zeta = 1$ and $\omega_B^* = \omega_P^*$.

Figure 2 Equilibrium Comparison: Baseline vs. Peer



Note: The left column represents the equilibrium in the baseline model, while the right column corresponds to the equilibrium in the peer model. For each combination of (γ_1, γ_2), each sub-figure includes two lines that represent the range of possible price differences. The top line illustrates the best response of E_1, while the bottom line represents E_2's best response. The solid and dashed lines represent the regions of price differences δ where the employees select S_H and S_L respectively. Finally, the slashed area represents the region where E_2 follows E_1, i.e., chooses S_H if he observes that E_1 chose S_H, and chooses S_L if E_1 chose S_L.

and two employees (employee 1 and employee 2).[9] The random re-matching in between rounds prevents punishment or reputation effects from carrying over from one round to the next. After the matching occurs, the director chooses a wage of either 25 or 40 points that is paid to both employees. Each employee then separately chooses between supplier L, whose price is $p_L = 20$, and supplier H, whose price is[10] $p_H = p_L + \delta$. The price difference between suppliers, δ, is randomly determined and takes one of three values, 10, 25, or 40, all with equal probability. As in the theoretical model, choosing supplier H results in an additional benefit for the employees, which is set to $r = 10$ points. We elicited employees' decisions by having them follow the strategy method

[9] In the instructions we refer to the employees as "employee A" and "employee B" respectively, to avoid inducing any perceptions of hierarchy.

[10] In the experiment, suppliers H and L are labeled supplier A and B respectively.

so that they would provide a full contingency plan, i.e., a decision for each combination of wage the director might offer them $\omega \in \{25, 40\}$ and price difference that might be randomly realized $\delta \in \{10, 25, 40\}$—six decisions in total.[11] The strategy method has the advantage that it allows us to elicit subjects' complete strategies, including their choices under those scenarios that arise less frequently in the experiment. In addition, previous literature has shown that subjects' strategies do not change significantly under the strategy method relative to the direct-response method (Brandts and Charness 2011), and this especially holds in the case of gift-exchange games (Falk and Kosfeld 2006, Gächter et al. 2013), which are similar to our procurement game.

At the end of each round, a price difference δ is randomly chosen by the computer for each organization and each subject's payoff is computed. The director's payoff is

$$\pi_D = 200 - 2 \cdot \omega - p_{\sigma_1(\omega,\delta)} - p_{\sigma_2(\omega,\delta)},$$

where $p_{\sigma_i(\omega,\delta)}$ is the price of the supplier selected by E_i, $i \in \{1, 2\}$, and each employee's payoff is

$$\pi_i = 50 + \omega + 10 \cdot \mathbb{1}_{\{\sigma_i(\omega,\delta)=S_H\}}.$$

Note that, to prevent negative payoffs and associated loss aversion effects, the director starts with an initial endowment of 200 points, while each employee starts with 50 points. After each round, the director learns the realized price difference δ between the suppliers, the decisions $\sigma_1(\omega, \delta)$ and $\sigma_2(\omega, \delta)$ made by each employee (based on the wage chosen by the director and the realized price difference), and her own total profit. Similarly, the employees learn the wage chosen by the director, the realized price difference, and their own payoff in the round.

Our experimental design consists of two treatments. In the *baseline* treatment both employees choose a supplier simultaneously (without observing each other's choices), as in the theoretical model. In the *peer* treatment, the employees make their decisions sequentially, with E_1 choosing first and E_2 *observing* E_1*'s choice in each situation* (i.e., for each pair ω, δ) before making his own decision. To keep a clear distinction between the roles of observed and observer employees, subjects play either in the role of E_1 or E_2 throughout a session. At the end of the experiment, one of the six rounds of the procurement game is randomly selected for payment and subjects are paid \$0.10 per point earned in that round.

We make the following important design considerations. 1) The values of the parameters for the price difference and the extra reward are carefully chosen to capture different focal circumstances. When $\delta = 10$, choosing either supplier results in the same total surplus (recall that $r = 10$), and

[11] We also included $\delta = 0$ to check for consistency. We omit these results from the analysis because all subjects in the baseline chose the expensive supplier, as expected.

the employee faces the dilemma of benefiting himself or the director. The values of $\delta = 25$ and 40 capture settings where choosing S_H maximizes the employees' own monetary payoff but is inefficient in terms of total surplus. In addition, the reward the employee gets from choosing the expensive supplier is relatively low compared to his wage (it is at most half the wage). This is consistent with our motivating examples, where the extra benefit an employee gets from delegated procurement is not his main source of compensation. 2) The director chooses between two wages, $\omega \in \{25, 40\}$, rather than from a continuum of possible wages. We make this simplification for two reasons: first, a simple choice set for the director allows us to use the strategy method for the employees' decisions, which are our main focus. Second, both wage alternatives, 25 and 40, are significantly higher than the reward and are thus likely to be perceived by the employees as being higher than the reference (fair) wage.[12] 3) The initial endowments are chosen so that there is large asymmetry between the director's and the employees' endowments. This is intended to emulate the actual relation between a large organization and its employees.

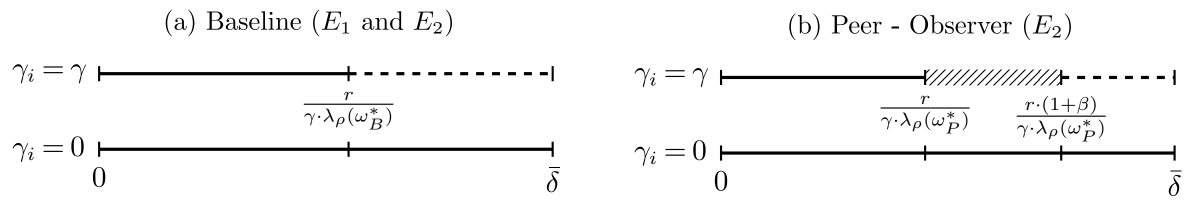
Additional Trust Game. After playing six rounds of the *procurement game*, subjects participate in an additional *trust game* (Berg et al. 1995). The trust game aims to measure preferences for trust and reciprocity. Since reciprocity is one of the main behavioral drivers in our model, we will use the outcome of this game to construct an exogenous measure of a subject's reciprocity. In this game, a sender and a receiver are initially endowed with 10 points. The sender moves first and decides how much of his endowment to send to the receiver. The amount sent is tripled by the experimenter. The receiver moves second and decides how much of the amount received to return to the sender. Following the strategy method, subjects make decisions as senders (how much to send) and as receivers (how much to return for each possible amount received, ranging from 0 to 30 in increments of 3 points). Subjects are then randomly matched and assigned a role for payment, which consists of \$0.10 per point earned. Only subjects who are assigned the role of employee in the procurement game participate in the trust game.

4.1. Hypotheses

Based on the predictions of the theoretical model, we derive the following hypotheses for the baseline and peer treatments. First, we expect reciprocal employees to be less likely to choose the expensive supplier compared to non-reciprocal employees, both in the baseline and peer treatments. The theory predicts that, while non-reciprocal employees choose the expensive supplier regardless

[12] The conjecture that both wage alternatives, 25 and 40, are (at least to some extent) perceived as "fair" is later confirmed by our experimental results. Recall that if $\omega = 25$ was not perceived as fair, our model of reciprocity predicts that all employees would choose S_H, regardless of the price difference between suppliers. On the contrary, we observe heterogeneity in employees' decisions when the wage is 25, with some of them choosing S_H (particularly when the price difference between suppliers is high).

Figure 3 Best Response: Baseline vs. Observer in Peer



Note: Figures 3a and 3b show the best-response function of employees in the baseline model (E_1 and E_2) and of observers (E_2) in the peer model respectively. Each figure includes two lines that represent the range of possible price differences. The top line illustrates the best response for reciprocal employees ($\gamma_i = \gamma$), while the bottom line corresponds to non-reciprocal employees ($\gamma_i = 0$). The solid and dashed areas represent the regions of price differences δ where the employees always select S_H and S_L respectively. The slashed area represents the region where E_2 follows E_1, i.e. chooses S_H if he observes that E_1 chose S_H, and chooses S_L if E_1 chose S_L.

of the wage and the price difference between suppliers, reciprocal employees are less likely to choose the expensive supplier as the wage and price difference increase. Therefore, we distinguish between reciprocal and non-reciprocal employees, based on the exogenous measure of reciprocity elicited in the trust game. We expect that the difference between reciprocal and non-reciprocal employees should be higher when the wage and the price difference are high.

Hypothesis 1 **(Effect of Reciprocity)**. *Reciprocal employees are less likely to choose S_H than non-reciprocal employees. The difference between reciprocal and non-reciprocal employees is higher when the wage and the price difference between suppliers are high.*

Our theoretical model also predicts changes in employees' behavior when transparency is introduced (peer treatment). We first consider the effects of transparency on employees who *observe* a peer's decision before making their own decision. The theory predicts the existence of negative spillovers, by which E_2 is more likely to choose S_H when he observes that E_1 chose S_H. Furthermore, our model specifies the behavioral mechanisms leading to this result. Figure 3 shows the best-response functions of E_1 and E_2 in the baseline (Figure 3a) and of E_2 in the peer treatment (Figure 3b), where the top lines correspond to reciprocal employees and the bottom lines correspond to non-reciprocal employees. The model predicts that a reciprocal E_2 in the peer treatment behaves differently depending on what he observes. Specifically, E_2 mimics E_1's decision when the price difference is between $\frac{r}{\gamma \cdot \lambda_\rho(w_P^*)}$ and $\frac{r(1+\beta)}{\gamma \cdot \lambda_\rho(w_P^*)}$ (slashed region in the top line of Figure 3b). Thus, we expect that a reciprocal E_2 in the peer treatment is more likely to choose the expensive supplier if he observes that the peer chose S_H than if he observes that the peer chose S_L. In addition, since in the slashed region a reciprocal E_2 in the peer treatment mimics the decision of E_1, the region where a reciprocal E_2 chooses S_H if he observes that the peer chose S_H is larger than the region where a reciprocal employee in the baseline chooses S_H. Therefore, we expect that the probability of choosing S_H is higher for a reciprocal employee who observes that his peer chose S_H than for a reciprocal employee in the baseline.[13] Hypothesis 2 summarizes these predictions.

HYPOTHESIS 2 **(Peer Effects on Observer Employees)**. *Observing that a peer chose S_H results in negative spillovers on a reciprocal E_2:*

(a) The probability of choosing S_H is higher for a reciprocal employee that observes that his peer chose S_H than for a reciprocal employee that observes that his peer chose S_L.

(b) The probability of choosing S_H is higher for a reciprocal employee that observes that his peer chose S_H in the peer treatment than for a reciprocal employee in the baseline.

Our theory predicts two additional results related to observer employees. First, an *absence of positive spillovers* on reciprocal employees; that is, a reciprocal E2 is not more likely to choose S_L when he observes that his peer did so compared to the baseline treatment. Second, an *absence of peer effects on non-reciprocal employees*—that is, a non-reciprocal E_2 who observes that his peer chose S_H is equally likely to choose S_H as a non-reciprocal E_2 who observes that his peer chose S_L (as shown by the bottom lines in Figure 3).

The final hypothesis focuses on the behavior of *observed* employees. The theoretical model predicts that there will be no peer effects on observed employees, as E_1's behavior remains unchanged in the peer treatment relative to the baseline.

HYPOTHESIS 3 **(Absence of Peer Effects on Observed Employees)**. *E_1 in the peer treatment is equally likely to choose S_H as an employee in the baseline treatment.*

5. Experimental Results

The experiment was conducted using z-Tree (Fischbacher 2007) at a public university in the Midwest of the USA, between September and November of 2017.[14] Average payoffs were \$15 (including a \$5 show-up fee) and each session lasted approximately one hour. In total, 165 students participated in the experiment, and no subject participated in more than one session.[15] Of these students, 48 participated in four sessions of the baseline treatment and 117 in ten sessions of the peer treatment.

[13] This should especially hold when the observer's income inequality aversion, β, is sufficiently high, as this increases the length of the interval where the negative spillovers occur.

[14] Subjects were undergraduate students. Average age was 21.98, 57.27% were female and 42.73% were male, and 17.27% were economics or business majors and 82.73% were other majors.

[15] Subjects were recruited using the online recruiting system ORSEE (Greiner 2004).

5.1. Preliminaries and General Results

As described in Section 4, we used the strategy method to elicit employees' decisions for each wage $\omega \in \{25, 40\}$ and for each price difference $\delta \in \{10, 25, 40\}$. Considering all the possible combinations of these parameters we obtain six situations, which we order lexicographically by wage and later by price difference.[16] We denote by $\sigma_{ist} \in \{S_H, S_L\}$ the decision made by employee i in situation s in round t, and we record it as a binary variable y_{ist} such that[17]

$$y_{ist} = \begin{cases} 1 \text{ if } \sigma_{ist} = S_H \\ 0 \text{ if } \sigma_{ist} = S_L. \end{cases}$$

For some of the analysis (indicated in the corresponding cases), we use the subject-level average decision (sometimes in a particular role or condition), $\bar{y}_{is}$, as an estimator of the overall probability that subject i chooses the expensive supplier in situation s.

Appendix C describes the general results, which we next summarize. Since the game in the baseline treatment is symmetric, we expect to find no differences in a subject's behavior in the roles of E_1 and E_2. This result is confirmed by the tests in Table 20 for all situations. Since there are no significant differences, for the rest of the analysis we pool the data from E_1 and E_2 in the baseline treatment. In the peer treatment, the probability of choosing the expensive supplier (S_H) is different depending on the role played (Table 21). More specifically, we find that E_2 is more likely to choose S_H compared to E_1, and these differences are significant in all cases where $\delta \geq 25$. Given these differences, in the peer treatment we analyze separately the behavior of employees who are *observed* (E_1) from those who are *observers* (E_2).

When we analyze the frequency of choosing the expensive supplier, S_H, aggregated at the subject level, we find that the probability of choosing the expensive supplier is decreasing in the wage offered by the director and in the price difference between suppliers, in both treatments (Appendix C). In the baseline treatment, the effect of wage is significant under all price differences, and the effect of price difference is significant, both when the wage is 25 and 40. In the peer treatment the probability of choosing S_H is decreasing in ω (statistically significant for observers and observed separately when $\delta = 25$ and marginally significant for observers when $\delta = 40$) and in price difference for both wages, for observed and observers separately.

In the next subsections we test our hypotheses, organizing the results as follows: Section 5.2 focuses on the effect of reciprocity, while Sections 5.3 and 5.4 examine the effect of transparency on *observer* and *observed* employees, respectively.

[16] That is, situations 1 to 3 consider a wage of 25 and an increasing price difference, and situations 4 to 6 consider a wage of 40 and an increasing price difference.

[17] At no risk of confusion we will sometimes omit the subindices s and t.

5.2. Effect of Reciprocity

The predictions derived from the theoretical model rely on the assumption that employees are heterogeneous in their reciprocity towards the director. Specifically, we assume that employees are either reciprocal ($\gamma_i = \gamma > 0$) or non-reciprocal ($\gamma_i = 0$). As stated in Hypothesis 1, we expect that reciprocal employees are less likely to choose the expensive supplier compared to non-reciprocal employees. This should especially hold for high wages and price differences, as non-reciprocal employees are expected to choose S_H regardless of the wage and price difference, while reciprocal employees choose S_L if the wage and the price difference are sufficiently high.

To test how reciprocity affects subjects' behavior in the *procurement game*, we elicit subjects' individual level of intrinsic reciprocity with an additional *trust game* that participants play at the end of the session.[18] Based on their decisions in this game, we create a measure of reciprocity for each subject by taking the difference between the maximum and the minimum of the amount returned (as in Beer et al. 2018).[19] The metric of reciprocity ranges between 0 and 30 and its distribution (presented in Figure 6 in Appendix B) confirms that there is high heterogeneity among subjects. We then characterize subjects as non-reciprocal if their reciprocity is within the lowest 30th percentile[20] (less than or equal to 8) and as reciprocal otherwise.

Table 1 presents the probability of choosing the expensive supplier (aggregated at the subject level, $\bar{y}_{is}$) for reciprocal and non-reciprocal employees separately. The table suggests that non-reciprocal employees are more likely to choose S_H than reciprocal employees in all conditions (that is, employees in the baseline treatment, and observers and observed employees in the peer treatment), providing support for Hypothesis 1. In addition, as predicted by Hypothesis 1, the differences between reciprocal and non-reciprocal employees are increasing in wage and price difference. In particular, these differences are significant when the price difference is high ($\delta = 40$), when the wage is high ($\omega = 40$) and the price difference intermediate ($\delta = 25$), and for observer employees in the peer treatment whenever the price difference is intermediate ($\delta = 25$).

We confirm the previous results with regressions in Table 9 in Appendix B. The table shows the results of panel probit models with subject random effects for each wage and price difference considering an employee's choice (i.e., y_{ist}) as a dependent variable, and as independent variable a binary variable that takes value 1 if subjects are reciprocal and 0 otherwise. Unless otherwise stated,

[18] Since subjects play the *procurement game* before the *trust game*, we check that the behavior in the second game is not affected by the treatment or by the role played in the first game. We find no significant differences in either the amount sent or the amount returned for each amount received depending on subjects' condition: baseline, observed, and observer (see Table 8 in Appendix B).

[19] The minimum amount returned is measured considering all possible amounts sent, excluding 0.

[20] The percentile for the cutoff is chosen based on the distribution of the metric of reciprocity presented in Figure 6 in Appendix B. The results remain qualitatively unchanged if the cutoff is set at the 10th or 25th percentile.

Table 1 Comparison Between Reciprocal and Non-Reciprocal Employees by Condition

		Probability of choosing S_H					
		$\omega = 25$			$\omega = 40$		
		$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Baseline	Reciprocal	0.89 (0.25)	0.61 (0.39)	0.39 (0.43)	0.73 (0.37)	0.34 (0.41)	0.27 (0.36)
	Non-Reciprocal	0.97 (0.11)	0.87 (0.27)	0.80 (0.35)	0.95 (0.16)	0.87 (0.20)	0.73 (0.42)
	Difference (p-value)	0.403	0.061	**0.015**	0.054	**0.001**	**0.009**
Observed	Reciprocal	0.79 (0.32)	0.55 (0.36)	0.25 (0.32)	0.75 (0.37)	0.32 (0.31)	0.19 (0.24)
	Non-Reciprocal	0.85 (0.25)	0.74 (0.30)	0.62 (0.39)	0.83 (0.22)	0.61 (0.40)	0.56 (0.38)
	Difference (p-value)	0.665	0.104	**0.004**	0.899	**0.030**	**0.003**
Observer	Reciprocal	0.88 (0.18)	0.72 (0.26)	0.53 (0.36)	0.79 (0.30)	0.56 (0.35)	0.46 (0.38)
	Non-Reciprocal	0.95 (0.11)	0.93 (0.16)	0.88 (0.26)	0.95 (0.11)	0.85 (0.21)	0.83 (0.26)
	Difference (p-value)	0.235	**0.008**	**0.003**	0.096	**0.017**	**0.005**

Note: Standard errors reported in parentheses. We report the p-values of Wilcoxon rank-sum tests comparing reciprocal and non-reciprocal employees for each condition. Bold values represent significant differences at the 5% level.

all regressions have errors clustered at the session level and control for round and demographics.[21] Panels 1, 2, and 3 present the results separately for employees in the baseline treatment, observed employees in the peer treatment, and observer employees in the peer treatment. We find that the coefficients are negative for most conditions, confirming the hypothesis that reciprocal subjects are less likely to choose the expensive supplier. In addition, we observe that the result is significant in all situations among subjects in the baseline treatment, while it is significant when the price differences are high (i.e., $\delta \geq 25$) among observed and observer employees in the peer treatment.

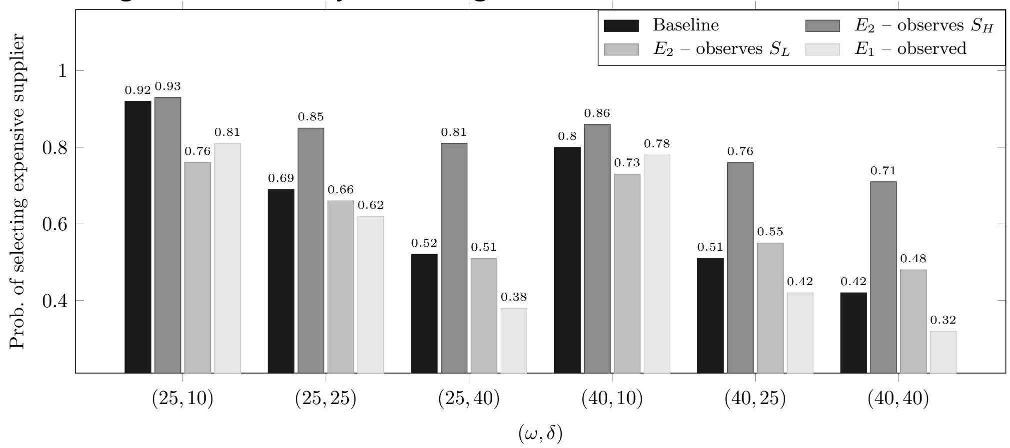
5.3. Spillover Effects

Hypothesis 2 predicts that observing that a peer chose S_H results in negative spillovers on a reciprocal E_2. To test this, we divide observers (E_2 in the peer treatment) into two subconditions: *observes H*, for those who observe that their peer chose supplier H; and *observes L*, for those who observe that their peer chose supplier L. Note that subjects in the role of E_2 (which is fixed throughout a session) may in some rounds observe that E_1 chose S_H and in other rounds observe that E_1 chose S_L. Therefore, for each E_2 in the peer treatment we compute the probability of choosing S_H when they observed that E_1 chose S_H and S_L separately, by taking the average of their decisions in all the rounds where they played in each of these two conditions respectively.

Figure 4 shows the estimated probability of choosing S_H for each pair (ω, δ) and condition ("baseline" and, in the peer treatment, "observes H," "observes L," and "observed"). In this section we focus on the analysis of the first three; the analysis of observed employees (E_1) in the peer

[21] Demographic controls include age, gender, race, income, and major.

Figure 4 **Probability of Choosing S_H across Treatments and Conditions**



treatment is presented in Section 5.4. The figure suggests that the probability of choosing S_H is higher for an employee who observes that the peer chose S_H than for an employee who observes that the peer chose S_L or for an employee in the baseline treatment, providing a first indication of the existence of negative spillovers on observer employees. Appendix D presents an analysis of how the probability of choosing S_H in each of these four conditions changes as rounds in a session elapse. Overall, we observe that the frequency of choosing S_H slightly increases with the rounds of play; however, in most conditions a steep increase occurs between rounds 1 and 2 and then remains relatively stable from round 2 onwards. In order to formally test the behavioral drivers derived from the theoretical model—the presence of negative spillovers on reciprocal observer employees (Hypotheses 2a and 2b)—we next examine separately the behavior of *reciprocal* and *non-reciprocal* observer employees in the peer treatment.

Hypothesis 2a predicts that a reciprocal employee who observes that his peer chose S_H is more likely to choose S_H than a reciprocal employee who observes that his peer chose S_L. Panel 1 in Table 2 pools data of all reciprocal observers in the peer treatment. The coefficients of the dummy variable "Observes H" are positive and significant for all wages and price differences, confirming that reciprocal employees are more likely to choose S_H if they observe that their peer chose S_H than if they observe that their peer chose S_L. This result provides support for Hypothesis 2a.

Hypothesis 2b predicts that a reciprocal employee who observes that his peer chose S_H is more likely to choose S_H than a reciprocal employee in the baseline treatment. Panel 2 in Table 2 presents the results of panel probit regressions with subject random effects, pooling the data of reciprocal employees in the baseline treatment and of reciprocal employees (E_2) in the peer treatment who observe that their peer chose S_H. For each situation (ω, δ), the dependent variable is the decision made by the employee in each round, y_{ist}, and the independent variable is a dummy that takes value 1 if the employee observes that S_H was chosen, and 0 if the employee is in the baseline treatment.

Table 2 Social Spillovers — Reciprocal Employees

Panel 1: Reciprocal — Observes H vs. Observes L

	Probability of choosing S_H					
	$\omega=25$			$\omega=40$		
	$\delta=10$	$\delta=25$	$\delta=40$	$\delta=10$	$\delta=25$	$\delta=40$
Observes H	0.566***	0.481*	0.846***	0.728**	0.576**	0.998***
	(0.117)	(0.280)	(0.319)	(0.327)	(0.293)	(0.318)
Constant	1.339	1.025	-0.061	1.931	1.120	1.178
	(0.911)	(0.664)	(1.113)	(1.450)	(0.787)	(1.067)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	174	174	174	174	174	174

Panel 2: Reciprocal — Baseline vs. Observes H

	Probability of choosing S_H					
	$\omega=25$			$\omega=40$		
	$\delta=10$	$\delta=25$	$\delta=40$	$\delta=10$	$\delta=25$	$\delta=40$
Observes H	-0.134	0.765	2.175**	0.582	1.626*	1.530**
	(0.525)	(0.521)	(0.980)	(0.405)	(0.833)	(0.662)
Constant	2.069	0.817	-0.832	1.308	-1.000	0.103
	(1.323)	(0.861)	(1.984)	(1.691)	(1.437)	(1.231)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	271	228	191	268	197	172

Note: Panel probit regressions with subject random effects. Standard errors clustered at the session level reported in parentheses. Panel 1 pools data from reciprocal employees who are observers in the peer treatment. Panel 2 pools data from reciprocal employees in the baseline and reciprocal employees who observe that their peer chose S_H in the peer treatment. We control for round and demographics. Significance reported: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$.

Note that the number of observations differ by situation as the frequency with which E_2 observes that his peer chose S_H changes with the wage and the price difference. We find that the coefficient of the dummy variable "Observes H" is positive and significant when $(\omega,\delta)\in\{(25,40),(40,40)\}$ and marginally significant when $(\omega,\delta)=(40,25)$, implying that the probability of choosing the expensive supplier after observing that the peer did so is significantly higher compared to the baseline, and that this holds particularly when the price difference is high. Hence, we conclude that Hypothesis 2b is supported by our data.

We test two additional predictions on observer employees derived from our theoretical model. First, we test the absence of positive spillovers on reciprocal employees. Table 10 in Appendix B pools data from reciprocal employees in the baseline treatment and reciprocal E_2 in the peer treatment who observe that the peer chose S_L. We find that the difference between these two conditions is not significant under any wage and price difference, confirming the absence of positive spillovers on reciprocal observer employees. Second, we test the absence of peer effects on non-reciprocal employees. The regressions in Table 11 in Appendix B pool data from non-reciprocal employees in the baseline and non-reciprocal E_2 in the peer treatment who observe S_H (Panel 1) and S_L (Panel 2). We find that among non-reciprocal employees, observing that the peer chose S_H results in a higher probability of choosing S_H in only one situation, while observing that the peer chose S_L does not result in a significant difference in the frequency of choosing S_H in any situation. These results confirm the absence of effects on non-reciprocal employees, for the most part.

Overall, the results in this section confirm that Hypothesis 2 is well supported by the experimental results, and that the underlying behavioral mechanisms obtained from the theoretical model explain the data well.[22]

[22] In Appendix D.2 we explore whether the behavior of E_2 in the peer treatment is affected by past observations of a peer. We find that E_2 primarily care about what they observe in the current round (over what they observed in previous rounds), suggesting that the peer effects arising in the current round are a more salient driver of behavior.

Table 3 Effect on Observed Employees

	Probability of choosing S_H					
	$\omega = 25$			$\omega = 40$		
	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Observed	-1.684***	-0.750**	-1.287*	-0.585	-0.413	-0.367
	(0.493)	(0.379)	(0.750)	(0.637)	(0.404)	(0.428)
Constant	0.208	-0.856	-1.990	-0.549	-1.988**	-1.993*
	(0.740)	(0.647)	(1.446)	(0.720)	(0.911)	(1.119)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	426	426	426	426	426	426

Note: Panel probit regressions with subject random effects. Standard errors clustered at the session level reported in parentheses. The table pools data from employees in the baseline and observed employees in the peer treatment. We control for round and demographics. Significance reported: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

5.4. Effects on Employees Who Are Observed by a Peer

Hypothesis 3 predicts no differences between employees in the baseline and observed employees in the peer treatment. However, as shown in Table 3, we find that the coefficient of the dummy variable "Observed," which is equal to 1 if the employee is observed in the peer treatment and 0 if the employee is in the baseline, is negative and significant when $\omega = 25$ for all δ. Furthermore, Figures 7a and 7b in Appendix D show a parallel downward shift in the frequency of choosing the expensive supplier for E_1 in the peer treatment relative to the baseline as rounds in a session elapse, suggesting that the positive effect on observed employees remains steady over rounds. This implies that, when the wage is low, observed employees in the peer treatment are less likely to choose the expensive supplier compared to the employees in the baseline.

One possible explanation of this result is that observed employees anticipate the negative spillovers associated with the decision of choosing the expensive supplier; i.e., an observed employee choosing the expensive supplier generates the "extra punishment" for the director of increasing the probability that the employee who observes his action will choose the expensive supplier as well. This concern for inflicting a double punishment on the director should be higher among reciprocal employees who were offered a high wage. To test whether this explains the difference between observed employees in the peer treatment and employees in the baseline treatment, in Table 4 we report the results of interacting the dummy variable "Observed" with the employees' reciprocity. If the differences in the probability of choosing S_H between observed employees in the peer treatment and employees in the baseline are driven mainly by reciprocal preferences, we would expect the differences in behavior to be significant among reciprocal employees (and to be particularly salient when the wage is high). However, we observe that the difference among reciprocal employees is only significant when $(\omega, \delta) = (25, 10)$. By contrast, the lower probability of an observed employee choosing S_H relative to the peer is significant in all situations where $\delta < 40$ among non-reciprocal employees. This suggests that non-reciprocal employees that are observed by a peer are significantly

Table 4 Effect on Observed Employees — Reciprocity

	Probability of choosing S_H					
	$\omega = 25$			$\omega = 40$		
	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Baseline × Recip	-2.208***	-2.111***	-3.905***	-3.166***	-3.005***	-2.383***
	(0.432)	(0.482)	(0.929)	(0.526)	(0.544)	(0.638)
Observed × Recip	-3.671***	-2.564***	-4.744***	-3.293***	-2.908***	-2.630***
	(0.766)	(0.524)	(1.114)	(0.662)	(0.402)	(0.559)
Observed × Non-Recip	-2.987***	-1.595**	-2.249*	-2.820***	-1.457***	-0.707
	(0.809)	(0.711)	(1.273)	(0.784)	(0.558)	(0.678)
Constant	1.261*	0.196	0.004	1.219	-0.341	-0.751
	(0.765)	(0.723)	(1.101)	(0.755)	(0.632)	(1.045)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	426	426	426	426	426	426
	Tests (p-value)					
(1) Baseline = Observed | Recip	**0.008**	0.200	0.133	0.834	0.855	0.611
(2) Baseline = Observed | Non-Recip	**0.000**	**0.050**	0.155	**0.001**	**0.018**	0.595

Note: Panel probit regressions with subject random effects. Standard errors clustered at the session level reported in parentheses. The table pools data from employees in the baseline and observed employees in the peer treatment. Bold values represent significant differences at the 5% level. We adjust p-values using the Holm method (Holm 1979) for multiple-hypothesis testing. We control for round and demographics. Significance reported: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

less likely to choose the expensive supplier compared to non-reciprocal employees in the baseline (tests 1 and 2 in Table 4).

Overall, our results show that there are significant differences in behavior between employees in the baseline and those employees whose decisions are observed in the peer treatment. This is especially true when the wage is low and employees are non-reciprocal, suggesting that the result is not driven by reciprocal employees' concern to avoid inflicting a double punishment on the director.

5.4.1. Alternative Model: Compliance with the Social Norm In this section we explore an alternative explanation of the difference in the behavior of non-reciprocal observed employees in the peer treatment relative to the no-transparency baseline case. We consider that observed employees may be less likely to choose the expensive supplier due to a desire to comply with *social norms*—defined as collective agreements about the appropriateness of different behaviors or actions (Fehr and Gächter 2000, Krupka and Weber 2013). In particular, we focus on *injunctive norms*, which are defined as what one "ought" to do, rather than *descriptive norms*, which are customs or actions that people regularly take (Krupka et al. 2016). Previous literature has explored the role of preferences for compliance with the social norm of appropriate behavior in settings related to ours, with a focus on its effect on the behavior on an *observer* of a peer's action (Gächter et al. 2013, Gächter et al. 2017). Building on this literature, we study whether a preference for compliance with the social norm provides a plausible explanation of the behavior of an *observed* employee. In particular, we conjecture that the appropriateness of choosing the expensive supplier changes when transparency is introduced. Note that the social norm of appropriate behavior is highly dependent on the context (Gächter et al. 2017). If transparency affects the social norm (i.e., choosing the expensive supplier is perceived as less appropriate when an employees is observed by a peer), a model that incorporates an observed employee's preference for compliance with the social norm of appropriate behavior could explain why observed employees in the peer treatment are less likely to choose the expensive supplier compared to the baseline. In addition, while one could conjecture that the social norm may incorporate reciprocal considerations (i.e., it is less appropriate to choose the expensive supplier when the wage and the price difference are high), the social norm should be empirically elicited as this may not necessarily be the case. In fact a social norm that (at least to some extent) deviates from reciprocity may explain why the difference in the behavior of observed employees is mostly present among non-reciprocal employees.

Model. We consider non-reciprocal employees' preference for compliance with the social norm as follows. Let $N : [\underline{\omega}, \infty) \times \left[0, \bar{\delta}\right] \times \{S_H, S_L\} \to \mathbb{R}_+$ be a function such that $N(\omega, \delta, \sigma_i)$ represents the perceived social appropriateness of choosing supplier σ_i when the wage is ω and the price difference is δ. Then, the utility of employee i is

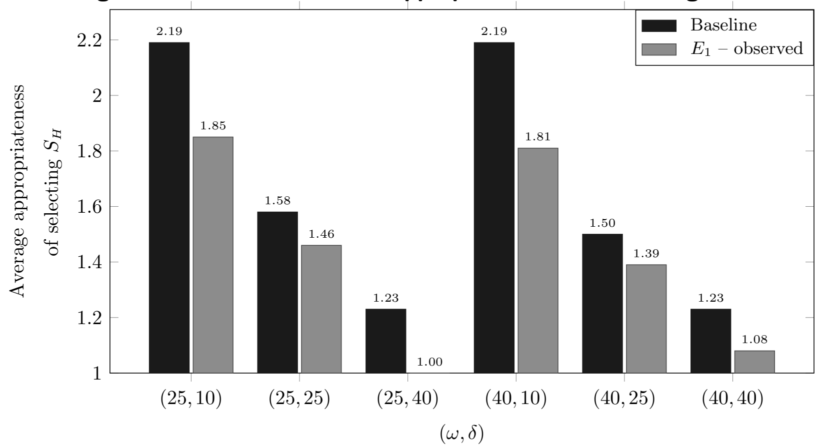
$$u_i(\omega, \delta, \sigma_i) = \omega + r \cdot \mathbb{1}_{\{\sigma_i = S_H\}} + \varphi_i \cdot N(\omega, \delta, \sigma_i), \tag{9}$$

where φ_i represents employee i's preference for complying with the social norm. Note that this utility function is the same for employees in the baseline and for those who are observed in the peer model. However, we conjecture that the social appropriateness of choosing the expensive supplier changes when transparency is introduced. Specifically, we expect that it is less appropriate for an employee to choose the expensive supplier if his decision is observed by a peer. To test this, the social norm needs to be *empirically derived* under each of these settings.

Hypothesis 4 **(Effect of Transparency on the Social Norm)**. *The social appropriateness of choosing S_H is lower when the employee's decision is observed by a peer than when there is no observability of employees' actions (baseline treatment).*

Social Norm Elicitation. To test Hypothesis 4, we design two incentivized norm elicitation treatments. In the first treatment, subjects are given a description of the setting of the procurement game baseline treatment, and in the second treatment, they are given a description of the setting of the procurement game peer treatment. After the setting is described, participants evaluate how socially appropriate it is to choose the expensive supplier for each situation $(\omega, \delta) \in \{25, 40\} \times \{10, 25, 40\}$, rating it as "very socially inappropriate," "somewhat socially inappropriate,"

Figure 5 Social Norm — Appropriateness of Choosing S_H



"somewhat socially appropriate," or "very socially appropriate."[23] For analysis, we later translate these answers into an appropriateness scale ranging from 1 to 4, corresponding to each of the four ratings, respectively.

To avoid experimenter demand effects, we use a between-subject design (subjects who participate in one norm elicitation treatment do not participate in the other) and we use the neutral labels "Employee A" and "Employee B," as in the original procurement game.

We incentivize the norm elicitation treatments following the procedure in Krupka and Weber (2013), by offering participants an extra \$10 (in addition to the \$5 show-up fee) if their rating in a randomly selected situation coincides with the mode among all participants' ratings in the session. This coordination game incentivizes participants to respond what they perceive is the most socially accepted answer rather than what they personally believe is most appropriate.

Results. A total of 52 students participated in the norm elicitation treatments; 26 of them participated in the *baseline norm elicitation* treatment, and 26 in the *peer norm elicitation* treatment. The number of subjects in each session was between 4 and 8, and each session lasted approximately 30 minutes.

Figure 5 shows the average social appropriateness of choosing supplier H for an employee in the baseline norm elicitation treatment and for the observed employee in the peer norm elicitation treatment. First, we observe that the appropriateness is decreasing in the price difference (Kruskal–Wallis test; $p < 0.001$ for all wages and conditions considered), but it does not vary significantly with wage (Wilcoxon signed-rank test; $p > 0.113$ for all prices and for both baseline and "observed"). This suggests, that the price difference between suppliers has higher influence than

[23] For completeness we also elicited the social norm when the employee observes that another employee previously chose supplier H or supplier L. In addition, we also elicited the social appropriateness of choosing supplier L. The results are consistent with the results of the procurement game.

Table 5 Procurement Game vs. Social Norm — Observed Employees

		Probability of choosing S_H					
		$\omega = 25$			$\omega = 40$		
		$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Procurement Game	Observed	-1.684*** (0.493)	-0.750** (0.379)	-1.287* (0.750)	-0.585 (0.637)	-0.413 (0.404)	-0.367 (0.428)
	Observations	426	426	426	426	426	426
		Appropriateness of choosing S_H					
Social Norm	Observed	-0.689* (0.360)	-0.240*** (0.090)	-4.762*** (0.393)	-0.576*** (0.195)	-0.247 (0.182)	-0.151 (0.476)
	Observations	52	52	52	52	52	52

Note: The top panel shows the results from the *procurement game* previously reported in Table 3. The bottom panel corresponds to the norm elicitation treatments, and reports the estimates of ordered probit regressions with errors clustered at the session level reported in parentheses. The dependent variable is the social appropriateness of choosing S_H (in a scale from 1 to 4). The independent variable is a dummy that takes value 1 when subjects evaluate the appropriateness of observed employees' decisions in the peer norm elicitation, and 0 when subjects evaluate the appropriateness of employees' decisions in the baseline norm elicitation. Significance reported: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

the wage on the changes in the appropriateness of choosing the expensive supplier. Second, we observe that the social appropriateness of choosing S_H is significantly lower for employees who are "observed" in the peer treatment than it is for employees in the baseline. The bottom panel in Table 5 reports the estimates of an ordered probit model regressing subjects' ratings of the appropriateness of choosing S_H on the dummy variable "Observed."[24] We find that choosing the expensive supplier is significantly less appropriate for an observed employee when the wage and the price difference are low (the coefficient is significant for $(\omega, \delta) \in \{(25, 25), (25, 40), (40, 10)\}$ and marginally when $(\omega, \delta) \in (25, 10)$). The top panel in Table 5 presents a result from the procurement game previously reported in Table 3. It shows panel probit regressions of the decision variable y_{ist} on a dummy variable "Observed." Comparing the results in the top and bottom panels of the table we find that, whenever observed employees are significantly less likely to choose S_H than employees in the baseline (procurement game), it is significantly less appropriate to do so (norm elicitation treatments). Tables 13 and 14 in Appendix B compare the changes in behavior in the procurement game with the changes in the social norm *separately for non-reciprocal and reciprocal employees*, respectively. For non-reciprocal employees we confirm that whenever observed employees are significantly less likely to choose S_H than employees in the baseline, it is significantly less appropriate to do so. On the contrary, for reciprocal employees, the difference in behavior in the procurement game is only significant in one situation, which does not coincide with the significant changes in

[24] Table 12 in Appendix B reports the estimates of OLS models. The results are qualitatively equivalent to those obtained with the ordered probit models, but their significance is weaker because no subject rates choosing supplier H as "very socially appropriate" and, therefore, we effectively have only three categories.

the social norm. The fact that the situations where there are significant differences in the purchasing decisions of non-reciprocal employees coincide with the situations where there are significant differences in the appropriateness of choosing S_H suggests that preferences for compliance with the social norm provide a more compelling explanation of the differences in behavior between observed employees in the peer treatment and those in the baseline.

5.5. Weak and Strong Overlap of Effects

Our previous results suggest that two effects arise from increased transparency. The first is a positive spillover effect by which observed employees are less likely to choose the expensive supplier, which is more salient among non-reciprocal employees. The second is a negative spillover effect by which observer employees become more likely to choose the expensive supplier when they observe that their peer did so, which affects mostly reciprocal employees. Isolating these effects was possible because our peer treatment was designed so that a subject could either be observed by a peer or observe a peer's decision, but not both. We now turn to study the behavior of an employee who is both an observer of a peer's action and observed by a peer. To do so, we conduct two new treatments, described next.

In the first treatment, two employees make their decisions sequentially with E_1 choosing first and E_2 observing E_1's choice before making his own decision, as in the peer treatment. After E_2 has made a decision, E_1 observes E_2's choice but cannot update his own decision. The roles remain fixed and subjects are randomly re-matched into new organizations for the following round. Thus, in this treatment, E_1 is not only an observed employee but he is also a "weak" observer: his decision in round two and onward may be influenced by his previous observation of E_2. However, this observation is of a peer he is no longer paired with (and within an organization he is no longer part of) at the time of his next decision. Similarly, E_2 is not only an observer but he is also "weakly" observed, as he is observed by a peer who does not make a new purchasing decision within the same organization. We denote this treatment *Weak Overlap of Effects* (WOE). Note that this treatment also allows E_1 to learn about his peers' decisions as rounds in a session elapse, whereas an E_1 in the peer treatment received no feedback about the decisions of other subjects in the session.

In the second treatment we consider an organization consisting of a director and three employees, E_1, E_2, and E_3. Employees make their decisions sequentially, with E_1 choosing first, E_2 choosing second after observing E_1's decision, and E_3 choosing last after observing E_2's decision.[25] Subjects keep their role and are randomly re-matched for the following round. Note that E_2 in this

[25] To provide a cleaner comparison with the peer treatment, we allow for E_3 to observe E_2's decision but not E_1's, keeping constant that each employee observes and/or is observed by at most *one* peer.

treatment is both a "strong" observer and "strongly" observed: he observes E_1 who has made a purchasing decision within the same organization, and is observed by E_3 who will subsequently make a purchasing decision within the same organization. Therefore, we denote this treatment *Strong Overlap of Effects* (SOE).

5.5.1. Hypotheses To test the weak overlap of effects, we first compare the decisions of E_1 in the peer treatment and E_1 in the WOE treatment. In both cases, E_1 is observed but in the WOE treatment he is also a "weak" observer. Based on our previous results, we conjecture that E_1 in the WOE treatment who in the previous round observed that his peer chose the expensive supplier is more likely to choose S_H than E_1 in the peer treatment, and that these "weak" negative spillovers across rounds will be more prominent among reciprocal employees. We next compare the decisions of E_2 in the peer treatment with those of E_2 in the WOE treatment. Both employees are observers, but E_2 in the WOE treatment is also "weakly" observed. Thus, by our previous findings, we expect E_2 in the WOE treatment to be less likely to choose S_H than E_2 in the peer treatment, and this effect to be more salient among non-reciprocal employees.

HYPOTHESIS 5 **(Weak Overlap of Effects)**.

(a) E_1 in the WOE treatment who observes that the last E_2 he was paired with chose S_H is more likely to choose S_H than E_1 in the peer treatment. The difference is larger among reciprocal employees.

(b) E_2 in the WOE treatment is less likely to choose S_H than E_2 in the peer treatment. The difference is larger among non-reciprocal employees.

To study the strong overlap of effects, we first compare the decisions of E_2 in the SOE treatment who observed that E_1 chose S_H with E_1 in the peer treatment. Both employees are "strongly" observed by a peer (the former by E_3 and the latter by E_2) but E_2 in the SOE treatment is also a "strong" observer who observed that E_1 chose S_H. Based on our previous results, we expect that the former will choose S_H more frequently than the latter, and that this should hold particularly among reciprocal employees. We next compare E_2 in the SOE treatment with E_2 in the peer treatment. Both employees are "strong" observers, but E_2 in the SOE treatment is also being strongly observed (by E_3), which we expect will make him less likely to choose S_H. We expect this to hold particularly among non-reciprocal employees.

HYPOTHESIS 6 **(Strong Overlap of Effects)**.

(a) E_2 in the SOE treatment who observed that E_1 chose S_H is more likely choose S_H than E_1 in the peer treatment. The difference is larger among reciprocal employees.

(b) E_2 in the SOE treatment is less likely to choose S_H than E_2 in the peer treatment. The difference is larger among non-reciprocal employees.

Table 6 E1 Peer vs. E1 WOE

	Probability of choosing S_H					
	$\omega = 25$			$\omega = 40$		
	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Peer × Recip	-0.598	-0.550	-1.612**	0.046	-1.200***	-1.616***
	(0.364)	(0.370)	(0.644)	(0.471)	(0.314)	(0.465)
Observed H × Recip	0.076	0.363	-0.750	0.515	-0.776*	-1.085**
	(0.289)	(0.437)	(0.622)	(0.485)	(0.416)	(0.438)
Observed H × Non-Recip	1.164**	0.890**	0.753**	0.685***	0.614	0.332
	(0.496)	(0.398)	(0.376)	(0.249)	(0.399)	(0.406)
Observed L × Recip	-	0.872	-1.082*	-0.306	-1.913***	-2.667***
	-	(0.635)	(0.553)	(0.703)	(0.552)	(0.508)
Observed L × Non-Recip	-	-0.443	-0.410	0.400	-0.623*	-0.409*
	-	(0.610)	(0.375)	(0.790)	(0.330)	(0.233)
Constant	-0.957	-0.773	-1.023	0.296	-1.461	-1.165
	(0.687)	(0.895)	(1.319)	(1.037)	(0.991)	(1.086)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	449	456	456	445	456	456
	Tests (p-value)					
(1) Peer vs. Observed H | Recip	0.170	**0.018**	**0.019**	0.471	0.212	0.110
(2) Peer vs. Observed H | Non-Recip	0.056	0.077	0.135	**0.024**	0.124	0.414
(3) Peer vs. Observed L | Recip	0.101	0.061	0.148	1.000	0.177	**0.001**
(4) Peer vs. Observed L | Non-Recip	-	0.468	0.274	0.613	0.176	0.158

Note: Panel probit regressions with subject random effects. Standard errors clustered at the session level reported in parentheses. The table pools data for E_1 in the peer and WOE treatments. Bold values represent significant differences at the 5% level. We adjust p-values using the Holm method (Holm 1979) for multiple hypothesis testing. We control for round and demographics. Missing values for Observed L×Non-Recip and Observed L×Recip are due to perfect separation (these variables perfectly predict that S_H is chosen). Significance reported: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

5.5.2. Results We conducted the new treatments between February and April of 2019 at the same university and with the same subject pool as the original treatments. A total of 111 subjects participated in the WOE treatment and 136 subjects participated in the SOE treatment.

Hypothesis 5a predicts that E_1 in the WOE treatment who in the previous round observed that E_2 chose S_H is more likely to choose S_H than E_1 in the peer treatment, and that the difference is more prominent among reciprocal employees. In Table 6 we compare the probability of choosing the expensive supplier for E_1 in the peer treatment and for E_1 in the WOE treatment who in the previous round observed that E_2 chose S_H or S_L, distinguishing between reciprocal and non-reciprocal employees. Note that for the situation $(\omega, \delta) = (25, 10)$ we drop the cell corresponding to the case where E_2 observed S_L, due to lack of enough observations for each reciprocity type. We find that, among reciprocal employees, E_1 in the WOE treatment who observed that E_2 in the previous round chose S_H are more likely to choose S_H compared to E_1 in the peer treatment (this holds directionally for all situations and is significant in two of them; see test 1 in Table 6). Moreover, these differences are also present among non-reciprocal employees, but only in one situation (test 2).

Hypothesis 5b predicts that E_2 in the WOE treatment is less likely to choose S_H than E_2 in the peer treatment, and that the difference is more prominent among non-reciprocal employees.

Based on our previous results, we test this by conditioning on what E_2 observed. Panel 1 in Table 15 (Appendix B) compares the purchasing decisions of E_2 who observes that E_1 chose S_H in the peer and WOE treatments, interacted with their reciprocity. We observe no significant differences between E_2 who observed S_H in the peer and WOE treatments, for either reciprocal or non-reciprocal employees (panel 1, tests 1 and 2, respectively). Similarly, we find no significant differences (except in one situation) between E_2 who observed that their peer chose S_L across the peer and WOE treatments, for either reciprocal or non-reciprocal employees (panel 2 in Table 15, tests 1 and 2, respectively). Together, these results suggest that there are no positive effects of being "weakly" observed by a peer who will not make a decision in the current round, indicating that Hypothesis 5b is not supported.

Table 7 Peer vs. E2 SOE — Observes H

	Probability of choosing S_H											
	Panel 1: E_1 Peer vs. E_2 SOE						Panel 2: E_2 Peer vs. E_2 SOE					
	$\omega = 25$			$\omega = 40$			$\omega = 25$			$\omega = 40$		
	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Peer × Recip	-0.579*	-0.777**	-1.936**	-0.493	-1.255***	-1.592***	-0.863**	-1.242***	-2.049***	-1.305**	-1.734***	-1.337**
	(0.337)	(0.317)	(0.809)	(0.374)	(0.478)	(0.398)	(0.339)	(0.270)	(0.764)	(0.632)	(0.457)	(0.590)
SOE × Recip	0.611	0.305	0.297	-0.382	-0.322	-0.305	-0.675	-1.266***	-2.037***	-1.499**	-2.118***	-1.520***
	(0.480)	(0.415)	(0.852)	(0.420)	(0.726)	(0.564)	(0.420)	(0.403)	(0.669)	(0.748)	(0.463)	(0.460)
SOE × Non-Recip	2.622***	0.962*	0.809	0.864	0.611	-0.178	-0.109	-0.705	-1.604***	-0.619	-1.209***	-1.383***
	(0.500)	(0.537)	(0.775)	(0.689)	(0.782)	(0.508)	(0.496)	(0.431)	(0.558)	(0.731)	(0.463)	(0.416)
Constant	-1.545**	-1.093	-0.933	0.141	-0.481	-0.183	1.758***	1.831***	2.963**	3.835***	3.198***	3.043**
	(0.717)	(0.690)	(1.188)	(0.720)	(1.077)	(1.030)	(0.464)	(0.669)	(1.174)	(1.153)	(0.973)	(1.552)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	421	392	355	389	336	312	376	302	210	338	201	154
	Tests (p-value)											
(1) Peer vs. SOE | Recip	**0.034**	**0.002**	**0.002**	0.832	0.121	**0.011**	1.000	0.929	0.986	0.662	0.419	0.738
(2) Peer vs. SOE | Non-Recip	**0.000**	0.073	0.297	0.421	0.435	0.725	0.825	0.203	**0.008**	0.794	**0.018**	**0.002**

Note: Panel probit regressions with subject random effects. Standard errors clustered at the session level reported in parentheses. Panel 1 pools data from E_1 in the peer treatment and E_2 who observe S_H in the SOE treatment. Panel 2 pools data from E_2 who observe S_H in the peer treatment and E_2 who observe S_H in the SOE treatment. Bold values represent significant differences at the 5% level. We adjust p-values using the Holm method (Holm 1979) for multiple-hypothesis testing. We control for round and demographics. Significance reported: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

We next analyze the case of a strong overlap of effects. Hypothesis 6a predicts that E_2 in the SOE treatment who observed that E_1 chose S_H (and is observed by E_3) is more likely to choose S_H than E_1 in the peer treatment (who is observed but not an observer). Panel 1 in Table 7 presents the decisions of E_1 in the peer treatment and E_2 who observed that E_1 chose the expensive supplier in the SOE treatment, separating between reciprocal and non-reciprocal employees. The tests at the bottom of the table confirm that observed employees are more likely to choose the expensive supplier when they observe that a peer chose S_H, as predicted by Hypothesis 6a. Furthermore, this effect is more salient among reciprocal employees (significant in four situations) than among non-reciprocal employees (significant in one situation).[26] These results suggest that the positive effects of being observed are diminished by the negative spillovers associated with observing that a peer chose the expensive supplier, providing support for Hypothesis 6a.

Hypothesis 6b predicts that the negative spillovers on observers may be attenuated when employees are also observed by a peer. To obtain a cleaner comparison, we test this by conditioning on what E_2 observed. Panel 2 in Table 7 compares the decisions of E_2 who observes that E_1 chose S_H in the peer and SOE treatments, distinguishing between reciprocal and non-reciprocal employees. We find that E_2 in the SOE treatment (who are observed by E_3) are significantly less likely to choose the expensive supplier than E_2 in the peer treatment (who are not observed by a peer). In addition, this effect is only present among non-reciprocal employees (significant in three situations), consistent with our previous results. Table 17 in Appendix B shows that the positive effects of being observed by E_3 are also significant in two situations when we compare E_2 who observe that E_1 chose S_L in the SOE and in the peer treatments. Both these results provide support for Hypothesis 6b.

Overall, these two additional treatments allow us to extend the scope of our findings in the previous subsection. First, these treatments suggest that the negative spillovers associated with observing that a peer chose the expensive supplier also affect employees who are themselves observed. Second, the positive effects associated with being observed by a peer are also present in employees who observe that a peer chose the expensive supplier. This is particularly true in the case with *strong* overlap of effects. In addition, we find that the main behavioral mechanisms we identified in the peer treatment—that negative spillovers affect primarily reciprocal employees, and the positive effects from being observed affect non-reciprocal employees—are still present when an employee both observes and is obseved.

6. Discussion

The experimental results largely confirm the main prediction derived from our theoretical model: the existence of negative spillover effects, by which reciprocal employees are more likely to choose the expensive supplier after observing that their peer did so. Our results also confirm the underlying mechanisms leading to this effect: the heterogeneity in employees' preferences for reciprocity and the employees' aversion to disadvantageous income inequality. Our additional treatments also suggest that negative spillovers are present even when employees are (weak or strong) observers. We also find support for the theoretical prediction that increased transparency does not induce positive

[26] Table 16 in Appendix B compares E_2 in the SOE treatment who observe that E_1 chose S_L with E_1 in the peer treatment and confirms the absence of positive spillovers in this case.

spillovers; i.e., employees are not more likely to choose the cheaper supplier after observing that their peer did so.

In addition, the experimental results show that transparency has positive effects on employees whose decisions are observed by a peer, by which they are significantly less likely to choose the expensive supplier than the employees in the baseline. These effects appear to be consistent with a preference for compliance with the social norm, a result that is not anticipated by the theory and that has been mostly overlooked in previous literature on peer effects in related three-person gift exchange and trust games.[27] Furthermore, being observed by a peer also mitigates the negative spillovers arising from observing that a peer chose the expensive option in larger organizations, as shown by the analysis of the SOE treatment.[28]

Finally, while the theoretical model predicts that, as a result of the negative spillovers, the average procurement cost per employee should be higher in the peer treatment than in the baseline (Corollary 1 in Appendix A), the experimental results show no significant differences in the average cost per employee between the two treatments ($c_B = 60.208$ vs. $c_P = 61.314$; Wilcoxon rank-sum test; $p = 0.335$). This mismatch is due to the fact that our theory does not account for the positive effects of transparency on observed employees: while negative spillover effects work in the direction of increasing the procurement cost, the effects on observed employees work in the opposite direction and contribute to reducing it. In fact, when comparing the overall probability that an employee will choose the expensive supplier in the baseline and peer treatments (we look at the average across E_1 and E_2 in each treatment) we find that there is no significant difference, except in one situation where this probability is lower in the peer treatment; see Table 19 in Appendix B. Consistent with this observation, the experimental results show that the average wage is not significantly different across baseline and peer treatments ($w_B = 28.125$ vs. $\omega_P = 28.269$; Wilcoxon rank-sum test; $p = 0.915$).

We are also interested in understanding how the average procurement cost changes in organizations where the same employees can observe and be observed. We first compare the overall probability that an employee will choose the expensive supplier in the peer, WOE, and SOE treatments (we look at the average across E_1 and E_2 in the peer and WOE treatments, and across E_1,

[27] Mittone and Ploner (2011) analyze peer-pressure effects on the first follower in a trust game. They find that peer pressure has a positive effect on reciprocity, but this effect is significant for the highest investment level only.

[28] To examine whether the positive effect on observed employees is consistent with preferences for compliance with the social norm, we conducted a norm elicitation treatment corresponding to the SOE setting. Twenty-three subjects participated in the social norm elicitation treatment, which was conducted in the same university with the same subject pool and the same recruiting protocol as the previous treatments. While we do not find evidence that E_2's behavior is consistent with the corresponding social norm, we do find that the higher frequency of choosing the expensive supplier of E_1 in the SOE treatment relative to E_1 in the peer treatment is consistent with the social norm: it is more appropriate for E_1 to choose the expensive supplier in the SOE treatment relative to the peer treatment, particularly in the situation where this behavior is observed (Table 18 in Appendix B). This suggests that a preference for compliance with the social norm is a robust result among E_1 (who are observed but not observers).

E_2, and E_3 in the SOE treatment). We observe that both treatments with overlap of effects lead to a higher frequency of choosing S_H, and the difference is significant when the wage is low; see Table 19 in Appendix B. However, we find no significant differences in the average wage between the peer treatment and the WOE and SOE treatments. Moreover, the difference in the average cost between the peer and SOE treatments is not significant and, while there is a significant difference in the average cost between the peer and WOE treatments, the latter is higher by only 3.8%.

7. Conclusions

Motivated by recent initiatives to increase transparency in procurement, we study the effects of disclosing information about previous purchases in a setting where an organization delegates its purchasing decisions to its employees. We develop a theoretical model that captures the main dynamics of delegated procurement and makes two behavioral considerations: that employees are heterogeneous in their reciprocity towards the employer and that they are averse to disadvantageous income inequality relative to their peer. We show the existence of a price region where increased transparency leads to negative spillovers on reciprocal employees, who in the absence of peer effects would have chosen the cheaper supplier to benefit their employer. Our model also predicts an absence of positive spillovers on reciprocal employees, and a lack of peer effects on employees who are observed by their peer.

We design a laboratory experiment to test these predictions and to shed light on the behavioral mechanisms driving decisions. To this end, we introduce a new game, the procurement game, that captures the setting analyzed in the theory. Our experimental results confirm the existence of negative spillovers on reciprocal employees, by which they are more likely to choose the expensive supplier after observing that their peer did so. Consistent with the theory, we also find that there are no positive spillovers; i.e., employees who observe that their peer chose the cheapest option are not more likely to do so than in the case with no transparency.

A result that is not predicted by our model is that employees whose decision will be observed by their peer are less likely to choose the expensive supplier. These effects are especially significant among non-reciprocal employees and when the wage is low, suggesting that reciprocity is not the main mechanism driving this behavior. We propose an alternative explanation based on employees' desire to comply with the social norm by taking the action that is socially perceived as most appropriate. To test this, we conduct two additional norm elicitation treatments to evaluate the appropriateness of choosing the expensive supplier in the context of our experimental setting, with and without transparency. We find that choosing the expensive supplier is less appropriate when employees' decisions are observed by a peer than in the no-transparency baseline, and that these differences in the social norm are consistent with the differences in purchasing behavior between employees in the baseline and those who are observed in the peer treatment. This result suggests that a model that incorporates the desire to comply with social norms provides a plausible explanation for the behavior of observed employees.

Finally, our additional treatments confirm that the two main effects that we identified (negative spillovers associated with observing that a peer chose the expensive supplier and positive effects associated with being observed by a peer) are still present when an employee both observes and is observed.

7.1. Managerial Implications

Our results provide valuable insights for organizations seeking to implement transparency initiatives in their procurement processes, and suggest some concrete recommendations to be incorporated when designing such procurement platforms.

First, we find that increased transparency affects employees' purchasing behavior: observing overspending negatively affects reciprocal employees and being observed by a peer positively affects non-reciprocal employees. Hence, firms' internal communication policies should emphasize that employees' decisions will be observed: this would help reduce overspending by non-reciprocal employees which in turn would mitigate the negative spillovers on reciprocal employees, leading to lower procurement costs. A second effect of increased transparency we identify is the change in the perceived appropriateness of choosing the expensive supplier. In particular, overspending is perceived to be less appropriate when an employee's decision will be observed by other employees. Our results also show that observed employees comply, to certain extent, with what is perceived as socially appropriate. Therefore, organizations should make communication efforts that reinforce what is perceived as appropriate spending behavior in an attempt to increase compliance with the social norm and to reduce procurement costs.

We believe that a great opportunity for future research will be to test the aforementioned ideas in a field experiment setting. For instance, in line with what we just described, two concrete suggestions arise. First, it would interesting to implement an intervention where employees are reminded of the fact that other employees will observe their decisions later on. Second, it would be interesting to elicit the perceived appropriateness of choosing the expensive suppliers, and make that visible to employees when they make their choices. This intervention would be in line with previous literature which has shown that communication that makes the social norm salient can be effective in influencing behavior in settings such as alcohol and cigarette use, energy consumption, and pro-environmental behavior (Reid et al. 2010, Schultz et al. 2007, and Cialdini et al. 2006). In addition, in public procurement settings, it would be of interest to study what is the impact of purchasing decisions being observed not only by peers but also by citizens.

Finally, while our experiment captures decision making in a procurement setting, we believe our findings can more broadly translate to the effects of increased transparency in other settings. As discussed in the literature review, our results are consistent with previous findings in other principal-agent problems (e.g., in gift-exchange games). Therefore, the behavioral mechanisms we identify and the consequent managerial implications could apply to those settings as well.

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Appendix A: Proofs

A.1. Proof of Proposition 1

Recall that the utility of employee $i \in \{1,2\}$ in the baseline model is given by

$$u_i(\omega,\delta,\gamma_i,\sigma_i) = \omega + r \cdot \mathbb{1}_{\{\sigma_i = S_H\}} + \gamma_i \cdot R_\rho(\omega,\delta,\sigma_i).$$

Suppose first that employee i is non-reciprocal, i.e. $\gamma_i = 0$. Then, as $r > 0$, it is optimal for employee i to always choose supplier A, regardless of the received wage and the realized price difference.

Next, suppose that $\gamma_i = \gamma$. If the wage offered is below the reference wage, then it is optimal for employee i to select supplier A regardless of the realized price difference, as $\lambda_\rho(\omega) < 0$ and therefore $R_\rho(\omega,\delta,S_H) \geq 0$. In contrast, if ω is above the reference wage, we have $\lambda_\rho(\omega) > 0$ and the utility of choosing S_H is $\omega + r - \gamma\lambda_\rho(\omega)\frac{\delta}{2}$, whereas the utility of choosing S_L is $\omega + \gamma\lambda_\rho(\omega)\frac{\delta}{2}$. Hence, it is optimal for employee i with $\gamma_i = \gamma$ to choose supplier A if and only if $\delta \leq \frac{r}{\gamma\lambda_\rho(\omega)}$.

We thus conclude that the optimal strategy for the employees is:

$$\sigma_i(\omega,\delta,0) = S_H, \quad \text{and} \quad \sigma_i(\omega,\delta,\gamma) = \begin{cases} S_H & \text{if } \lambda_\rho(\omega) \leq 0 \\ S_H & \text{if } \lambda_\rho(\omega) > 0 \text{ and } \delta \leq \frac{r}{\gamma\lambda_\rho(\omega)} \\ S_L & \text{if } \lambda_\rho(\omega) > 0 \text{ and } \delta > \frac{r}{\gamma\lambda_\rho(\omega)} \end{cases}. \tag{10}$$

Anticipating the employees' behavior, the director will choose a wage in order to minimize her cost function given by (3). We consider two cases:

1. If $\omega \in \left[\underline{\omega}, \rho + \frac{r}{\gamma\bar{\delta}}\right]$, then all reciprocal employees will choose supplier A regardless of the price difference, as $\bar{\delta} \leq \frac{r}{\gamma\lambda_\rho(\omega)}$ for any wage in this interval. As non-reciprocal employees always choose supplier A (regardless of the wage and price difference), the cost of the director will be:

$$c_D^B(\omega) = 2\omega + 2p_L + 2\mathbb{E}_\delta[\delta] = 2\omega + 2p_L + \bar{\delta},$$

where for the second equality we use the fact that δ is uniformly distributed in $[0,\bar{\delta}]$.

2. If $\omega \in \left(\rho + \frac{r}{\gamma\bar{\delta}}, \infty\right)$, then whether an employee will choose supplier A or not depends on his reciprocity type and the realized price difference. In this case, using the fact that the utility of employee i is independent of the type and strategy of the other employee, we have that:

$$\begin{aligned} c_D^B(\omega) &= 2\omega + 2p_L + \sum_{i=1}^{2} \mathbb{E}_{\delta,\gamma_i}\left[\mathbb{1}_{\{\sigma_i = S_H | \omega,\delta,\gamma_i\}}\delta\right] \\ &= 2\omega + 2p_L + 2\Big((1-q)\mathbb{E}_\delta\left[\mathbb{1}_{\{\sigma_i = S_H|\omega,\delta,\gamma_i=0\}}\delta\right] + q\mathbb{E}_\delta\left[\mathbb{1}_{\{\sigma_i=S_H|\omega,\delta,\gamma_i=\gamma\}}\delta\right]\Big) \\ &= 2\omega + 2p_L + 2(1-q)\frac{\bar{\delta}}{2} + 2q\int_0^{\frac{r}{\gamma\lambda_\rho(\omega)}} \frac{\delta}{\bar{\delta}}d\delta \\ &= 2\omega + 2p_L + (1-q)\bar{\delta} + \frac{q}{\bar{\delta}}\left(\frac{r}{\gamma\lambda_\rho(\omega)}\right)^2 \end{aligned}$$

where the first equality is obtained by noting that both employees are ex-ante symmetrical, the second one by conditioning on the reciprocity type of the employee, and the third equality follows by replacing the strategies of the employees in the indicator by those in (10).

We therefore conclude that the cost of the director is given by the following piecewise function:

$$c_D^B(\omega) = \begin{cases} 2\omega + 2p_L + \bar{\delta} & \text{if } \omega \in \left[\underline{\omega}, \rho + \frac{r}{\gamma\bar{\delta}}\right] \\ 2\omega + 2p_L + (1-q)\bar{\delta} + \frac{q}{\bar{\delta}}\left(\frac{r}{\gamma\lambda_\rho(\omega)}\right)^2 & \text{if } \omega \in \left(\rho + \frac{r}{\gamma\bar{\delta}}, \infty\right) \end{cases}. \tag{11}$$

It is easy to check that $c_D^B(\omega)$ is convex in $\left[\rho + \frac{r}{\gamma\bar{\delta}}, \infty\right)$ and that it is increasing for ω big enough, so it has a unique minimizer in this range.

Let ω_B^* be the optimal wage for the director. Given that the director's cost is a piece-wise function, with one of the pieces being linear and the other convex, to find the optimal wage we can optimize over the two pieces separately and compare these costs.

As the cost function is linear and increasing in $\left[\underline{\omega}, \rho + \frac{r}{\gamma\bar{\delta}}\right]$, it follows that, in this range, the cost function is minimized at $\underline{\omega}$.

Let $\hat{\omega}$ be the minimizer of the cost function in $\left[\rho + \frac{r}{\gamma\bar{\delta}}, \infty\right)$. Note that

$$(c_D^B)'(\omega) = \frac{\partial c_D^B(\omega)}{\partial \omega} = 2 - \frac{2q}{\bar{\delta}} \cdot \frac{1}{\lambda_\rho(\omega)^3} \cdot \left[\frac{r}{\gamma}\right]^2 = 2 - \frac{2q}{\bar{\delta}} \cdot \frac{1}{(\omega-\rho)^3} \cdot \left[\frac{r}{\gamma}\right]^2.$$

When $(c_D^B)'\left(\rho + \frac{r}{\gamma\bar{\delta}}\right) \geq 0$ (which happens if and only if $r \geq q\gamma\bar{\delta}^2$), it follows by convexity that $\hat{\omega} = \rho + \frac{r}{\gamma\bar{\delta}}$. It can be easily verified that $c_D^B(\hat{\omega}) > c_D^B(\underline{\omega})$. Otherwise, $\hat{\omega}$ must satisfy the first order conditions and thus:

$$\hat{\omega} = \rho + \left[\frac{q}{\bar{\delta}}\right]^{\frac{1}{3}} \cdot \left[\frac{r}{\gamma}\right]^{\frac{2}{3}} = \rho + \psi$$

Therefore, for ω_B^* to be in $(\underline{\omega}, \infty)$ it must be the case that $r < q\gamma\bar{\delta}^2$ and $c_D^B(\hat{\omega}) < c_D^B(\underline{\omega})$, which holds only if $2(\rho - \underline{\omega}) < q\bar{\delta} - 3\psi$. Thus,

$$\omega_B^* = \begin{cases} \rho + \psi & \text{if } r < q\gamma\bar{\delta}^2,\ 2(\rho - \underline{\omega}) < q\bar{\delta} - 3\psi \\ \underline{\omega} & \text{if } otherwise \end{cases},$$

as desired. □

A.2. Proof of Proposition 2

As employee 1's utility only depends on his reciprocity towards the director and does not depend on employee 2's decision, it is direct that his optimal strategy is the same as in the baseline model (without transparency), so it is given by (4).

On the other hand, employee 2 incorporates distributional preferences as he is able to infer employee 1's payoff from his decisions. Noticing that

$$(\pi_i - \pi_j) = \begin{cases} r & \text{if } \sigma_i = S_H,\ \sigma_j = S_L \\ 0 & \text{if } \sigma_i = \sigma_j \\ -r & \text{if } \sigma_i = S_L,\ \sigma_j = S_H \end{cases}$$

the utility of employee 2 in Equation (6) can be re-written as:

$$u_2(\omega, \delta, \gamma_2, \sigma_1, \sigma_2) = \omega + \gamma_2 \cdot R_\rho(\omega, \delta, \sigma_2) + \mathbb{1}_{\{\sigma_2 = S_H\}} \cdot r \cdot [(1-\alpha) + (\alpha+\beta) \cdot \mathbb{1}_{\{\sigma_1 = S_H\}}] - \beta \cdot r \cdot \mathbb{1}_{\{\sigma_1 = S_H\}} \tag{12}$$

Suppose first that $\gamma_2 = 0$. Then, as $r > 0$ and $\alpha \leq 1$, the benefit from choosing supplier A (equal to r) is always greater than or equal to the disutility resulting from the aversion to advantageous income inequality (equal to $\alpha \cdot r$), so it is optimal for employee 2 to choose supplier A for all combinations of wage and price difference, regardless of employee 1's decision.

Next, suppose that $\gamma_2 = \gamma$. If the wage offered is below the reference wage, then $R_\rho(\omega, \delta, S_H) \geq 0$ and employee 1 chooses S_H, so it is optimal for employee 2 to always choose the expensive supplier.

In contrast, if ω is above the reference wage (and thus $\lambda_\rho(\omega) > 0$), the decision of employee 2 depends on the decision made by employee 1. In particular, if $\sigma_1 = S_H$, the utility that employee 2 gets from choosing S_H is $\omega + r - \gamma\lambda_\rho(\omega)\frac{\delta}{2}$, while his utility from choosing S_L is $\omega + \gamma\lambda_\rho(\omega)\frac{\delta}{2} - \beta r$. Hence, it is optimal for employee 2 to choose supplier A if and only if $\delta \leq \frac{r(1+\beta)}{\gamma \cdot \lambda_\rho(\omega)}$. On the other hand, if $\sigma_1 = S_L$, the utility that employee 2 gets from choosing S_H is $\omega + r - \gamma\lambda_\rho(\omega)\frac{\delta}{2} - \alpha r$, whereas his utility from choosing S_L is $\omega + \gamma\lambda_\rho(\omega)\frac{\delta}{2}$, so it is optimal for him to choose S_H if and only if $\delta \leq \frac{r(1-\alpha)}{\gamma \cdot \lambda_\rho(\omega)}$. Combining these cases, the optimal strategy of employee 2 becomes

$$\sigma_2(\omega, \delta, 0, \sigma_1) = S_H, \quad \text{and} \quad \sigma_2(\omega, \delta, \gamma, \sigma_1) = \begin{cases} S_H \text{ if } \lambda_\rho(\omega) \leq 0 \text{ or } \lambda_\rho(\omega) > 0 \text{ and } \delta < \frac{r}{\gamma\lambda_\rho(\omega)}, \\ S_H \text{ if } \lambda_\rho(\omega) > 0 \text{ and } \delta \in \left[\frac{r}{\gamma\lambda_\rho(\omega)}, \frac{r(1+\beta)}{\gamma\lambda_\rho(\omega)}\right] \text{ and } \sigma_1 = S_H, \\ S_L \text{ if } \lambda_\rho(\omega) > 0 \text{ and } \delta \in \left[\frac{r}{\gamma\lambda_\rho(\omega)}, \frac{r(1+\beta)}{\gamma\lambda_\rho(\omega)}\right] \text{ and } \sigma_1 = S_L, \\ S_L \text{ if } \lambda_\rho(\omega) > 0 \text{ and } \delta > \frac{r\cdot(1+\beta)}{\gamma\lambda_\rho(\omega)}, \end{cases} \tag{13}$$

As in the baseline model, the director anticipates the behavior of the employees and chooses a wage to minimize her expected cost, which is given by:

$$\begin{aligned} c_D^P(\omega) &= 2\omega + 2p_L + \mathbb{E}_{\delta,\gamma_1,\gamma_2}\left[\left(\mathbb{1}_{\{\sigma_1 = S_H | \omega,\delta,\gamma_1\}} + \mathbb{1}_{\{\sigma_2 = S_H | \omega,\delta,\gamma_2,\sigma_1\}}\right)\delta\right] \\ &= 2\omega + 2p_L + (1-q)^2\mathbb{E}_\delta\left[\left(\mathbb{1}_{\{\sigma_1 = S_H | \omega,\delta,\gamma_1 = 0\}} + \mathbb{1}_{\{\sigma_2 = S_H | \omega,\delta,\gamma_2 = 0,\sigma_1\}}\right)\delta\right] \\ &\quad + q(1-q)\mathbb{E}_\delta\left[\left(\mathbb{1}_{\{\sigma_1 = S_H | \omega,\delta,\gamma_1 = 0\}} + \mathbb{1}_{\{\sigma_2 = S_H | \omega,\delta,\gamma_2 = \gamma,\sigma_1\}}\right)\delta\right] \\ &\quad + q(1-q)\mathbb{E}_\delta\left[\left(\mathbb{1}_{\{\sigma_1 = S_H | \omega,\delta,\gamma_1 = \gamma\}} + \mathbb{1}_{\{\sigma_2 = S_H | \omega,\delta,\gamma_2 = 0,\sigma_1\}}\right)\delta\right] \\ &\quad + q^2\mathbb{E}_\delta\left[\left(\mathbb{1}_{\{\sigma_1 = S_H | \omega,\delta,\gamma_1 = \gamma\}} + \mathbb{1}_{\{\sigma_2 = S_H | \omega,\delta,\gamma_2 = \gamma,\sigma_1\}}\right)\delta\right] \end{aligned} \tag{14}$$

We consider three cases:

1. If $\omega \in \left[\underline{\omega}, \rho + \frac{r}{\gamma\bar{\delta}}\right]$, then both employees will always choose supplier A regardless of the price difference, as $\alpha \leq 1$ and $\bar{\delta} \leq \frac{r}{\gamma\lambda_\rho(\omega)}$ for any wage in this interval. Then, similar to the baseline, the cost for the director will be:
$$c_D^P(\omega) = 2\omega + 2p_L + 2\mathbb{E}_\delta[\delta] = 2\omega + 2p_L + \bar{\delta}.$$
2. If $\omega \in \left[\rho + \frac{r}{\gamma\bar{\delta}}, \rho + \frac{r(1+\beta)}{\gamma\bar{\delta}}\right]$, a reciprocal employee 2 will follow the decision made by employee 1 regardless of the price difference, as $\bar{\delta} \leq \frac{r(1+\beta)}{\gamma\lambda_\rho(\omega)}$ for any wage in this interval. In contrast, if employee 2 is non-reciprocal he will always choose S_H regardless of employee 1's decision (as $\alpha \leq 1$). Using these facts, the expected cost for the director in (14) can be written as
$$\begin{aligned} c_D^P(\omega) &= 2\omega + 2p_L + 2(1-q)^2\frac{\bar{\delta}}{2} + 2q(1-q)\frac{\bar{\delta}}{2} + q(1-q)\left[\int_0^{\frac{r}{\gamma\lambda_\rho(\omega)}} \frac{\delta}{\bar{\delta}}d\delta + \frac{\bar{\delta}}{2}\right] + 2q^2\int_0^{\frac{r}{\gamma\lambda_\rho(\omega)}} \frac{\delta}{\bar{\delta}}d\delta \\ &= 2\omega + 2p_L + \bar{\delta} - \frac{q(1+q)\bar{\delta}}{2} + \frac{q(1+q)}{2\bar{\delta}}\left(\frac{r}{\gamma\lambda_\rho(\omega)}\right)^2 \end{aligned}$$
where the first equality is obtained by replacing the indicator functions in Equation (14) with the strategies of the employees.

3. If $\omega \in \left[\rho + \frac{r(1+\beta)}{\gamma\bar{\delta}}, \infty\right)$, then a reciprocal employee 2 will follow a non-reciprocal employee 1 and choose S_H if $\delta \le \frac{r(1+\beta)}{\gamma\lambda_\rho(\omega)}$, whereas he will choose the cheapest supplier if the price difference exceeds this threshold. Then, replacing the indicator functions by the employees' best responses, the expected cost in (14) becomes:

$$\begin{aligned} c_D^P(\omega) &= 2\omega + 2p_L + 2(1-q)^2\frac{\bar{\delta}}{2} + q(1-q)\left[\frac{\bar{\delta}}{2} + \int_0^{\frac{r(1+\beta)}{\gamma\lambda_\rho(\omega)}} \frac{\delta}{\bar{\delta}} d\delta\right] + q(1-q)\left[\int_0^{\frac{r}{\gamma\lambda_\rho(\omega)}} \frac{\delta}{\bar{\delta}} d\delta + \frac{\bar{\delta}}{2}\right] + 2q^2 \int_0^{\frac{r}{\gamma\lambda_\rho(\omega)}} \frac{\delta}{\bar{\delta}} d\delta \\ &= 2\omega + 2p_L + (1-q)\bar{\delta} + \frac{q(1+q)}{2\bar{\delta}}\left(\frac{r}{\gamma\lambda_\rho(\omega)}\right)^2 + \frac{q(1-q)}{2\bar{\delta}}\left(\frac{r(1+\beta)}{\gamma\lambda_\rho(\omega)}\right)^2 \end{aligned}$$

Therefore, the expected cost of the director is given by the following piecewise function:

$$c_D^P(\omega) = \begin{cases} 2\omega + 2p_L + \bar{\delta} & \text{if } \omega \in \left[\underline{\omega}, \rho + \frac{r}{\gamma\bar{\delta}}\right] \\ 2\omega + 2p_L + \bar{\delta} - \frac{q(1+q)\bar{\delta}}{2} + \frac{q(1+q)}{2\bar{\delta}}\left(\frac{r}{\gamma\lambda_\rho(\omega)}\right)^2 & \text{if } \omega \in \left[\rho + \frac{r}{\gamma\bar{\delta}}, \rho + \frac{r(1+\beta)}{\gamma\bar{\delta}}\right] \\ 2\omega + 2p_L + (1-q)\bar{\delta} + \frac{q(1+q)}{2\bar{\delta}}\left(\frac{r}{\gamma\lambda_\rho(\omega)}\right)^2 + \frac{q(1-q)}{2\bar{\delta}}\left(\frac{r(1+\beta)}{\gamma\lambda_\rho(\omega)}\right)^2 & \text{if } \omega \in \left[\rho + \frac{r(1+\beta)}{\gamma\bar{\delta}}, \infty\right) \end{cases} \quad (15)$$

Notice that if $\beta = 0$ (i.e. no aversion to disadvantageous income inequality) or $q \in \{0,1\}$ (i.e. no heterogeneity in reciprocity), the best response function of employee 2 is the same as for employee 1, and the expected cost for the director is equivalent to that in the baseline case. Thus, from now on we focus on the case where $\beta > 0$ and $q \in (0,1)$. Then, the cost function $c_D^P(\omega)$ is linear in $\omega \in \left[\underline{\omega}, \rho + \frac{r}{\gamma\bar{\delta}}\right]$ and convex in both $\left[\rho + \frac{r}{\gamma\bar{\delta}}, \rho + \frac{r(1+\beta)}{\gamma\bar{\delta}}\right]$ and $\left[\rho + \frac{r(1+\beta)}{\gamma\bar{\delta}}, \infty\right)$, and it is easy to see that this function is increasing for ω large enough. Hence, $c_D^P(\omega)$ has a unique minimizer in each piece, and therefore to find the globally optimal solution it is enough to optimize over the three pieces separately and compare the resulting costs.

Let ω_1, ω_2 and ω_3 be the minimizers of $c_D^P(\omega)$ in each piece, and let ω_P^* be the optimal wage for the director. As the cost function is linear and increasing in $\left[\underline{\omega}, \rho + \frac{r}{\gamma\bar{\delta}}\right]$, it is direct that $\omega_1 = \underline{\omega}$. In addition, note that in $\left[\rho + \frac{r}{\gamma\bar{\delta}}, \rho + \frac{r(1+\beta)}{\gamma\bar{\delta}}\right]$,

$$(c_D^P)'(\omega) = \frac{\partial c_D^P(\omega)}{\partial\omega} = 2 - \frac{q(1+q)}{\bar{\delta}} \cdot \left[\frac{r}{\gamma}\right]^2 \cdot \frac{1}{(\omega-\rho)^3}.$$

By convexity it follows that $\omega_2 = \rho + \frac{r}{\gamma\bar{\delta}}$ if $(c_D^P)'\left(\rho + \frac{r}{\gamma\bar{\delta}}\right) \ge 0$. This holds if $q\gamma\bar{\delta}^2\xi^3 \le r$, or equivalently if $\xi \le \frac{r}{\gamma\bar{\delta}\psi}$, where $\xi = \left[\frac{1+q}{2}\right]^{\frac{1}{3}}$ and $\psi = \left[\frac{q}{\bar{\delta}}\right]^{\frac{1}{3}} \cdot \left[\frac{r}{\gamma}\right]^{\frac{2}{3}}$. On the other hand, if $(c_D^P)'\left(\rho + \frac{r(1+\beta)}{\gamma\bar{\delta}}\right) \le 0$ (which happens if $q\gamma\bar{\delta}^2\left[\frac{\xi}{1+\beta}\right]^3 \ge r \Leftrightarrow \xi \ge \frac{r(1+\beta)}{\gamma\bar{\delta}\psi}$) then it must be that $\omega_2 = \rho + \frac{r(1+\beta)}{\gamma\bar{\delta}}$ because convexity implies that $c_D^P(\omega)$ is decreasing in this interval. Otherwise, ω_2 satisfies first order conditions, and therefore:

$$\omega_2 = \rho + \left[\frac{q}{\bar{\delta}}\right]^{\frac{1}{3}} \cdot \left[\frac{r}{\gamma}\right]^{\frac{2}{3}} \cdot \left[\frac{1+q}{2}\right]^{\frac{1}{3}} = \rho + \psi \cdot \xi.$$

Finally, for $\omega \in \left[\rho + \frac{r(1+\beta)}{\gamma\bar{\delta}}, \infty\right)$ note that

$$(c_D^P)'(\omega) = \frac{\partial c_D^P(\omega)}{\partial\omega} = 2 - \frac{q(1+q)}{\bar{\delta}} \cdot \left[\frac{r}{\gamma}\right]^2 \cdot \frac{1}{(\omega-\rho)^3} - \frac{q(1-q)}{\bar{\delta}} \cdot \left[\frac{r(1+\beta)}{\gamma}\right]^2 \cdot \frac{1}{(\omega-\rho)^3}.$$

As before, convexity implies that $\omega_3 = \rho + \frac{r(1+\beta)}{\gamma\bar{\delta}}$ if $(c_D^P)'\left(\rho + \frac{r(1+\beta)}{\gamma\bar{\delta}}\right) \ge 0$. This holds if $q\gamma\bar{\delta}^2\left[\frac{\zeta}{1+\beta}\right]^3 \le r$, which can also be written as $\zeta \le \frac{r(1+\beta)}{\gamma\bar{\delta}\psi}$, where $\zeta = \left[\frac{(1+q)+(1+\beta)^2\cdot(1-q)}{2}\right]^{\frac{1}{3}}$ and ψ is defined as before. Otherwise, ω_3 must satisfy first order conditions, and therefore:

$$\omega_3 = \rho + \left[\frac{q}{\bar{\delta}}\right]^{\frac{1}{3}} \cdot \left[\frac{r}{\gamma}\right]^{\frac{2}{3}} \cdot \left[\frac{(1+q)+(1+\beta)^2\cdot(1-q)}{2}\right]^{\frac{1}{3}} = \rho + \psi \cdot \zeta.$$

For ω_i to be globally optimal it must be the case that $c_D^P(\omega_i) < \min_{j\in\{1,2,3\}\setminus\{i\}}\{c_D^P(\omega_j)\}$. It is easy to check that for ω_2 to be globally optimal it must be the case that $\omega_2 = \rho + \psi\xi$ (which holds if $\xi \in \left(\frac{r}{\gamma\bar{\delta}\psi}, \frac{r(1+\beta)}{\gamma\bar{\delta}\psi}\right)$), as $c_D^P(\underline{\omega}) = c_D^P(\omega_1) < c_D^P(\rho + \frac{r}{\gamma\bar{\delta}})$ and $c_D^P(\omega_3) \le c_D^P(\rho + \frac{r(1+\beta)}{\gamma\bar{\delta}})$. Moreover, it must also be the case that $c_D^P(\omega_2) < c_D^P(\omega_1) = c_D^P(\underline{\omega})$, which follows if $2(\rho - \underline{\omega}) < \frac{q(1+q)\bar{\delta}}{2} - 3\psi\xi$.

Similarly, for ω_3 to be globally optimal it must be the case that $\omega_3 = \rho + \psi\zeta$ (which holds if $\zeta > \frac{r(1+\beta)}{\gamma\bar{\delta}\psi}$), as $c_D^P(\omega_2) \le c_D^P(\rho + \frac{r(1+\beta)}{\gamma\bar{\delta}})$. Also it must be that $c_D^P(\omega_2) < c_D^P(\omega_1) = c_D^P(\underline{\omega})$, which holds if $2(\rho - \underline{\omega}) < q\bar{\delta} - 3\psi\zeta$.

Finally, suppose that $\omega_2 = \rho + \psi\xi$ and $\omega_3 = \rho + \psi\zeta$. Then, $c_D^P(\omega_2) \le c_D^P(\omega_3)$ if and only if $3(\zeta - \xi) \ge \frac{q(1-q)\bar{\delta}}{2}$, so the optimal wage ω_P^* can be written as,

$$\omega_P^* = \begin{cases} \rho + \psi\zeta \text{ if } \xi \in \left(\frac{r}{\gamma\bar{\delta}\psi}, \frac{r(1+\beta)}{\gamma\bar{\delta}\psi}\right), \zeta \in \left(\frac{r(1+\beta)}{\gamma\bar{\delta}\psi}, \infty\right), 2(\rho - \underline{\omega}) < q\bar{\delta} - 3\psi\zeta,\ 3\psi(\zeta - \xi) < \frac{q(1-q)\bar{\delta}}{2} \\ \rho + \psi\zeta \text{ if } \xi \notin \left(\frac{r}{\gamma\bar{\delta}\psi}, \frac{r(1+\beta)}{\gamma\bar{\delta}\psi}\right), \zeta \in \left(\frac{r(1+\beta)}{\gamma\bar{\delta}\psi}, \infty\right), 2(\rho - \underline{\omega}) < q\bar{\delta} - 3\psi\zeta \\ \rho + \psi\xi \text{ if } \xi \in \left(\frac{r}{\gamma\bar{\delta}\psi}, \frac{r(1+\beta)}{\gamma\bar{\delta}\psi}\right), \zeta \in \left(\frac{r(1+\beta)}{\gamma\bar{\delta}\psi}, \infty\right), 2(\rho - \underline{\omega}) < \frac{q(1+q)\bar{\delta}}{2} - 3\psi\xi,\ 3\psi(\zeta - \xi) \ge \frac{q(1-q)\bar{\delta}}{2} \\ \rho + \psi\xi \text{ if } \xi \in \left(\frac{r}{\gamma\bar{\delta}\psi}, \frac{r(1+\beta)}{\gamma\bar{\delta}\psi}\right), \zeta \le \frac{r(1+\beta)}{\gamma\bar{\delta}\psi}, 2(\rho - \underline{\omega}) < \frac{q(1+q)\bar{\delta}}{2} - 3\psi\xi. \\ \underline{\omega} \qquad \text{if } \textit{otherwise} \end{cases}$$

□

A.3. Procurement cost in the baseline and peer setting

Proposition 3 shows that if the director decides to offer a wage above the reference wage, then the expected cost in the peer model is greater than or equal to the cost in the baseline model.

Proposition 3. *For any $\omega \in [\underline{\omega}, \infty)$, $c_D^B(\omega) \le c_D^P(\omega)$.*

C onsider Equations 11 and 15. If $\omega \in \left[\underline{\omega}, \rho + \frac{r}{\gamma\bar{\delta}}\right]$, then it is direct that $c_D^B(\omega) = c_D^P(\omega)$. If $\omega \in \left[\rho + \frac{r}{\gamma\bar{\delta}}, \rho + \frac{r(1+\beta)}{\gamma\bar{\delta}}\right]$, then

$$\begin{aligned} c_D^P(\omega) - c_D^B(\omega) &= \bar{\delta} - \frac{q(1+q)\bar{\delta}}{2} + \frac{q(1+q)}{2\bar{\delta}}\left(\frac{r}{\gamma\lambda_\rho(\omega)}\right)^2 - (1-q)\bar{\delta} - \frac{q}{\bar{\delta}}\left(\frac{r}{\gamma\lambda_\rho(\omega)}\right)^2 \\ &= q\bar{\delta}\left(\frac{1-q}{2}\right) - \left(\frac{r}{\gamma\lambda_\rho(\omega)}\right)^2 \frac{q}{\bar{\delta}}\left(\frac{1-q}{2}\right) \\ &\ge q\bar{\delta}\left(\frac{1-q}{2}\right) - \left(\frac{r}{\gamma\lambda_\rho(\rho + \frac{r}{\gamma\bar{\delta}})}\right)^2 \frac{q}{\bar{\delta}}\left(\frac{1-q}{2}\right) \\ &= q\bar{\delta}\left(\frac{1-q}{2}\right) - \left(\frac{r}{\gamma\cdot\frac{r}{\gamma\bar{\delta}}}\right)^2 \frac{q}{\bar{\delta}}\left(\frac{1-q}{2}\right) \\ &= 0, \end{aligned} \tag{16}$$

where the inequality comes from the fact that $\lambda_\rho(\omega) = \omega - \rho$ is increasing in ω and the assumption that $\omega \in \left[\rho + \frac{r}{\gamma\bar{\delta}}, \rho + \frac{r(1+\beta)}{\gamma\bar{\delta}}\right]$.

Finally, if $\omega \in \left[\rho + \frac{r(1+\beta)}{\gamma\bar{\delta}}, \infty\right)$,

$$\begin{aligned} c_D^P(\omega) - c_D^B(\omega) &= \frac{q(1+q)}{2\bar{\delta}}\left(\frac{r}{\gamma\lambda_\rho(\omega)}\right)^2 + \frac{q(1-q)}{2\bar{\delta}}\left(\frac{r(1+\beta)}{\gamma\lambda_\rho(\omega)}\right)^2 - \frac{q}{\bar{\delta}}\left(\frac{r}{\gamma\lambda_\rho(\omega)}\right)^2 \\ &= \frac{q(1-q)}{2\bar{\delta}}\left[\left(\frac{r(1+\beta)}{\gamma\lambda_\rho(\omega)}\right)^2 - \left(\frac{r}{\gamma\lambda_\rho(\omega)}\right)^2\right] \\ &\ge 0, \end{aligned} \tag{17}$$

where the inequality comes from $\beta \ge 0$. □

From Proposition 3 it follows directly that $c_D^B(\omega_B^*) \le c_D^P(\omega_P^*)$. This result is formalized in the next corollary.

COROLLARY 1. *Suppose that* $\alpha, \beta \in [0,1]$. *Let* ω_B^*, ω_P^* *be the optimal wages in the baseline and peer models respectively. Then,* $c_D^B(\omega_B^*) \le c_D^P(\omega_P^*)$.

Appendix B: Additional Tables and Figures

Figure 6 Reciprocity Distribution

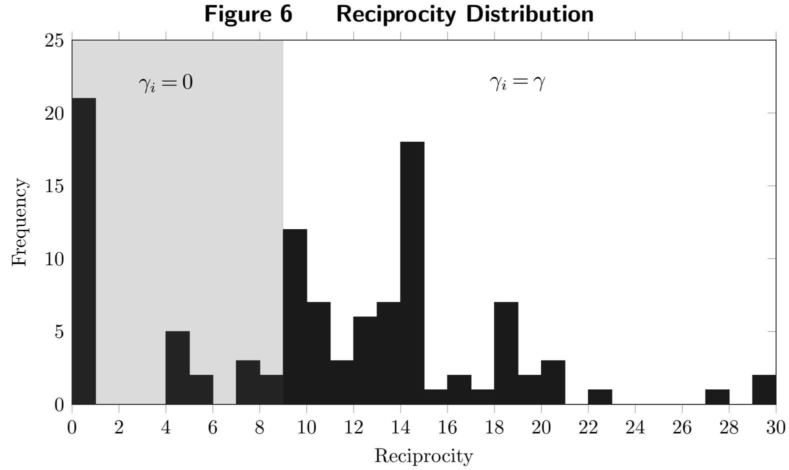


Table 8 Results Trust Game

		Amount Returned									
	Amount Sent	1	2	3	4	5	6	7	8	9	10
Baseline	4.50 (3.06)	0.69 (0.64)	1.81 (1.33)	2.78 (1.90)	3.91 (2.72)	4.63 (3.37)	5.50 (4.24)	6.28 (4.95)	7.34 (5.61)	8.31 (6.32)	9.97 (7.34)
Peer - E_1	5.00 (3.46)	0.85 (0.74)	1.85 (1.41)	2.79 (1.98)	3.95 (2.77)	4.85 (3.54)	5.95 (4.37)	7.08 (5.27)	8.18 (6.02)	9.33 (6.73)	11.13 (7.95)
Peer - E_2	4.72 (2.70)	0.96 (0.80)	2.21 (1.45)	3.22 (2.11)	4.21 (2.75)	5.37 (3.58)	6.82 (4.19)	7.86 (5.02)	8.87 (5.62)	9.65 (6.58)	10.64 (7.43)
		Tests (p-value)									
(1) Baseline vs. Peer - E_1	0.752	0.407	0.943	0.943	0.832	0.760	0.686	0.453	0.618	0.539	0.618
(2) Baseline vs. Peer - E_2	0.627	0.147	0.263	0.371	0.809	0.501	0.317	0.224	0.409	0.583	0.925
(3) Peer - E_1 vs. Peer - E_2	0.911	0.489	0.294	0.406	0.988	0.663	0.485	0.529	0.657	0.900	0.758

Note: Standard errors reported in parentheses. Wilcoxon rank-sum p-values reported.

Table 9 Effect of Reciprocity

	Probability of choosing S_H																	
	Panel 1: Baseline						Panel 2: Peer - Observed						Panel 3: Peer - Observer					
	$\omega = 25$			$\omega = 40$			$\omega = 25$			$\omega = 40$			$\omega = 25$			$\omega = 40$		
	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Reciprocity	-2.506*** (0.351)	-2.229** (1.074)	-5.570*** (1.621)	-3.923*** (1.240)	-3.754*** (1.371)	-2.789** (1.273)	-0.703* (0.417)	-0.966** (0.399)	-2.136*** (0.829)	-0.510 (0.511)	-1.456*** (0.417)	-1.831*** (0.430)	-0.702 (0.505)	-1.648*** (0.273)	-2.670*** (0.480)	-1.532*** (0.504)	-1.490*** (0.301)	-1.954*** (0.510)
Constant	5.185*** (1.690)	2.435** (1.052)	2.614 (1.629)	2.300 (2.131)	1.471 (1.117)	0.159 (0.992)	-2.532*** (0.738)	-2.608*** (0.797)	-2.822* (1.556)	-1.451 (1.214)	-2.443*** (0.891)	-1.577 (1.024)	2.089* (1.264)	2.906*** (0.670)	3.010*** (0.913)	3.458*** (1.226)	2.545*** (0.593)	3.034*** (1.063)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	168	192	192	168	192	192	234	234	234	234	234	234	234	234	234	234	234	234

Note: Panel probit regressions with subject random effects. Standard errors clustered at the session level reported in parentheses. Panel 1 pools data from employees in the baseline. Panel 2 pools data from employees who are observed in the peer treatment. Panel 3 pools data from employees who are observers in the peer treatment. Missing observations for $\delta = 10$ in Panel 1 are due to perfect separation. We control for round and demographics. Significance reported: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

Table 10 Reciprocal Employees — Baseline vs. Observes L

	Probability of choosing S_H											
	Panel 1: Round						Panel 2: Round and Demographics					
	$\omega = 25$			$\omega = 40$			$\omega = 25$			$\omega = 40$		
	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Observes L	-1.126	0.042	0.318	-0.374	0.908	0.649	-1.524***	0.125	0.547	-0.652	0.888	0.604
	(0.729)	(0.719)	(0.742)	(0.532)	(0.604)	(0.501)	(0.590)	(0.672)	(0.678)	(0.748)	(0.686)	(0.540)
Constant	2.737***	0.068	-1.464**	0.657	-1.428***	-1.513***	3.992**	1.278	-0.914	-1.544	-0.551	-0.356
	(0.884)	(0.537)	(0.688)	(0.704)	(0.533)	(0.353)	(1.690)	(0.919)	(1.494)	(1.205)	(1.149)	(1.217)
Round	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Demographics	No	No	No	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes
Observations	167	210	247	170	241	256	167	210	247	160	241	256

Note: Panel probit regressions with subject random effects. Standard errors clustered at the session level reported in parentheses. The table pools data from reciprocal employees in the baseline and reciprocal employees who observe that S_L was chosen in the peer treatment. In Panel 1 we control for round, and in Panel 2 we control for round and demographics. The missing observations for $(\omega, \delta) = (40, 10)$ in Panel 2 are due to perfect separation when including the demographic controls. Significance reported: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

Table 11 Social Spillovers — Non-Reciprocal Employees

Panel 1: Non-Reciprocal — Baseline vs. Observes H

	Probability of choosing S_H					
	$\omega = 25$			$\omega = 40$		
	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Observes H	0.110	1.726**	3.650	-0.339	0.493	1.028
	(0.856)	(0.858)	(30.23)	(0.605)	(0.393)	(1.069)
Constant	6.517*	1.171	3.577	3.166**	0.231	0.181
	(3.328)	(1.533)	(18.36)	(1.307)	(0.459)	(.)
Round	Yes	Yes	Yes	Yes	Yes	Yes
Demographics	No	No	No	No	No	No
Observations	110	108	90	107	94	86

Panel 2: Non-Reciprocal — Baseline vs. Observes L

	Probability of choosing S_H					
	$\omega = 25$			$\omega = 40$		
	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Observes L	-2.289*	-0.957	0.269	-0.988	-0.855	0.461
	(1.260)	(2.286)	(1.278)	(1.077)	(0.819)	(1.346)
Constant	3.499*	2.248***	1.108	3.998**	0.412	0.241
	(1.791)	(0.861)	(1.807)	(1.868)	(0.372)	(1.139)
Round	Yes	Yes	Yes	Yes	Yes	Yes
Demographics	No	No	No	No	No	No
Observations	70	72	90	73	86	94

Panel 3: Non-Reciprocal — Baseline vs. Observes H

	Probability of choosing S_H					
	$\omega = 25$			$\omega = 40$		
	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Observes H	-	0.224	-1.639	-5.869***	0.062	-0.224
	-	(1.226)	(21.50)	(0.068)	(0.503)	(.)
Constant	-	-3.578***	-3.953	0.571	-1.207***	-5.554
	-	(1.201)	(16.76)	(1.058)	(0.458)	(.)
Round	Yes	Yes	Yes	Yes	Yes	Yes
Demographics	Yes	Yes	Yes	Yes	Yes	Yes
Observations	-	77	67	35	88	80

Panel 4: Non-Reciprocal — Baseline vs. Observes L

	Probability of choosing S_H					
	$\omega = 25$			$\omega = 40$		
	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Observes L	-	-	-0.936	-6.311***	-1.460**	0.402
	-	-	(1.235)	(1.382)	(0.712)	(1.682)
Constant	-	-	-3.488	0.854	-0.917	-2.049
	-	-	(3.357)	(1.823)	(0.583)	(2.889)
Round	Yes	Yes	Yes	Yes	Yes	Yes
Demographics	Yes	Yes	Yes	Yes	Yes	Yes
Observations	-	-	65	14	74	82

Note: Panel probit regressions with subject random effects. Standard errors clustered at the session level reported in parentheses. Panels 1 and 3 pool data from non-reciprocal employees in the baseline and non-reciprocal employees that choose S_H in the peer treatment. Panels 2 and 4 pool data from non-reciprocal employees in the baseline and non-reciprocal employees that choose S_L in the peer treatment. Note that since non-reciprocal employees are only 30% of our sample, including demographic controls results in a significant drop in the number of observations due to perfect separation. Therefore, we report separately the regressions with (panels 3 and 4) and without (panels 1 and 2) demographic controls. Significance reported: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

Table 12 Social Norm — Appropriateness of Choosing S_H — OLS

	$\omega = 25$			$\omega = 40$		
	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Observed	-0.346*	-0.115*	-0.231	-0.385**	-0.115	-0.154
	(0.159)	(0.0511)	(0.155)	(0.121)	(0.113)	(0.134)
Constant	2.192***	1.577***	1.231***	2.192***	1.500***	1.231***
	(0.0975)	(0.0386)	(0.155)	(0.0628)	(0.0831)	(0.116)
Observations	52	52	52	52	52	52

Note: OLS regressions with errors clustered at the session level reported in parentheses. The dependent variable is the social appropriateness of choosing S_H (in a scale from 1 to 4). The independent variable is a dummy that takes value 1 when subjects evaluate the appropriateness of observed employees' decisions in the peer norm elicitation, and 0 when subjects evaluate the appropriateness of employees' decisions in the baseline norm elicitation. Significance reported: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

Table 13 Procurement Game vs. Social Norm — Observed Employees (Non-Reciprocal)

		Probability of choosing S_H					
		$\omega = 25$			$\omega = 40$		
		$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Procurement Game	Observed	-	-1.741**	-2.655*	-1.730***	-1.452	-0.413
		-	(0.823)	(1.512)	(0.582)	(0.970)	(1.097)
	Constant	2.207***	-1.022	-2.132	1.901***	-0.832	-1.492
		(0.412)	(1.215)	(2.272)	(0.620)	(1.404)	(2.106)
	Controls	Yes	Yes	Yes	Yes	Yes	Yes
	Observations	72	132	132	132	144	144
		Appropriateness of choosing S_H					
Social Norm	Observed	-0.689*	-0.240***	-4.762***	-0.576***	-0.247	-0.151
		(0.360)	(0.090)	(0.393)	(0.195)	(0.182)	(0.476)
	Observations	52	52	52	52	52	52

Note: The top panel reports the results of panel probit regression considering as dependent variable y_{ist} (from the *procurement game*), pooling data from non-reciprocal employees in the baseline and observed conditions only. Missing observations when $\omega = 25$ and $(\omega, \delta) = (40, 10)$ are due to perfect separation. The bottom panel corresponds to the norm elicitation treatments, and reports the estimates of ordered probit regressions with errors clustered at the session level reported in parentheses. The dependent variable is the social appropriateness of choosing S_H (in a scale from 1 to 4). The independent variable is a dummy that takes value 1 when subjects evaluate the appropriateness of observed employees' decisions in the peer norm elicitation, and 0 when subjects evaluate the appropriateness of employees' decisions in the baseline norm elicitation. We control for round and demographics. Significance reported: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

Table 14 Procurement Game vs. Social Norm — Observed Employees (Reciprocal)

		Probability of choosing S_H					
		$\omega = 25$			$\omega = 40$		
		$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Procurement Game	Observed	-1.561**	-0.283	-0.448	-0.027	0.289	-0.040
		(0.619)	(0.393)	(0.574)	(0.745)	(0.595)	(0.481)
	Constant	-1.150	-1.039	-2.422*	-3.194**	-3.139***	-2.210*
		(1.099)	(0.785)	(1.278)	(1.527)	(1.015)	(1.244)
	Controls	Yes	Yes	Yes	Yes	Yes	Yes
	Observations	282	282	282	282	282	282
		Appropriateness of choosing S_H					
Social Norm	Observed	-0.689*	-0.240***	-4.762***	-0.576***	-0.247	-0.151
		(0.360)	(0.090)	(0.393)	(0.195)	(0.182)	(0.476)
	Observations	52	52	52	52	52	52

Note: The top panel reports the results of panel probit regression considering as dependent variable y_{ist} (from the *procurement game*), pooling data from reciprocal employees in the baseline and observed conditions only. The bottom panel corresponds to the norm elicitation treatments, and reports the estimates of ordered probit regressions with errors clustered at the session level reported in parentheses. The dependent variable is the social appropriateness of choosing S_H (in a scale from 1 to 4). The independent variable is a dummy that takes value 1 when subjects evaluate the appropriateness of observed employees' decisions in the peer norm elicitation, and 0 when subjects evaluate the appropriateness of employees' decisions in the baseline norm elicitation. We control for round and demographics. Significance reported: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

Table 15 E2 Peer vs. E2 WOE

	Probability of choosing S_H											
	Panel 1: E_2 Peer vs. E_2 WOE - Observes H						Panel 2: E_2 Peer vs. E_2 WOE - Observes L					
	$\omega = 25$			$\omega = 40$			$\omega = 25$			$\omega = 40$		
	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Peer × Recip	-1.127***	-1.525***	-2.952***	-1.443**	-2.131***	-2.328**	0.417	-1.506	-2.047***	-1.504**	-0.794	-1.699***
	(0.372)	(0.227)	(1.008)	(0.563)	(0.442)	(1.113)	(0.873)	(0.998)	(0.489)	(0.726)	(0.563)	(0.511)
WOE × Recip	-0.489	-1.529***	-3.726***	-0.619	-2.479***	-3.179**	-	-1.212	-1.865***	-7.701**	-1.208**	-2.318***
	(0.406)	(0.316)	(1.126)	(0.530)	(0.399)	(1.314)	-	(1.189)	(0.650)	(3.031)	(0.602)	(0.574)
WOE × Non-Recip	-0.647	-0.818*	-0.046	-0.542	-0.600	0.113	-	0.098	-0.645	-0.868	0.614	-0.050
	(0.490)	(0.486)	(1.042)	(0.678)	(0.519)	(1.355)	-	(1.088)	(0.594)	(1.153)	(0.664)	(0.613)
Constant	2.394***	1.739***	1.785	3.447***	2.177**	3.064*	2.111*	3.757***	2.073***	1.249	1.276	2.165**
	(0.557)	(0.497)	(1.396)	(1.277)	(0.987)	(1.851)	(1.260)	(1.376)	(0.622)	(0.911)	(0.821)	(0.891)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	403	316	220	377	219	177	45	140	236	79	237	279
	Tests (p-value)											
(1) Peer vs. WOE | Recip	0.064	0.986	0.789	0.270	0.343	0.543	0.633	1.000	0.702	**0.035**	0.400	0.483
(2) Peer vs. WOE | Non-Recip	0.187	0.185	0.965	0.424	0.495	0.934	-	0.928	0.555	0.451	0.709	0.936

Note: Panel probit regressions with subject random effects. Standard errors clustered at the session level reported in parentheses. The table pools data for E_2 who observe S_H in the peer and WOE treatments. Bold values represent significant differences at the 5% level. We adjust p-values using the Holm method (Holm (1979)) for multiple hypothesis testing. We control for round and demographics. Missing values for WOE×Recip and WOE×Non-Recip in Panel 2, when $(\omega, \delta) = (25, 10)$, are due to perfect separation. Significance reported: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

Table 16 E1 Peer vs. E2 SOE — Observes L

	Probability of choosing S_H					
	$\omega = 25$			$\omega = 40$		
	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Peer × Recip	-0.695*	-0.911***	-1.897**	-0.337	-1.365***	-1.871***
	(0.362)	(0.338)	(0.772)	(0.515)	(0.465)	(0.419)
SOE × Recip	-0.993	-0.866	-0.293	-1.839**	-1.715**	-1.699**
	(0.732)	(0.660)	(0.889)	(0.898)	(0.747)	(0.682)
SOE × Non-Recip	-	0.008	-0.135	0.074	-0.209	-0.915
	-	(0.695)	(0.734)	(1.034)	(0.691)	(0.678)
Constant	-1.930***	-1.243*	-0.161	-0.300	-0.859	-0.522
	(0.710)	(0.752)	(1.107)	(0.943)	(1.090)	(1.126)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	243	280	317	283	336	360
	Tests (p-value)					
(1) Peer vs. SOE | Recip	0.678	1.000	**0.023**	0.170	0.957	0.742
(2) Peer vs. SOE | Non-Recip	-	0.990	0.854	0.943	0.762	0.355

Note: Panel probit regressions with subject random effects. Standard errors clustered at the session level reported in parentheses. We pool data for E_1 in the peer treatment and E_2 who observe that their peer chose S_L in the SOE treatment. Bold values represent significant differences at the 5% level. We adjust p-values using the Holm method (Holm 1979) for multiple hypothesis testing. Missing values when $(\omega, \delta) = (25, 10)$ are due to perfect separation and lack of observations. We control for round and demographics. Significance reported: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

Table 17 E2 Peer vs. E2 SOE — Observes L

	Probability of choosing S_H					
	$\omega = 25$			$\omega = 40$		
	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Peer × Recip	-0.174	-1.173	-2.395***	-1.216	-1.007*	-1.976***
	(0.913)	(1.037)	(0.569)	(0.941)	(0.530)	(0.474)
SOE × Recip	1.213	-1.387	-1.850**	-2.201	-1.974**	-2.863***
	(1.002)	(1.145)	(0.811)	(1.456)	(0.834)	(0.641)
SOE × Non-Recip	-	-0.210	-1.672**	-0.476	-0.470	-2.022***
	-	(1.120)	(0.701)	(1.191)	(0.644)	(0.580)
Constant	3.001**	2.668**	2.788***	2.033	2.299***	2.986***
	(1.230)	(1.148)	(0.807)	(1.477)	(0.690)	(1.004)
Observations	54	136	228	96	237	284
	Tests (p-value)					
(1) Peer vs. SOE | Recip	**0.016**	1.000	0.335	0.712	0.308	0.172
(2) Peer vs. SOE | Non-Recip	-	0.851	**0.034**	0.689	0.465	**0.001**

Note: Panel probit regressions with subject random effects. Standard errors clustered at the session level reported in parentheses. We pool data for E_2 who observe that their peer chose S_L in the peer and SOE treatments. Bold values represent significant differences at the 5% level. We adjust p-values using the Holm method (Holm 1979) for multiple hypothesis testing. Missing values when $(\omega, \delta) = (25, 10)$ are due to perfect separation and lack of observations. We control for round and demographics. Significance reported: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

Table 18 Procurement Game vs. Social Norm — Observed Employees, E1 (Peer vs. SOE)

		Probability of choosing S_H					
		$\omega = 25$			$\omega = 40$		
		$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Procurement Game	SOE	0.440	0.641*	1.040**	-0.173	0.332	0.061
		(0.368)	(0.350)	(0.430)	(0.507)	(0.433)	(0.405)
	Constant	-0.325	-1.367**	-2.982***	-0.669	-3.117***	-3.757***
		(0.736)	(0.531)	(1.088)	(0.848)	(0.750)	(0.878)
	Controls	Yes	Yes	Yes	Yes	Yes	Yes
	Observations	438	438	438	438	438	438
		Appropriateness of choosing S_H					
Social Norm	SOE	0.958***	1.019***	5.249***	1.138***	0.378	0.581
		(0.254)	(0.206)	(0.319)	(0.305)	(0.331)	(0.436)
	Observations	49	49	49	49	49	49

Note: The top panel reports the results of panel probit regression considering as dependent variable y_{ist} (from the *procurement game*), pooling data from observed employees (E_1) in the peer and SOE treatments. The bottom panel corresponds to the norm elicitation treatments, and reports the estimates of ordered probit regressions with errors clustered at the session level reported in parentheses. The dependent variable is the social appropriateness of choosing S_H (in a scale from 1 to 4). The independent variable is a dummy that takes value 1 when subjects evaluate the appropriateness of observed employees' decisions in the SOE norm elicitation, and 0 when subjects evaluate the appropriateness of employees' decisions in the peer norm elicitation. Significance reported: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

Table 19 Differences across Treatments — General Results

	Probability of choosing S_H							
	$\omega = 25$			$\omega = 40$				
	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$	Avg. Wage	Avg. Cost
Baseline	0.917	0.693	0.516	0.802	0.505	0.417	28.125	60.208
	(0.216)	(0.374)	(0.443)	(0.329)	(0.433)	(0.430)	(6.124)	(11.651)
Peer	0.853	0.694	0.502	0.806	0.530	0.440	28.269	61.314
	(0.243)	(0.313)	(0.392)	(0.296)	(0.366)	(0.384)	(6.206)	(12.363)
WOE	0.964	0.809	0.646	0.874	0.577	0.484	28.529	63.660
	(0.100)	(0.270)	(0.367)	(0.224)	(0.337)	(0.377)	(6.378)	(10.870)
SOE	0.938	0.799	0.629	0.809	0.515	0.376	28.514	63.581
	(0.119)	(0.248)	(0.343)	(0.281)	(0.372)	(0.370)	(6.367)	(12.721)
	Tests (p-value)							
(1) Peer vs. Baseline	**0.042**	0.653	0.741	0.543	0.774	0.734	0.915	0.335
(2) Peer vs. WOE	**0.000**	**0.013**	**0.016**	0.193	0.399	0.404	0.748	**0.043**
(3) Peer vs. SOE	**0.017**	**0.025**	**0.033**	0.878	0.712	0.262	0.775	0.121

Note: Standard errors reported in parentheses. The first six columns consider the average probability of choosing S_H. The next column considers the average wage, and the last column considers the average cost per employee. In each case we consider the data that is aggregated at the subject level. Tests 1, 2, and 3 are Wilcoxon rank-sum tests. Bold values represent significant differences at the 5% level.

Appendix C: Effect of Wage and Price

C.1. Effect of Wage and Price

Baseline Treatment Table 20 reports the mean and standard deviation of the probability of choosing S_H aggregated at the individual level when subjects play in each role. Since the game in the baseline treatment is symmetric, we expect to find no differences in a subject's behavior in the roles of E_1 and E_2. This result is confirmed by the tests in Table 20. Since there are no significant differences, for the rest of the analysis we pool the data from E_1 and E_2 in the baseline treatment.

Table 20 shows that the probability of choosing S_H decreases as the wage, ω, and the price difference, δ, increase. The effect of wage is significant for all price differences (Wilcoxon signed-rank test, p-value ≤ 0.009) and the effect of price difference is significant, both when the wage is 25 and 40 (Wilcoxon signed-rank test p-value ≤ 0.001 when $\omega = 25$ and p-value ≤ 0.030 when $\omega = 40$ for all pairs of price differences; Kruskal-Wallis test $p \leq 0.001$).

Table 20 Baseline — Frequency of Choosing S_H by Role

	$\omega = 25$			$\omega = 40$		
	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
E_1	0.93	0.73	0.52	0.82	0.50	0.44
	(0.22)	(0.36)	(0.46)	(0.34)	(0.46)	(0.47)
E_2	0.91	0.69	0.51	0.81	0.55	0.43
	(0.23)	(0.42)	(0.47)	(0.33)	(0.46)	(0.44)
Difference (p-value)	0.622	0.449	0.629	0.703	0.276	0.863

Note: Standard errors reported in parentheses. We report the p-values of Wilcoxon signed-ranked tests for subject-level pairwise comparisons when subjects play as E_1 and E_2.

Peer Treatment Table 21 presents the subject level average of the probability of choosing S_H across the six rounds played, separately for subjects in the role of E_1 and E_2. We observe that, in the peer treatment, the probability of choosing the expensive supplier (S_H) is different depending on the role played. The last row of the table shows that subjects who play in the role of E_2 are more likely to choose S_H compared to those who play in the role of E_1, and these differences are significant in all cases where $\delta \geq 25$. Given these differences, we analyze separately the behavior of employees who are *observers* (play in the role of E_2) from those who are *observed* (play in the role of E_1).

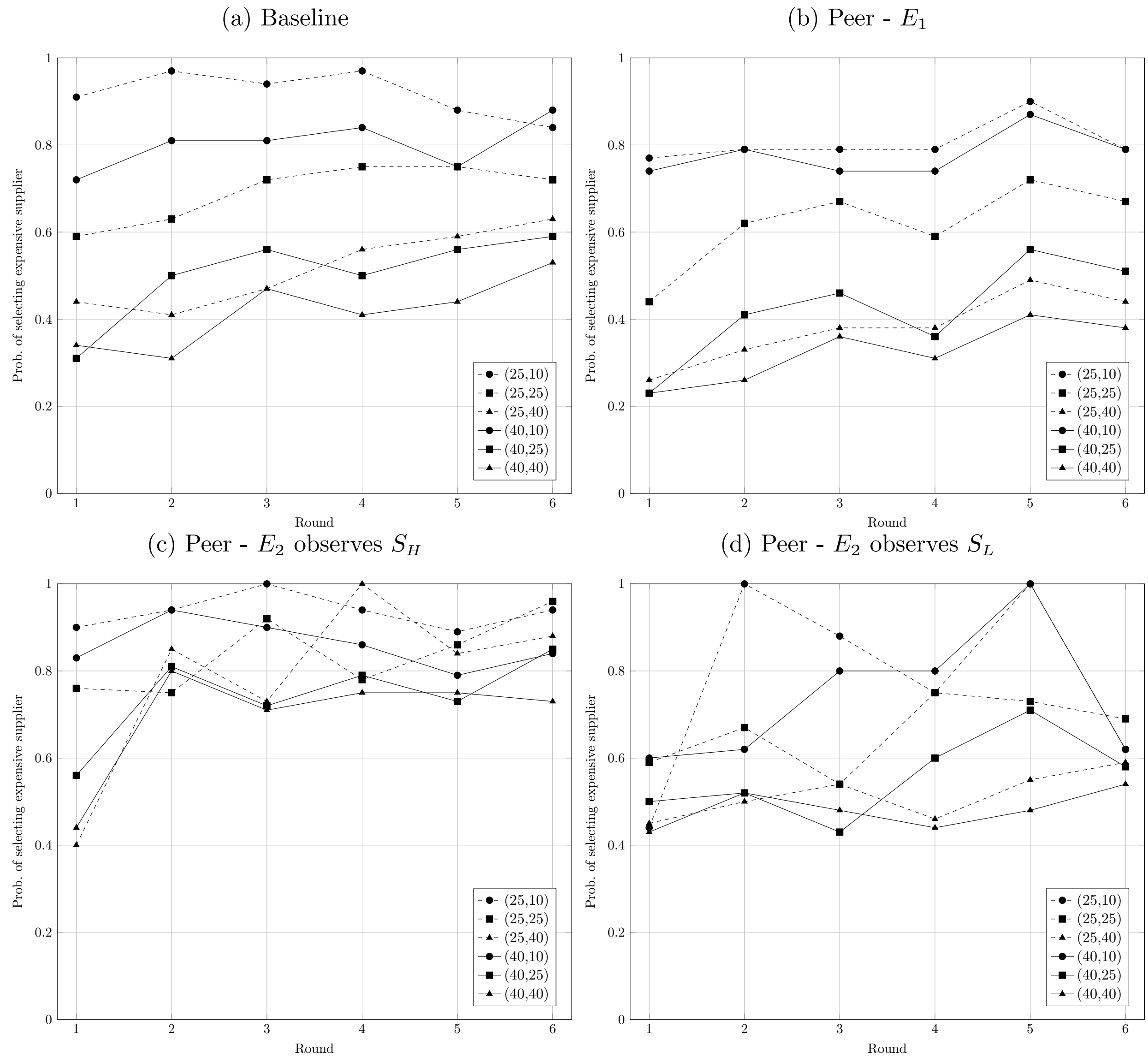
Table 21 Peer — Frequency of Choosing S_H by Role

	$\omega = 25$			$\omega = 40$		
	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
E_1	0.81	0.62	0.38	0.78	0.42	0.32
	(0.29)	(0.34)	(0.38)	(0.32)	(0.36)	(0.34)
E_2	0.90	0.77	0.62	0.83	0.64	0.56
	(0.17)	(0.26)	(0.36)	(0.27)	(0.34)	(0.39)
Difference (p-value)	0.243	**0.048**	**0.007**	0.513	**0.009**	**0.011**

Note: Standard errors reported in parentheses. We report the p-values of Wilcoxon rank-sum tests for differences between E_1 and E_2. Bold values represent significant trends at the 5% level.

Similar to what we find in the baseline, we observe that the probability of choosing S_H is decreasing in ω (statistically significant for observers and observed separately when $\delta = 25$, Wilcoxon signed-rank p-value ≤ 0.003 , and marginally significant for observers when $\delta = 40$, p-value $= 0.086$) and in price difference δ (signed-rank p-value ≤ 0.002 when $\omega = 25$ and p-value ≤ 0.024 when $\omega = 40$ for all pairs of price differences; Kruskal-Wallis test p-value ≤ 0.001 for observed and observers separately). Overall, the results indicate that the probability of choosing the expensive supplier is decreasing in wage and price difference in both treatments.

Figure 7 Probability of choosing S_H by Period and Condition



Appendix D: Dynamics of Play in the Baseline and Peer Treatments

D.1. Changes in Behavior with Rounds

We are interested in studying whether behavior changes as rounds in a session elapse. Figure 7 shows the evolution of the probability of choosing S_H among (a) employees in the baseline treatment, (b) E_1 in the peer treatment, (c) E_2 in the peer treatment who observe S_H, and (d) E_2 in the peer treatment who observe S_L. Note that employees in the baseline treatment and E_1 in the peer treatment do not learn about the population as rounds in a session elapse. On the other hand, E_2 in the peer treatment observes the decision of a different peer in each round, therefore, his later decisions may be affected by his observations of peers in earlier rounds.

We observe that there is a slight upward trend in the probability of choosing the expensive supplier both for employees in the baseline treatment and for E_1 in the peer treatment (Figures 7a and 7b), suggesting that subjects become more likely to choose the expensive supplier even in the absence of learning about the population. In addition, we observe a parallel downward shift for E_1 in the peer treatment relative to the baseline. This confirms that the positive effect on observed employees remains steady over rounds (for all combinations of wage and price difference). Figure 7c presents the dynamics of play for E_2 in the peer treatment who observes S_H. In this case there seems to be steep increase in the probability of choosing S_H from round 1 to 2, and then this probability remains relatively stable in rounds 2 onwards for all combinations of wage and price difference. Finally, Figure 7d shows the dynamics of play for E_2 who observes S_L. In this case, the probability of choosing S_H is relatively stable, with spikes for $\delta = 10$ (where the number of observations is smaller).

To formally test whether subjects' behavior presents trends across rounds, Table 22 reports the average probability of choosing S_H in each round and for each combination of wage and price difference in the four cases analyzed in Figure 7. Tests (1) and (2) at the bottom of each panel correspond to a nonparametric test for trends across ordered groups.[29] The tests show under various conditions a significant trend of increasing propensity to choose the expensive supplier as rounds elapse. Nevertheless, a large part of this trend is attributed to the learning between rounds 1 and 2—when round 1 is excluded from the analysis (test 2), the trends are no longer significant in most conditions.

In Table 23 we compare the probability of choosing S_H in rounds 1 to 3 vs. 4 to 6 for each treatment and role. We observe that in most situations the difference between the first and second half of the rounds is negative, indicating that employees are more likely to choose S_H in the last three rounds of play. Note, however, that these differences are only significant when $(\omega, \delta) = (25, 40)$ for employees in the baseline treatment and E_1 in the peer treatment. We confirm that the main results in the paper remain directionally the same if we consider only the last three rounds of play.

D.2. Cumulative Effect of Learning on Observers

We next focus on E_2 in the peer treatment, who observes the decisions of a peer in each round. We examine whether past observations affect the behavior of an E_2 in the peer treatment, separating those who in the current round observe that the peer chose S_H from those who in the current round observe that a the peer chose S_L. In Table 24, we consider rounds 2 to 6 and examine whether the probability of choosing the expensive supplier changes with the interaction between what the employees observed in the previous period (observed H or observed L) and what they observe in the current one (observes H or observes L). The tests at the bottom of the table show that there is a significant negative effect of having observed H in the previous round in only one of the six situations for employees who observe that their peer chose S_H in the current round, and there are no significant effects for employees who observe S_L in the current round. This suggests that employees mostly care about what they observe in the current round (i.e., within their current organization), and that this effect outweighs the effect of what they observed in the previous round.

[29] The nonparametric test for trend across ordered groups developed by Cuzick (1985) is an extension of the Wilcoxon rank-sum test.

Table 22 Frequency of Choosing S_H by Role and Treatment

	Panel 1: Baseline						Panel 2: Peer - E_1					
	$\omega = 25$			$\omega = 40$			$\omega = 25$			$\omega = 40$		
Round	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
1	0.91	0.59	0.44	0.72	0.31	0.34	0.77	0.44	0.26	0.74	0.23	0.23
	(0.30)	(0.50)	(0.50)	(0.46)	(0.47)	(0.48)	(0.43)	(0.50)	(0.44)	(0.44)	(0.43)	(0.43)
2	0.97	0.63	0.41	0.81	0.50	0.31	0.79	0.62	0.33	0.79	0.41	0.26
	(0.18)	(0.49)	(0.50)	(0.40)	(0.51)	(0.47)	(0.41)	(0.49)	(0.48)	(0.41)	(0.50)	(0.44)
3	0.94	0.72	0.47	0.81	0.56	0.47	0.79	0.67	0.38	0.74	0.46	0.36
	(0.25)	(0.46)	(0.51)	(0.40)	(0.50)	(0.51)	(0.41)	(0.48)	(0.49)	(0.44)	(0.51)	(0.49)
4	0.97	0.75	0.56	0.84	0.50	0.41	0.79	0.59	0.38	0.74	0.36	0.31
	(0.18)	(0.44)	(0.50)	(0.37)	(0.51)	(0.50)	(0.41)	(0.50)	(0.49)	(0.44)	(0.49)	(0.47)
5	0.88	0.75	0.59	0.75	0.56	0.44	0.90	0.72	0.49	0.87	0.56	0.41
	(0.34)	(0.44)	(0.50)	(0.44)	(0.50)	(0.50)	(0.31)	(0.46)	(0.51)	(0.34)	(0.50)	(0.50)
6	0.84	0.72	0.63	0.88	0.59	0.53	0.79	0.67	0.44	0.79	0.51	0.38
	(0.37)	(0.46)	(0.49)	(0.34)	(0.50)	(0.51)	(0.41)	(0.48)	(0.50)	(0.41)	(0.51)	(0.49)
						Tests (p-value)						
(1) 1 to 6	0.17	0.13	**0.03**	0.29	**0.04**	0.09	0.41	**0.03**	**0.04**	0.38	**0.01**	0.06
(2) 2 to 6	**0.04**	0.39	**0.04**	0.77	0.50	0.14	0.60	0.53	0.22	0.54	0.22	0.20

	Panel 3: Peer - E_2 observes S_H						Panel 4: Peer - E_2 observes S_L					
	$\omega = 25$			$\omega = 40$			$\omega = 25$			$\omega = 40$		
Round	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
1	0.90	0.76	0.40	0.83	0.56	0.44	0.44	0.59	0.45	0.60	0.50	0.43
	(0.31)	(0.44)	(0.52)	(0.38)	(0.53)	(0.53)	(0.53)	(0.50)	(0.51)	(0.52)	(0.51)	(0.50)
2	0.94	0.75	0.85	0.94	0.81	0.80	1.00	0.67	0.50	0.62	0.52	0.52
	(0.25)	(0.44)	(0.38)	(0.25)	(0.40)	(0.42)	(0.00)	(0.49)	(0.51)	(0.52)	(0.51)	(0.51)
3	1.00	0.92	0.73	0.90	0.72	0.71	0.88	0.54	0.54	0.80	0.43	0.48
	(0.00)	(0.27)	(0.46)	(0.31)	(0.46)	(0.47)	(0.35)	(0.52)	(0.51)	(0.42)	(0.51)	(0.51)
4	0.94	0.78	1.00	0.86	0.79	0.75	0.75	0.75	0.46	0.80	0.60	0.44
	(0.25)	(0.42)	(0.00)	(0.35)	(0.43)	(0.45)	(0.46)	(0.45)	(0.51)	(0.42)	(0.50)	(0.51)
5	0.89	0.86	0.84	0.79	0.73	0.75	1.00	0.73	0.55	1.00	0.71	0.48
	(0.32)	(0.36)	(0.37)	(0.41)	(0.46)	(0.45)	(0.00)	(0.47)	(0.51)	(0.00)	(0.47)	(0.51)
6	0.94	0.96	0.88	0.84	0.85	0.73	0.62	0.69	0.59	0.62	0.58	0.54
	(0.25)	(0.20)	(0.33)	(0.37)	(0.37)	(0.46)	(0.52)	(0.48)	(0.50)	(0.52)	(0.51)	(0.51)
						Tests (p-value)						
(1) 1 to 6	0.88	0.07	**0.01**	0.41	0.29	0.31	0.63	0.37	0.36	0.42	0.23	0.64
(2) 2 to 6	0.38	0.11	0.53	0.13	0.77	0.85	0.09	0.58	0.58	0.84	0.28	0.91

Note: Standard errors reported in parentheses. Tests 1 and 2 are NP-trend tests considering all rounds and rounds two to six, respectively. Bold values represent significant trends at the 5% level.

It is also possible that learning in a session occurs over several rounds of play. We next examine whether longer cumulative effects play a role, by examining the decisions of E_2 in the peer treatment in rounds 5 and 6—where they have experienced at least four rounds of learning. Table 25 presents a probit regression of the probability of choosing S_H on the cumulative number of times the employee has seen his peers chose S_H in the first four rounds, controlling for his observation in the current round and his own previous decisions. In particular, we create a dummy variable, observed H ($\geq K$), that is equal to 1 if the employee observed that his peers chose S_H in at least K of the first four rounds, and 0 otherwise, for $K \in \{2, 3, 4\}$. The tests at the bottom of the table compare the frequency of choosing S_H among employees who saw that their peers chose S_H at least K times and emploees who saw that their peers chose S_H less than K times, conditioning on what they observe in the current round. The results show that, for most situations, there are no significant differences between employees who observe that their peers chose the expensive supplier in at least K of the periods and those who observe the opposite, regardless of whether they observe S_H or S_L in the current

Table 23 Frequency of Choosing S_H by Role and Treatment — First and Second Half

		$\omega = 25$			$\omega = 40$		
		$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Baseline	Rounds 1 to 3	0.94	0.65	0.44	0.78	0.46	0.38
		(0.24)	(0.48)	(0.50)	(0.42)	(0.50)	(0.49)
	Rounds 4 to 6	0.90	0.74	0.59	0.82	0.55	0.46
		(0.31)	(0.44)	(0.49)	(0.38)	(0.50)	(0.50)
	Difference (p-value)	0.384	0.092	0.029	0.521	0.112	0.143
Peer - E_1	Rounds 1 to 3	0.79	0.57	0.32	0.76	0.37	0.28
		(0.41)	(0.50)	(0.47)	(0.43)	(0.48)	(0.45)
	Rounds 4 to 6	0.83	0.66	0.44	0.80	0.48	0.37
		(0.38)	(0.48)	(0.50)	(0.40)	(0.50)	(0.48)
	Difference (p-value)	0.219	0.174	0.037	0.168	0.097	0.248
Peer - E_2 Observes S_H	Rounds 1 to 3	0.95	0.82	0.68	0.89	0.72	0.67
		(0.23)	(0.39)	(0.47)	(0.32)	(0.45)	(0.48)
	Rounds 4 to 6	0.92	0.87	0.90	0.83	0.79	0.74
		(0.28)	(0.34)	(0.30)	(0.38)	(0.41)	(0.44)
	Difference (p-value)	0.509	0.169	0.105	0.163	0.166	0.226
Peer - E_2 Observes S_L	Rounds 1 to 3	0.76	0.6	0.49	0.68	0.49	0.48
		(0.44)	(0.49)	(0.50)	(0.48)	(0.50)	(0.50)
	Rounds 4 to 6	0.75	0.73	0.53	0.78	0.62	0.49
		(0.44)	(0.45)	(0.50)	(0.42)	(0.49)	(0.50)
	Difference (p-value)	0.699	0.550	0.944	0.367	0.241	0.716

Note: Standard errors reported in parentheses. We report the p-values of Wilcoxon signed-rank tests comparing Rounds 1 to 3 - Rounds 4 to 6: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

Table 24 Effect of Learning from Previous Round

	Probability of choosing S_H					
	$\omega = 25$			$\omega = 40$		
	$\delta = 10$	$\delta = 25$	$\delta = 40$	$\delta = 10$	$\delta = 25$	$\delta = 40$
Observed L × Observes H	-0.049	0.391	0.961*	0.533	0.823*	1.261**
	(0.531)	(0.337)	(0.531)	(0.443)	(0.491)	(0.527)
Observed H × Observes L	-0.291	-0.212	0.289	-0.360	-0.511	0.110
	(0.538)	(0.473)	(0.408)	(0.445)	(0.356)	(0.393)
Observed H × Observes H	0.828	0.883	1.992***	0.308	0.372	0.672*
	(0.669)	(0.587)	(0.572)	(0.494)	(0.468)	(0.404)
Constant	1.425*	0.765	0.562	2.639**	0.633	1.200
	(0.770)	(0.955)	(1.731)	(1.268)	(1.209)	(1.606)
Observations	135	160	195	195	195	195
	Tests (p-value)					
(1) Observed H vs. Observed L | Observes H	**0.000**	0.573	0.073	0.601	0.286	0.348
(2) Observed H vs. Observed L | Observes L	0.589	0.654	0.479	0.837	0.302	0.780

Note: Panel probit regressions with subject random effects. Standard errors clustered at the session level reported in parentheses. We consider data for E_2 in the peer treatment in rounds 2 to 6. Bold values represent significant differences at the 5% level. We adjust p-values using the Holm method (Holm 1979) for multiple hypothesis testing. Missing values when $(\omega, \delta) = (25, 10)$ and $(25, 25)$ are due to perfect separation. We control for round and demographics. Significance reported: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

round. Overall, these results suggest that there is a weak cumulative effect over rounds and that employees are mostly affected by what they observe in the current period.

Table 25 Effect of Cumulative Learning

	Probability of choosing S_H																	
	Panel 1: $K=2$						Panel 2: $K=3$						Panel 3: $K=4$					
	$\omega=25$			$\omega=40$			$\omega=25$			$\omega=40$			$\omega=25$			$\omega=40$		
	$\delta=10$	$\delta=25$	$\delta=40$	$\delta=10$	$\delta=25$	$\delta=40$	$\delta=10$	$\delta=25$	$\delta=40$	$\delta=10$	$\delta=25$	$\delta=40$	$\delta=10$	$\delta=25$	$\delta=40$	$\delta=10$	$\delta=25$	$\delta=40$
Observed H ($\geq K$)	-0.014	0.194	-0.354*	0.010	-0.005	-0.069	0.058	-0.184	0.028	0.067	0.182*	-0.014	-0.017	0.146	0.068	-0.127	0.080	0.089
× Observes L	(0.074)	(0.255)	(0.188)	(0.047)	(0.180)	(0.113)	(0.168)	(0.191)	(0.159)	(0.110)	(0.095)	(0.113)	(0.063)	(0.131)	(0.114)	(0.083)	(0.075)	(0.081)
Observed H ($< K$)	0.339*	0.310	-0.094	-0.059	0.224	0.095	-0.073	0.088	0.070	-0.242***	0.095	0.122	0.181*	0.211	-0.017	-0.038	0.231	-0.042
× Observes H	(0.180)	(0.290)	(0.137)	(0.217)	(0.150)	(0.101)	(0.107)	(0.155)	(0.126)	(0.083)	(0.097)	(0.091)	(0.100)	(0.136)	(0.138)	(0.109)	(0.174)	(0.076)
Observed H ($\geq K$)	0.067	0.288	0.054	-	-0.024	0.048	0.189	0.009	0.144	0.087	0.109	0.018	0.261***	0.200	0.197	0.062	0.152	0.226
× Observes H	(0.064)	(0.249)	(0.142)	-	(0.131)	(0.078)	(0.141)	(0.137)	(0.138)	(0.120)	(0.103)	(0.064)	(0.082)	(0.186)	(0.205)	(0.103)	(0.120)	(0.287)
Num. times	0.141**	0.058*	0.186***	0.235***	0.196***	0.224***	0.135***	0.063**	0.161***	0.241***	0.185***	0.231***	0.115**	0.052	0.160***	0.230***	0.177***	0.219***
chose S_H	(0.061)	(0.030)	(0.034)	(0.020)	(0.028)	(0.014)	(0.045)	(0.026)	(0.034)	(0.022)	(0.036)	(0.017)	(0.051)	(0.033)	(0.033)	(0.020)	(0.039)	(0.021)
Constant	1.111***	0.454	0.496	1.382***	0.607	0.720**	1.298***	0.479	0.763	1.526***	0.874***	0.739**	1.644***	0.558	0.828**	1.651***	0.951***	0.806***
	(0.381)	(0.515)	(0.498)	(0.450)	(0.415)	(0.303)	(0.497)	(0.483)	(0.488)	(0.436)	(0.330)	(0.315)	(0.464)	(0.460)	(0.397)	(0.500)	(0.333)	(0.251)
Observations	78	78	78	78	78	78	78	78	78	78	78	78	78	78	78	78	78	78
	Tests (p-value)																	
(1) Observes H ($\geq K$) vs ($< K$)	0.073	0.882	0.138	1.000	**0.030**	0.690	**0.018**	0.552	0.485	**0.000**	0.899	0.069	**0.000**	0.403	0.759	0.133	0.470	0.651
(2) Observes L ($\geq K$) vs ($< K$)	0.845	0.896	0.119	0.831	0.978	1.000	0.727	0.336	0.862	0.541	0.113	0.902	0.070	0.238	0.900	0.727	0.369	1.000

Note: Panel probit regressions with subject random effects. Standard errors clustered at the session level reported in parentheses. All panels consider only data for E_2 in the peer treatment in rounds 5 and 6. Panel 1 interacts a dummy variable equal to 1 if E_2 has observed a peer choosing S_H at least twice in the first 4 rounds with a dummy variable that is 1 if he observes S_H in the current round. Panel 2 interacts a dummy variable equal to 1 if E_2 has observed a peer choosing S_H at least three times in the first 4 rounds with a dummy variable that is 1 if he observes S_H in the current round. Panel 3 interacts a dummy variable equal to 1 if E_2 has observed a peer choosing S_H exactly four times in the first 4 rounds with a dummy variable that is 1 if he observes S_H in the current round. Missing values for $K = 2, (\omega, \delta) = (40, 10)$ are due to the absence of observations for observed H ($< K$) × observes L, so observed H ($\geq K$) × observes H is omitted because of collinearity. The tests report the p-values of the comparison between Observed H ($\geq K$)×Observes H vs. Observed H ($< K$)×Observes H (Test 1), and Observed H ($\geq K$)×Observes L vs. Observed H ($< K$)×Observes L (Test 2). Bold values represent significant differences at the 5% level. We adjust p-values using the Holm method (Holm 1979) for multiple hypothesis testing. We control for round and demographics. Significance reported: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$.

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School Choice in Chile

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Centralized school admission mechanisms are an attractive way of improving social welfare and fairness in large educational systems. In this paper we report the design and implementation of the newly established school choice system in Chile, where over 274,000 students applied to more than 6,400 schools. The Chilean system presents unprecedented design challenges that make it unique. On the one hand, it is a simultaneous nationwide system, making it one of the largest school choice problems worldwide. On the other hand, the system runs at all school levels, from Pre-K to 12th grade, raising at least two issues of outmost importance; namely, the system needs to guarantee their current seat to students applying for a school change, and the system has to favor the assignment of siblings to the same school. As in other systems around the world, we develop a model based on the celebrated Deferred Acceptance algorithm. The algorithm deals not only with the aforementioned issues, but also with further practical features such as soft-bounds and overlapping types. In this context, we analyze new stability definitions, present the results of its implementation and conduct simulations showing the benefits of the proposed innovations.

Key words: school choice, matching, two-sided market

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1. Introduction

According to the Duncan Index of segregation, Chilean schools are extremely socially segregated (Bellei 2013, Valenzuela et al. 2014). Several authors have shown that the costs of school segregation are important, including low social cohesion and lack of equal opportunities and social mobility (Villalobos and Valenzuela 2012, Wormald et al. 2012). While the drivers of school segregation include societal aspects well beyond school choice, social movements and politicians were probably right at blaming features of the admission system.

The School Inclusion Law marks a breaking point in the organization and functioning of the school system. The Law, promulgated in 2015, changed the old admission process drastically by (i) eliminating co-payments in publicly subsidized schools, (ii) forbidding publicly subsidized schools from selecting their students based on social, religious, economic, or academic criteria, and (iii) defining priorities that must be used to assign students to schools. [1]

In this paper we report the results of an ongoing collaboration with the Chilean Ministry of Education (MINEDUC) addressing the practical challenges of implementing the School Inclusion Law. To this end, we designed and implemented a centralized system that (i) provides information about schools—seats available, mission, values, educational project, among others—to help parents and students in building their preferences; (ii) collects families' preferences through an online platform, reducing the time and cost that visiting each school involved in the past; and (iii) assigns students to schools using a transparent, fair and efficient procedure.

One of the distinctive features of the new school choice system in Chile is its broadness, as it runs nationwide and throughout all school levels. Being nationwide, the system accommodates the needs of both urban and rural families. Since the system runs throughout all school levels, it needs to favor the joint allocation of siblings. These two aspects are unique to our problem and bring new design challenges for school choice.

At the core of the system is the assignment algorithm, which adapts the celebrated Deferred Acceptance (DA) algorithm—introduced in the seminal paper by Gale and Shapley (1962)—to incorporate all the elements required by law and by MINEDUC. In particular, the system considers a set of priority groups—students with siblings in the school, students with parents that work in the school and former students of the school—that are served in strict order of priority. The system also includes quotas for students (i) from disadvantageous environments, (ii) with special educational needs and disabilities, and (iii) with high academic performance. Within each priority group, ties are broken randomly.

The admission system runs throughout all educational levels from the highest (12th grade) to the lowest (Pre-K). This makes patent the need to secure their current enrollment to students that want to change to another school. Additionally, some families may have two or more of their children simultaneously applying to schools and may naturally want their children to attend the same school. Two features of our implementation favor the assignment of siblings to the same school. First, the tie-breaking lotteries are run over families rather than over students. Lotteries over families create correlation in the priority ranks of siblings applying to different levels in a given school. As our results and examples show, this correlation makes more likely siblings end up assigned to the same school. Second, families can express their willingness to have their children assigned to the same school by filling a *family application* (FA). A family application ensures that once the oldest child is assigned to some school, the application of the younger siblings are modified to put that school as their most preferred one.

In summary, our design presents at least four innovative features:

- It deals with multiple overlapping quotas.
- It allows currently enrolled students to apply to a different school while guaranteeing her current seat.
- It runs tie-breaking lotteries over families, significantly increasing the fraction of siblings that end up assigned to the same school.
- It adds a family application heuristic that improves the chances of having siblings assigned to the same school.

The results reported in this paper consider the current state of the system, which includes all regions, except the Metropolitan area of Santiago, reaching 274,990 students and 6,421 schools in the main round. In the current admission process—for students who started their academic year in March, 2019—students applied to 3.4 schools on average, and 59.2% of students were assigned to their top preference. Moreover, 82.5% of students were assigned to one of the schools in their application list, 8.6% were assigned by secured enrollment to their current schools, and only 8.9% resulted unassigned. In addition, there were 10,301 family applications involving 21,424 students and 65.3% of these were successful, i.e., siblings got assigned to the same school, while 3% were partially successful, i.e., only a subset of siblings got assigned together.[2] We also provide simulations evaluating different elements of our design.

Designing, implementing and improving the Chilean school choice system has resulted in many lessons that could be useful for other practitioners designing large-scale clearinghouses. From a theoretical standpoint, we contribute to the existing literature by introducing the notion of family applications. We show that a stable matching may not exist, and we provide heuristics that are successful at increasing the fraction of siblings assigned to the same school. Finally, our results show that having lotteries over families (considering students applying to a given school at all levels simultaneously) significantly increases the fraction of siblings assigned to the same school. From a practical standpoint, a key lesson is that having a continuous communication and collaboration with policy-makers is essential, as many aspects evolve over time and must be incorporated in the design. In addition, fragmenting the implementation in a given number of steps allowed us to gain experience, solve unexpected problems and continuously improve the system. As centralized procedures to assign students to schools are becoming the norm in many countries, we expect that the lessons and solutions offered in this work are deemed useful in other implementations.

The remainder of the paper is organized as follows. In Section 2 we describe the school choice problem in Chile, with the main features requested by law and by MINEDUC. In Section 3 we discuss how this paper relates to several strands of the literature. In Section 4 we present our model and describe its implementation. In Section 5 we present the results, focusing on the admission process of 2018. In addition, we evaluate the effects of (i) quotas for disadvantaged students and (ii) family applications via simulations. Finally, in Section 6 we conclude and provide directions for future work.

2. The problem in Chile

The Chilean school choice system considers fourteen levels, ranging from pre-kindergarten to 12th grade. There are five entry levels: pre-kindergarten, kindergarten, 1st, 7th and 9th grade, which are the levels where a school can start. Depending on their type of funding, schools can be classified in three types: (1) private, for those schools that are independent and privately funded; (2) voucher, where families make co-payments to complement state subsidies; and (3) public, for those schools that are fully funded and operated by local governments. Voucher and public schools, which are the focus of this paper, account for more than 90.3% of the total number of students in primary and secondary education (MINEDUC 2018).

Before the introduction of the School Inclusion Law, schools ran their admission processes independently, often selecting their students based on arbitrary rules, such as interviews with the students and their parents, results of unofficial admission exams, past academic records, among many others. Since the admission processes were not coordinated, in many cases parents were forced to strategically decide whether to accept an offer or to reject it and wait until other schools released their admission offers. Moreover, many schools used “first-come first-served” rules to prioritize students, resulting in many parents waiting in long overnight queues to secure a seat for their children. Overall, the freedom of schools to choose their students and the existence of voucher schools are considered among the main reasons that explain the polarization and segregation of the Chilean school system (Valenzuela et al. 2014).

To address these problems, the School Inclusion Law forbids any sort of discrimination in the admission processes of schools that receive (partial or full) government funding, and mandates schools to use a centralized system that collects families' and students' preferences and returns a fair allocation. In this system, students and families can access a platform where they can collect information—number of open seats, number of students per classroom and level, educational project, rules and values, co-payments required, among others—to build their preferences, and later they can use it to apply to as many schools as they want by submitting a strict order of preferences. The system collects all these applications and runs a mechanism that aims to assign each student to their top preference provided that there are enough seats available. More specifically, if the number of applicants is less than the number of open seats, the law requires that all students applying to that school are admitted, unless they can be allocated in a school they prefer over it. On the other hand, for schools that are over-demanded the law defines a set of priority groups that are used to order students. In particular, there are three priority groups, which are processed in strict order of priority:

1. *Siblings.* For students that have a sibling already enrolled at the school.
2. *Working parent.* For students that have a parent working at the school.
3. *Former students.* For students that were enrolled at the school in the past and were not expelled from it.

Within each priority group students are randomly ordered and each school uses a different random tie-breaker. In addition to these priorities, the law specifies three different types of quotas:

1. *Special needs.* This quota serves students with disabilities. It reserves at most two seats per classroom per school and it is processed before any other priority group or quota. The quota only applies to schools that have a validated special program.
2. *High-achieving.* This quota applies to students with high academic performance. It is processed right after the special needs quota and considers between 30% to 85% of the total number of seats depending on the school. Only a subset of pre-selected schools can implement this quota in 7th and 9th grade, and students can only be ranked based on an admission exam.
3. *Disadvantaged.* This quota prioritizes the most vulnerable students (bottom third in terms of income according to the Social Registry of Homes). At each level in every school, 15% of seats are reserved for disadvantaged students, and this group is processed right after the first priority group, i.e., students with siblings.

There are two additional features that are relevant in the design of the system. First, students that are currently enrolled at a school and apply in the system aiming to change their allocation have a *secured enrollment* in their current school, i.e., in case of not being assigned to a school of their preference they can keep their current assignment. Second, families having two or more children that participate in the centralized system can choose to *apply as a family*, which means

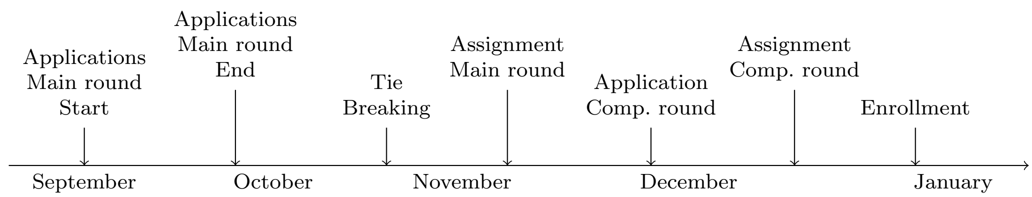

Figure 1 Timeline of the Admission Process

that they prioritize having their children assigned to the same school over a better school in their reported preferences.

The timing of the admission process is summarized in Figure 1. After collecting information, families submit their application between September and October. After all applications are received, the centralized mechanism generates the lotteries that will be used to order students in over-demanded schools and executes the main round of the process. For those students that result unassigned or did not apply in the main round, there is a complementary process where they submit a new application list that only includes schools with available seats. Finally, students that result unassigned in the complementary round are assigned to the closest school with available seats that does not charge a co-payment. We refer to this as *distance assignment*. In case that there are no schools with seats available within 17km, students remain unassigned and MINEDUC gives them a solution. The process ends in late December with the enrollment.

3. Literature

This paper is related to four strands of the literature: school choice, affirmative action, assignment of families and tie-breaking.

School Choice. In the last two decades, and starting from the theoretical formalization of the school choice problem by Abdulkadiroğlu and Sönmez (2003), there have been reforms to the school choice system in many places around the world. The first major reform was introduced in New York, where a variation of the Deferred Acceptance (DA) algorithm with restricted lists was implemented (Abdulkadiroğlu et al. 2005a). In 2005, the Boston Public School system decided to switch from the so called *Boston Mechanism* (BM), also known as *Immediate Acceptance* (IA) mechanism, to DA in order to address the strategic incentives introduced by the former algorithm (Abdulkadiroğlu et al. 2005b). Despite its lack of strategy-proofness, BM has been revisited in the last few years since it better captures cardinal preferences and therefore can lead to higher social welfare (Abdulkadiroğlu et al. 2011). Since then, other systems such as Barcelona (Calsamiglia and Güell 2018), Amsterdam (Gautier et al. 2016), New Orleans (Abdulkadiroglu et al. 2017), among others, have implemented centralized school choice systems using some variant of DA, BM or *top-trading cycles* (TTC). This paper contributes to this literature by adding a new case study with some additional features that have not been explored in previous literature, such as the admission of siblings in different levels and the secured enrollment problem. In addition, this is one of the first papers that describes the implementation of a system at a country level.

Priorities and Affirmative Action. Many school choice systems include affirmative action policies to promote diversity in the classrooms. Ehlers (2010) explores DA under type-specific quotas, finding that the student-proposing DA is strategy-proof for students if schools' preferences satisfy responsiveness. Kojima (2012) studies the implementation of majority quotas and shows that this may actually hurt minority students. Consequently, Hafalir et al. (2013) propose the use of minority reserves to overcome this problem, showing that DA with minority reserves Pareto dominates the one with majority quotas. Ehlers et al. (2014) extend the previous model to account for multiple disjoint types, and propose extensions of DA to incorporate soft and hard bounds. Other types of constraints are considered by Kamada and Kojima (2015), who study problems with distributional constraints motivated by the Japanese Medical Residency. Dur et al. (2016a) analyze the Boston school system and Dur et al. (2016b) analyze the impact of these policies using Chicago's system data. However, these models consider disjoint types of students. Kurata et al. (2017) study the overlapping types problem and show that, even in the soft-bound minority quotas scenario, a stable matching might not exist. As a solution, they propose a model in which stability is recovered by counting each student towards the seat type he was assigned to. We contribute to this literature by implementing a new case study and analyzing the effect of different minority quotas in the implementation. Moreover, we show that quotas depend on students truly being a minority and that there can also be some auto-segregation of minorities.

Assignment of families. The family application combines features of a many-to-many matching between families and schools and the existence of coalitions. A similar structure can be found in the labor market of medical residents, where couples prefer (in general) to be allocated in the same city. Roth (1984) shows that one cannot guarantee the existence of a matching without justified envy when couples have arbitrary preferences over pairs of hospitals. Kojima et al. (2013) show that a stable matching exists if the number of couples is relatively small and preference lists are sufficiently short relative to the size of the market. Another positive result is presented by Ashlagi et al. (2014), who introduce a new algorithm that finds a stable matching with high probability (in large matching markets) and where truth-telling becomes an approximate equilibrium for the induced game. We contribute to this literature by showing that a stable matching may not exist when there are family applications, and by introducing a new heuristic that can solve this problem.

Tie-breaking. A common approach to deal with ties in school choice is to use random tie-breaking rules, such as *single tie-breaking* (STB)—all schools use the same ordering for breaking ties—and *multiple tie-breaking* (MTB)—each school uses its own random order. Abdulkadiroğlu et al. (2009)

are the first to empirically compare these tie-breaking rules, and they find that there is no stochastic dominance in NYC. A similar pattern is found in Amsterdam, as described by De Haan et al. (2015). These findings are in line with the theoretical results by Ashlagi et al. (2019), who find that when there is low competition there is no stochastic dominance between random assignments. However, they also show that when there is a shortage of seats, STB almost dominates MTB and also leads to lower variance in students' rankings. Moreover, Arnosti (2015) shows that STB can lead to more matches when preferences are short and random. We contribute to this literature by introducing and studying the effect of family applications and breaking ties between families instead of students.

4. Model and Implementation

The Chilean school choice problem can be formalized as follows. Let K be the set of all levels, including pre-kindergarten and kindergarten in preschool, 1st to 8th grade in primary school and 9th to 12th grade in secondary school. Each school offers some or all of these levels, and students can only apply to schools that offer their level. For simplicity, we will first focus on a fixed level to define the basic setting, and later (in Section 4.3) we will introduce how levels interact with each other through the family application.

At a fixed level, let $S = \{s_1, ..., s_n\}$ be the set of students and $C = \{c_1, c_2, ..., c_m\}$ be the set of schools. Each school c has a capacity $q_c \in \mathbb{N}$ that accounts for the number of available seats. Students have a strict preference profile $\succ_S = (\succ_{s_1}, ..., \succ_{s_n})$ over schools, where $c \succ_s c'$ means that student s strictly prefers school c to c'. Students who rank a subset of schools implicitly declare that they prefer to be unassigned over being assigned to a school that is not in their preference list.

Schools, on the other hand, have a weak priority profile $\succsim_C = (\succsim_{c_1}, ..., \succsim_{c_m})$ over students, where $s \succsim_c s'$ means that at school c, student s has a higher or equal priority ranking than student s'. Students within a priority group are randomly ordered, creating a strict priority profile $\succ_C = (\succ_{c_1}, ..., \succ_{c_m})$ over students.

A **matching** μ is a function from the set $C \cup S$ to the subsets of $C \cup S$ such that:

(i) $\mu(s) \subseteq C, |\mu(s)| = 1$ or $\mu(s) = \emptyset$ for every student $s \in S$.

(ii) $\mu(c) \subseteq S$ and $|\mu(c)| \leq q_c$ for every school $c \in C$.

(iii) $c \in \mu(s)$ if and only if $s \in \mu(c)$, for every student $s \in S$ and school $c \in C$.

To simplify notation, for the case of a student $s \in S$ we write $\mu(s) = c$ to represent $\mu(s) = \{c\}$.

This definition formalizes that a feasible matching cannot assign more students to a school than its capacity, and it cannot assign a student to more than one school.

In what follows, we extend this model to address the particular features of the Chilean problem, namely: (1) secured enrollment, for students already enrolled at a school; (2) quotas for students of different types; (3) family applications; and (4) the tie breaking rule.

4.1. Secured enrollment.

The system allows students of any level to apply to schools of their choice, provided that those schools offer their level. In particular, some students may want to change to another school they prefer, but are better off at their current school than remaining unassigned or assigned to other schools they prefer less. The School Inclusion Law gives students the right to keep a seat at their current school in case they do not get a better assignment.

To address this requirement, we add to each student who seeks to change to another school a preference over their current school at the bottom of their preference list. In addition, we consider the secured enrollment of a student as a priority criterion with a higher priority than any other priority group. With this extra criterion, the complete list of priority groups is given by:

$$\text{Secured Enrollment} \succ_c \text{Siblings} \succ_c \text{Working Parent} \succ_c \text{Former Students} \quad \forall c \in C. \tag{1}$$

4.2. Quotas.

In order to promote diversity within schools, the School Inclusion Law includes affirmative action policies for financially disadvantaged students and children with special needs. Furthermore, a limited number of schools are allowed to reserve seats for students with high-achieving records.

Let $T = \{\textit{Special needs}, \ \textit{High-achieving}, \ \textit{Disadvantaged}, \textit{Regular}\}$ be the set of all possible types that students may belong to. In general, for each student, these types are school-dependent (as an exception, the disadvantaged type is school-independent) and may overlap. Each student belongs to at least one type, being *Regular* the default (i.e., *Regular* encodes the absence of type). We define a mapping $\tau : S \times C \to 2^T$ that maps students to their types on each school.

A function $p : C \times T \to \mathbb{N}$ defines type-specific quotas for each school, where p_{ct} represents a soft lower bound for school c, i.e., a flexible limit that regulates school c's priorities dynamically, giving higher priority to students of type t up to filling p_{ct} seats. Furthermore, we assume that in each school quotas can be met without violating its capacity, i.e., $\sum_{t \in T} p_{ct} \leq q_c$ for every school $c \in C$.

As shown by Kurata et al. (2017), when student types overlap the general concepts of stability for a matching with soft lower bounds proposed in literature (Hafalir et al. (2013), Ehlers et al. (2014)) are insufficient to guarantee the existence of a stable matching. To overcome this difficulty, they propose a new model based on the framework of matching with contracts due to Hatfield and Milgrom (2005). In this model, schools provide distinct reserved seats for each student type, and assignments are interpreted as contracts that explicitly state that a student is assigned to a particular reserved seat at a school, in contrast to previous models where a student accounts for seats of multiple types.

Following this literature we extend our model as follows. Every student s has now strict preferences $\succ_s$ over contracts of the form $(c,t) \in C \times T$, and every school c has now a weak priority profile $\succsim_c$ over contracts of the form $(s,t) \in S \times T$. Then, a **matching** μ is a function from $(S \cup C) \times T$ to the subsets of $(S \cup C) \times T$ such that:

(i) $\mu(s) \subseteq C \times T, |\mu(S)| = 1$ or $\mu(s) = \emptyset$, for every student $s \in S$.

(ii) $\mu(c) \subseteq S \times T$ and $|\mu(c)| \leq q_c$ for every school $c \in C$.

(iii) $\mu(s) = \{(c,t)\}$ if and only if $(s,t) \in \mu(c)$, for every student $s \in S$, school $c \in C$ and type $t \in T$.

In other words, a student s is either unassigned or assigned to a seat of type t in school c, $\mu(c)$ is the set of students assigned at school c, each one to a type-specific seat, and student s is assigned to a seat of type t in school c if and only if school c's assignment contains s assigned to a seat of type t. Note that this definition does not require that type t students must be matched to seats of type t.

Let $\mu_t(c) := \{s \in S : \ \mu(s) \in \{c\} \times T \text{ and } t \in \tau(s,c)\}$ be the set of students of type t assigned to school c. Two well-known and desirable properties of a matching are to be fair (or justified envy-free) and non-wasteful. In our setting, a student s has **justified envy** towards a student s' with assignment $\mu(s') = (c',t')$ in matching μ if there exists a type $t \in T$ such that:

(i) $(c',t) \succ_s \mu(s)$,

(ii) $(s,t) \succ_{c'} (s',t')$,

(iii) and either $t' = t$ or $|\mu_{t'}(c')| > q_{c't'}$.

That is, s has justified envy towards s' assigned to school c' in a seat of type t' if either s prefers (c',t') to his assignment and is preferred by the school on that seat, or for some type $t \neq t'$, s prefers (c',t) to his assignment, school c' has exceeded the quota of type t' students and prefers s in a seat of type t to s' in a seat of type t'.

A student s **claims an empty seat** of a school c in matching μ if there exists $t \in T$ such that $(c,t) \succ_s \mu(s)$ and one of the following conditions hold:

(i) $|\mu(c)| < q_c$ or

(ii) $\mu(s) = (c,t')$ for some type $t' \in T$, $(s,t) \succ_c (s,t')$ and $|\mu_{t'}(c)| > q_{ct'}$.

A student s **claims an empty seat by type** in school c in matching μ if there exists $t \in T$ such that $(c,t) \succ_s \mu(s)$ and the following condition holds:

(iii) $|\mu_t(c)| < q_{ct}$.

Table 1 Weak priorities by type-specific seats. Lower numbers indicate higher priority.

Priority	$\succsim_{c,\text{special needs}}$	$\succsim_{c,\text{high-achieving}}$	$\succsim_{c,\text{disadvantaged}}$	$\succsim_{c,\text{regular}}$
1	Secured enrollment	Secured enrollment	Secured enrollment	Secured enrollment
2	Special needs	High-achieving	Siblings	Siblings
3	Siblings	Siblings	Disadvantaged	Working parent
4	Working parent	Working parent	Working parent	Former students
5	Former students	Former students	Former students	No priority
6	No priority	No priority	No priority	

Namely, s claims an empty seat of type t at c if s prefers that contract to her assignment, and either c has empty seats, or s is assigned to c in a seat that exceeded its quota and the school prefers having s in a seat of type t, or the quota of type t students has not been met at c.

When no student claims an empty seat or claims an empty seat by type at any school, we say the matching is **non-wasteful**. Finally, a matching is **stable** if it is non-wasteful and eliminates justified envy for all students. This notion of stability matches the one proposed by Kurata et al. (2017), even when we assume some students might remain unassigned.

In Table 1 we describe the schools' weak preference profiles over contracts $(s,t) \in S \times T$ for each fixed type $t \in T$. At every school c, students currently enrolled at the school have the highest priority in all types of seats. Then, for the special needs and high-achieving seats, students of the corresponding type are given the second highest priority, and the rest of the students are given priorities according to (1). As required by law, students that have siblings currently enrolled at the school have higher priority than disadvantaged students, even in seats reserved for that type.

Priorities of schools over contracts $(s,t) \in S \times T$ and preferences of students over contracts $(c,t) \in C \times T$ also need to be defined to fully state our model, even though neither schools nor students have real preferences over the type of seat defined by the contract. The way to break ties is not straightforward: as shown by Dur et al. (2016b), different tie-breaking rules might favor some type of students. To reduce over-representation of quotas, we break ties in a way that students and schools favor assignments of students to seats of one of their corresponding types.

4.3. Family application.

Families that have two or more children participating in the system are given the option to prioritize assigning them to the same school of their preference list to the detriment of better schools reported in each individual preference list. We refer to this as *family application*.

It is important to emphasize the different roles that siblings play in the system. On the one hand, there is a sibling's priority that gives special priority to students applying to schools where they have siblings already enrolled at. On the other hand, there is the family application, which addresses siblings (either in the same or in a different level) that are all participating in the admission process.

We implement family applications by defining an equivalence relation on S that captures the sibling relationship among students that are participating in the system. Thereby, families correspond to equivalence classes of size two or more induced by this relation. We only consider families that have at least one school in common in their members' preference lists.

Similar to the matching with couples problem, a stable matching might not exist in the school choice problem with family applications. To explain why, we must first define stability in this context. Consider the simplest version of the school choice problem: without families, types or quotas. A stable matching is an assignment μ where there is no pair student-school (s,c) such that $c \succ_s \mu(s)$ and $|\{s' \in \mu(c) : s' \succ_c s\}| \leq q_c - 1$. Now, a school c in level k has capacity q_c^k and a family A has preferences over the possible assignments of its members. In particular, we consider that a family A has a strict order of preferences $\succ_A$ over a set of acceptable schools, and that they strictly prefer assignments where $A \subseteq \mu(c)$ for some c, and the rest of comparisons are given by the Pareto partial ordering induced by $\succ_A$. Then, we say that a matching μ is stable if (i) there are no triplets of student, school and level s, c, k where s is not matched to the same school as the rest of the family $A \ni s$, such that $c \succ_s \mu(s)$ and $|\{s' \in \mu(c,k) : s' \succ_c s\}| \leq q_c^k - 1$; and (ii) there are no pairs A, c, such that $|\{s' \in \mu(c,k) : s' \succ_c A\}| \leq q_c^k - |A_k|$ for all $k \in K$ and either $c \succ_A \mu(A)$ if $|\mu(A)| = 1$ or c is just acceptable for A if $|\mu(A)| > 1$.

Proposition 1. *If there are family applications, a stable matching might not exist. This is true even in the case where the families have at most two siblings, the preferences of schools are over families, and students of the same family have the same preferences.*

In Figure 2 we present an instance with two levels, four families $\{A, B, C, D\}$ and four schools $\{c_1, c_2, c_3, c_4\}$. In this example, each school has one seat in each level, families A and C have only one child (in levels 1 and 2 respectively), and families B and D have two children, one in each level.

For each level we represent the instance as a graph, where each row corresponds to a family and each column to a school, and preferences are captured through arrows pointing towards more preferred options. Notice that each family is the most preferred one for some school, so no student can result unassigned and siblings must be assigned in the same school in any stable matching. However, we claim that there is no stable matching. To see this, suppose that there is a stable matching μ. Then, both children from family B must be assigned in the same school $c \in \{c_1, c_2\}$, which we denote by $B \in \mu(c)$. Hence, there are two cases:

1. if $B \in \mu(c_1)$, then family C will be assigned to c_2 (its top choice) in level 2, family D will take its favorite school—c_3—in both levels, and family A will take its favorite school—c_4—in level 1. As a result, no family with higher priority than B is assigned to c_2, and since c_2 is family B's top choice in both levels we conclude that μ is not stable.

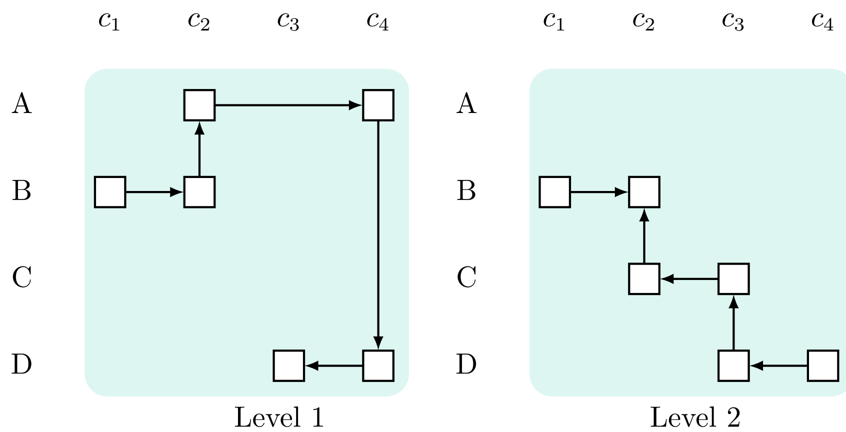

Figure 2 Instance with two levels where no stable matching exist because of the family applications.

2. if $B \in \mu(c_2)$, then family C is assigned to school c_3 in level 2, family D can only be accommodated in school c_4 (as both children must be assigned to the same school in any stable matching), and thus family A results unassigned. This contradicts the fact that all families should be allocated in any stable matching, and thus we conclude that there is no stable matching.

Due to this result, we implement the following heuristic to find a feasible solution to the problem with family applications.

1. Start from level $k = 12$, i.e., the highest level.
2. Obtain an assignment for level k. Call it μ_k.
3. Using μ_k, update the preferences of students whose siblings where assigned in Step (2) and that applied as a family, so that their top choice becomes the school where their elder sibling was assigned.
4. Update $k \leftarrow k - 1$, and go back to Step (2). If there are no levels left, stop.

4.4. Tie breaking rule.

As discussed earlier in this section, students within a priority group are randomly ordered to create a strict preference profile for each possible contract at each school. By law, schools are allowed to have their own lotteries, so we implement a variant of the multiple tie-breaking rule to account for this requirement.

As explained before, the system seeks to give siblings a higher chance of being assigned together. In order to do so, a second feature we implement is to break ties at the family level—which we call *family lotteries*—as opposed to having single student lotteries. Under this new approach, ties between families are broken first, and later a single student lottery is run within each family. In Proposition 2 we show that using family lotteries improves the probability that families are assigned together.

Proposition 2. *(Informal statement) Consider a family with two children, Alice and Bob, applying to different levels. Fix the preference profiles of all students applying to these two levels. Also fix the priorities of all students on all schools except those of Alice and Bob. Then the probability that Alice and Bob get assigned to the same school is larger under family lotteries than under student lotteries.*

To ease exposition we defer the formal statement and proof of the latter proposition to Appendix A.2. Interestingly, the proof involves a novel lemma establishing that the cutoffs determining the allocation of any given student are independent of her preferences.

4.5. Example: Family Applications and Lotteries.

To illustrate the benefits of the family application and the lotteries by family we present the following example. Consider two schools, c and c', that have a single seat in levels H and L, and suppose that level H is processed first. In addition, suppose that there are four students: a_1 and a_2 who are siblings and apply to H and L respectively, b_1 who is a single student applying to H and d_2 who is a single student applying to L. Finally suppose all students prefer school c over school c'. We consider four scenarios: with or without family application, and with student or family lotteries. To illustrate the impact of the proposed policies, in each scenario we compute the probability that the family is assigned together.

(i) *Student lotteries, no family application.* The probability that the siblings are assigned together is equal to the probability that they are both assigned to either school c or c'. Since the probability that a_1 is assigned to c (or c') is $\frac{1}{2}$ and the probability that a_2 is assigned to c (or c') is also $\frac{1}{2}$, then the overall probability of the family being assigned together is $\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$.

(ii) *Lotteries by family, no family application.* There are three possible lottery outcomes for the family in school c, namely, being ranked first, second or last. Each outcome has probability $\frac{1}{3}$. If the family is ranked first in c, a_1 and a_2 are assigned together in school c. If the family is ranked last in school c, both are assigned to school c'. Finally, if the family is ranked second in school c, one of the children is assigned to school c and the other to c'. Then, the overall probability that the family is assigned together is $\frac{1}{3} \cdot 1 + \frac{1}{3} \cdot 1 + \frac{1}{3} \cdot 0 = \frac{2}{3}$.

(iii) *Lotteries by student, with family application.* Here the probability that the family is assigned together in school c is $\frac{1}{4}$ as in case (i). However, notice that the probability that the family is assigned to school c' is now $\frac{1}{2}$. The reason is that once student a_1 is assigned to c' (which happens with probability $\frac{1}{2}$), then a_2 updates her preferences and now prefers school c' and gets a seat for sure. Therefore the overall probability that the family is assigned together is $\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot 1 = \frac{3}{4}$.

(iv) *Lotteries by family, with family application.* As in (ii), in school c the family may be ranked ranked first, second or last. Also, we know that the family is assigned together if it is ranked first or last. However, when the family is ranked second, either b_1 or d_2 are ranked first. In the former case the family is assigned together because in level H, a_1 is assigned to school c', and then a_2 updates her preferences also getting school c'. In the latter case the family is not assigned together because in level H, a_1 is assigned to school c, but in level L student a_2 is assigned to c'. As a result, the overall probability that the family is assigned together is $\frac{1}{3} \cdot 1 + \frac{1}{3} \cdot 1 + \frac{1}{3} \cdot \frac{1}{2} = \frac{5}{6}$.

4.6. Implementation.

Finally, we present the algorithm that determines the matching of students to schools. For each level $k \in K$, from the highest (12th grade) down to the lowest (Pre-K) do:

1. Update preferences of students in family application
2. Create the weak priority list for each school, updating sibling priority to applicants whose siblings were assigned in a higher level.
3. Create the strict priority list for each type of seats at each school, using random tie breaking by family.
4. Create student preferences over seats in each school.
5. Run the student-optimal *Deferred Acceptance* algorithm for overlapping types on all students and schools that belong to level k.

Our implementation of the Deferred Acceptance algorithm is based in the approach of directed graphs first proposed by Balinski and Ratier (1997) for one-to-one stable matchings and later extended by Baïou and Balinski (2004) for many-to-one matchings, which is efficient and allows us to find an assignment in a few seconds. At each level, we encode the instance as a graph in which every node represents a student applying to a type-specific seat in a school, and every directed edge connects either two prefences or two priorities, from the least preferred (or prioritized) one to the most. Then, the algorithm eliminates nodes that do not belong to any stable matching, until no further eliminations are possible. Finally, in order to find the student-optimal matching, we pick the top preference of each student that has at least one preference remaining in the graph. This algorithm allows us to solve all levels of the nationwide instance of the school admission problem in just a few seconds.

5. Results

In this section we report part of the implementation results. We start by describing how the system evolved from 2016 to 2018. Then, we focus on the current admission process and report the results of the main and complementary rounds in Sections 5.1 and 5.2 respectively. In Section 5.3 we

Table 2 Evolution of the System - Main Round

	2016	2017	2018
Regions	1	5	15
Schools	63	2,174	6,421
Students	3,436	76,821	274,990
% assigned 1st preference	58.0	56.2	59.2
% assigned any preference	86.4	83.0	82.5
% unassigned	9.0	8.7	8.9

analyze the effect of the quota for disadvantaged students. Finally, in Section 5.4 we study the impact of the family application.

In Table 2 we summarize how the admission system evolved. In 2016, we considered only the entry levels of the Magallanes region, located in the extreme south of the country. In 2017, the system was extended to all levels in Magallanes, and to four more regions considering only their entry levels. For the 2018 process all the levels of the aforementioned regions were added, and all the remaining regions (except for the Metropolitan area) were included at their entry levels. By 2020, the plan is to implement the system in the entire country and considering all levels, i.e., from Pre-K to 12th grade. As the table shows, most of the relevant performance metrics of the main round—fraction of students assigned to their top choice and unassigned—have remained stable over time.

5.1. Main Round.

In 2018, 274,990 students and 6,421 schools—divided in 32,198 sections, i.e., school-level pairs—participated in the system, with a total of 522,859 available seats (average of 16.2 seats per section). In Table 3 we classify students based on (1) their gender, (2) whether they have any type of priority in the schools they applied to, and (3) whether they were eligible for any quota in the schools of their choice. Notice that the percentage of disadvantaged students exceeds 50% of the total number of applicants. As the quota for this group is only 15%, an interesting design question is whether having a quota has any impact when the targeted population is relatively large. We analyze this in Section 5.3.

Analyzing the submitted preferences we observe that students apply on average to 3.4 schools. Among these applications, 73.1% are to public schools and 26.9% to voucher schools, although only 11% of the total seats available are of the latter type. Out of the 485,905 applications made by disadvantaged students, 22.0% were made to voucher schools, which is significantly less than the general population. These differences are not surprising considering that disadvantaged students have less resources, and therefore their willingness to pay is probably lower.

Table 3 Characterization of Applicants - Main Round

		# applicants	% of total applicants
Gender	Female	134,973	49.1%
	Male	140,016	50.9%
Priority	Siblings	66,743	24.3%
	Working parent	3,700	1.3%
	Former students	9,165	3.3%
Quota	Special needs	1,631	0.6%
	Academic excellence	6,534	2.4%
	Disadvantaged	150,287	54.7%

In Figure 3 we present the distribution of assignments by preference. We observe that 59.2% and 12.8% of the applicants are assigned to their first and second preference respectively. In addition, 8.6% are assigned to their current school via secured enrollment, and 8.9% are left unassigned (recall that these students —the unassigned— have the chance to participate in the complementary process, whose results are described in Section 5.2).

Figure 4 shows the fraction of students that (1) are assigned to one of their preferences, (2) are assigned to their current school by secured enrollment, and (3) result unassigned, conditional on the number of reported preferences. We observe that when the number of declared preferences increases so does the probability of being assigned, but the average preference of assignment also increases. Moreover, we find that students who result unassigned apply on average to fewer schools (3.36, with std. dev. 1.49) than those who result assigned (3.42, with std. dev. 1.83). Applicants assigned by secured enrollment usually submit even less preferences (3.05, with std. dev. 1.49), which is expected as they have a secured option.

5.2. Complementary Round.

A total of 46,698 students participated in the complementary round including unassigned students from the main round and new applicants. In Table 4 we characterize these students based on their gender, priority type and eligibility for disadvantaged quota—the other quotas are not considered in the complementary round. In general we observe that there are no significant differences relative to the main round.

In Figure 5 we present the distribution of preference of assignment in the complementary round. We observe that the results are not as good as in the main round, as 47% are assigned to their top choice, 28% are assigned by distance, and 3.6% resulted unassigned. This result can be explained by the number of submitted preferences, as students that get assigned apply to 3.49 (std. 1.83) schools on average, compared to 2.57 (std. 0.98) for students with secured enrollment, 3.28 (std. 1.47) for students assigned by distance, and 2.79 (std. 2.08) for unassigned students.

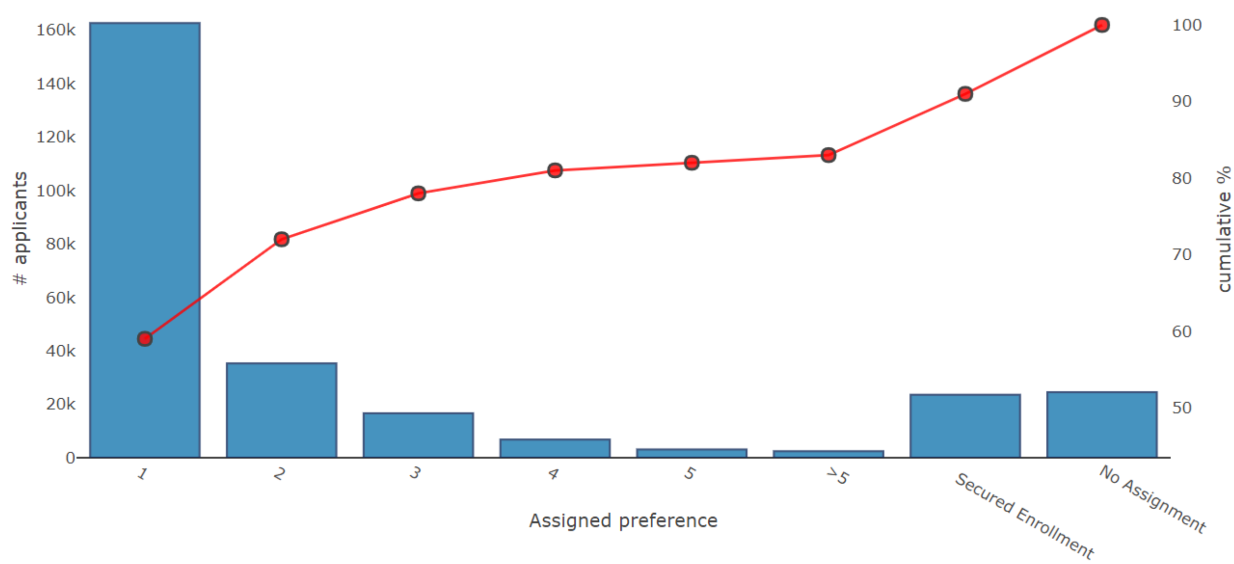

Figure 3 Number of assigned students according to their preferences and the cumulative percentage it represents, including students assigned by secured enrollment and students that were not assigned - Main Round

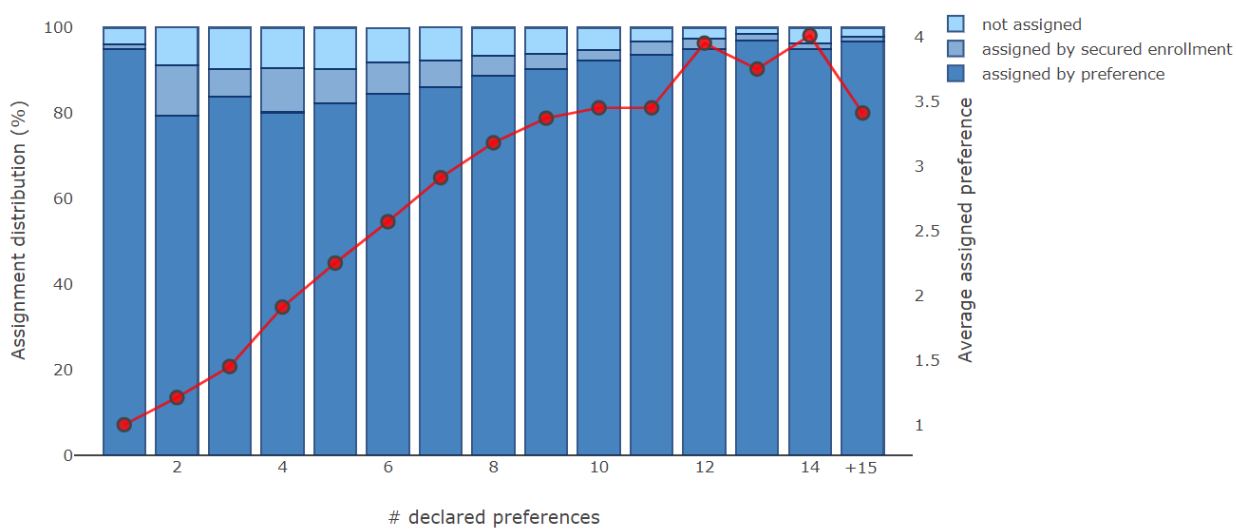

Figure 4 Assignment distribution and average rank distribution by number of declared preferences - Main Round

Table 4 Characterization of Applicants - Complementary Round

		# applicants	% of total applicants
Gender	Female	23,063	49.4%
	Male	23,635	50.6%
Priority	Siblings	5,443	11.7%
	Working parent	328	0.7%
	Former students	2,441	5.2%
Quota	Disadvantaged	23,414	50.1%

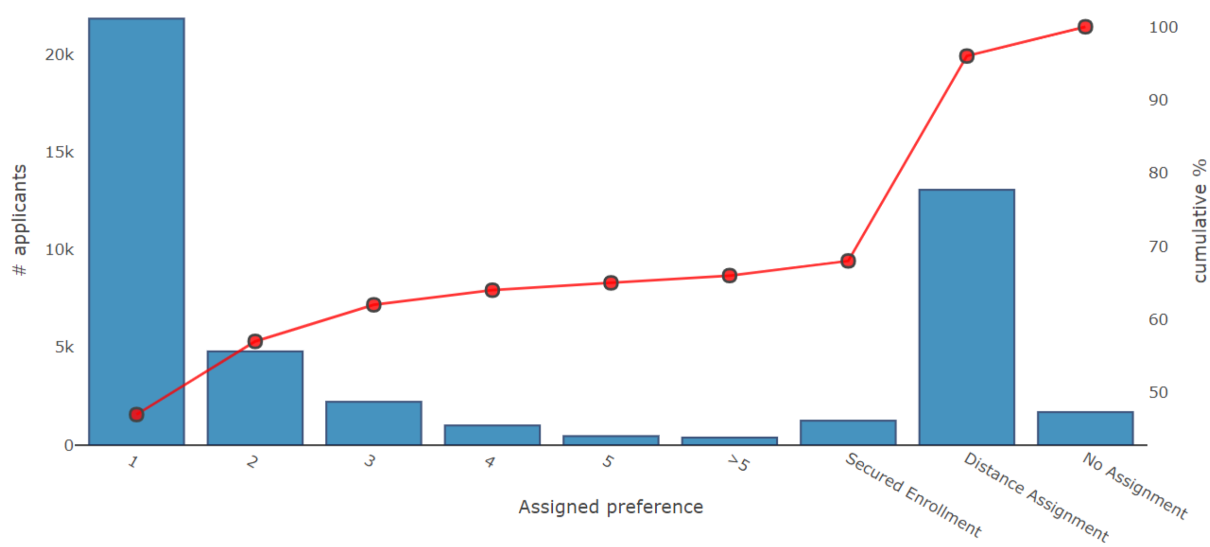

Figure 5 Number of assigned students according to their preferences and the cumulative percentage it represents, including students assigned by secured enrollment, distance and students that were not assigned - Complementary Round

Recall that those students who are not assigned to any of their preferences in the complementary round may be allocated to the nearest public school with remaining open seats within 17km, i.e., distance assignment. Indeed, 13,064 students were assigned by distance, and the average distance for these students was 2.17km, compared to 2.19km for those students who were assigned to one of their preferences in the complementary process and 3.35km for those assigned by secured enrollment. Finally, only 1,691 students—0.6% of the total number of applicants considering both rounds—resulted unassigned and were manually allocated by MINEDUC.

5.3. Quotas.

An important design question is whether quotas for disadvantaged students should be considered, especially when these students are the majority (54.7%) of those who participate in the system and when random numbers are used to break ties. In Table 5 we compare how well these students perform compared to those who are not eligible for this quota. The results for the other quotas, i.e., for students with special needs and for high-achieving students, are reported in Appendix B.1. As expected, students eligible for the quota perform better than those who are not, with more disadvantaged students being assigned to their top choice and less of them resulting unassigned.

In addition to helping students from disadvantageous environments, a second goal of the quota is to reduce segregation and achieve a higher level of socioeconomic heterogeneity in all the classrooms.

Table 5 Results for disadvantaged and non-disadvantaged students - Main Round

	Disadvantaged	Non disadvantaged
Number of applicants	150,287	124,703
1st preference	66.0%	51.0%
Other preference	21.1%	26.1%
Secured enrollment	7.3%	10.1%
Not assigned	5.7%	12.8%
Average rank	1.53	1.82
Average applications	3.02	3.37

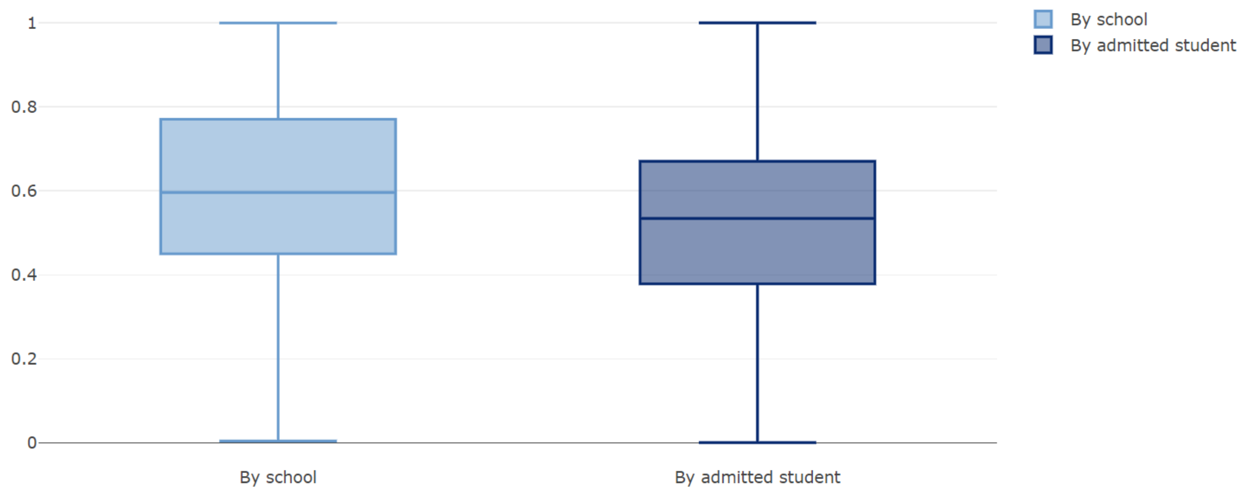

Figure 6 Distribution of the percentage of disadvantaged students in Pre-K - Main Round

To analyze this, in Figure 5.3 we show the distribution of the fraction of disadvantaged students assigned in pre-kindergarten (the lowest level). We focus on this level because there are no students with secured enrollment.

From this figure we observe that most school sections have a significant fraction of disadvantaged students, ranging from 45% to 77%. In addition, if we analyze this at the student level, we observe that 75% of students are assigned to a school that has between 38% and 67% of disadvantaged students. These results suggest that the disadvantaged quota has a positive effect in making diverse sections.

To complement this analysis, we conducted a simulation study to better understand the effect of having this quota. In particular, we performed 20,000 simulations, where half of them were done considering the quota of 15% and the other half were done eliminating this quota. In the latter case, seats were assigned regularly by priority groups.

Table 6 Results with and without socioeconomic quota - Simulations

	With quota		Without quota	
	Disadvantaged	Non-disadv.	Disadvantaged	Non-disadv.
1st preference	66.1%	50.1%	65.8%	51.3%
Other preference	21.1%	26.1%	21.0%	26.1%
Secured enrollment	7.2%	10.0%	7.4%	10.0%
Not assigned	5.6%	12.8%	5.8%	12.6%
Heterogeneity of schools				
	Mean	Std. Dev.	Mean	Std. Dev.
Fraction of disadv. classmates	0.5661	0.2173	0.5650	0.2198

In Table 6 we show the results of the simulations. As expected, disadvantaged students perform better when there is a quota, but the differences are not significant compared to the case with no quota. The fact that there is almost no change in performance when removing the quota could be due to auto-segregation, i.e., disadvantaged students apply to different schools than non disadvantaged students. Table 6 also shows the mean and standard deviation of the fraction of disadvantaged classmates that each student has (see to Appendix B.2 for details on the computation). We observe that the results are practically the same with and without quota, and therefore we conclude that having the quota has no major impact in the heterogeneity of schools.

5.4. Family Application.

Another distinctive feature of the Chilean school choice problem is the family application, which aims to increase the number of siblings that get assigned to the same school. In 2018, a total of 21,424 students were part of 10,301 family applications in the main round, with 2,869 (27.9%) formed by students that belong to the same level, and 7,432 (72.1%) having at least two students of different levels. For concreteness, we focus on the results for the main round.

Note that the overall number of siblings in the same level is less than 2%. However, family applications with siblings applying to the same level are over-represented mostly because (i) families can decide whether or not to apply as a family, and (ii) only PK, K, 1st, 7th and 9th grade are considered in the system, making it more likely to have siblings in the same level.

We say that a family application is successful if all of its members are assigned to the same school. Similarly, for families that include three or more students, we say that a family application is partially successful if at least two of its students, but not all of them, are assigned to the same school. Overall we observe that 6,725 (65.3%) family applications were successful, while 307 (3%) were partially successful. Figure 7 shows the success of family applications by size. As expected, larger families are less likely to be successful, as they require more students being allocated to the

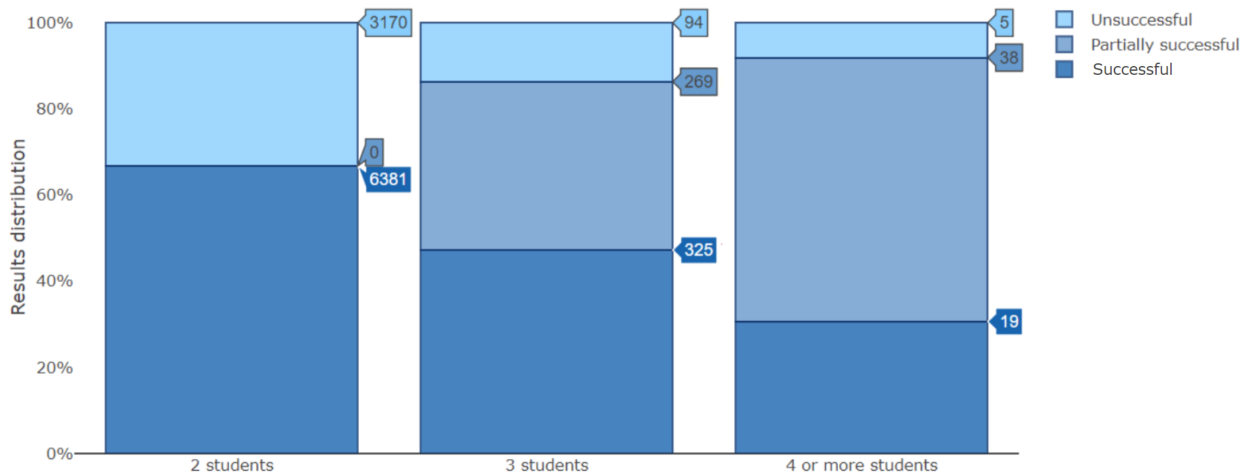

Figure 7 Number of successful and partially successful families by size - Main Round

same school. We refer to Appendix B.3 for results on family applications depending on the number of common schools a family declares.

As described in Section 4.3, to implement applications we consider lotteries by families and update students' preferences. However, there may be other approaches that could lead to more successful families. To explore the benefits of our approach, we conduct a simulation study to compare our current approach with three other alternatives: (i) lotteries by student in each school and updating preferences (ii) lotteries by families without updating preferences, and (iii) lotteries by student without updating preferences. Notice that the latter is the standard approach used in other school choice settings.

In Table 7 we report the results of 10,000 simulations for each approach. We observe that the fraction of successful families is larger when both components—lottery per family and updating preferences—are implemented. The mechanism cannot guarantee that all family applications will be successful. For example, siblings may apply to different schools, and younger siblings may not be eligible in some schools where their elder siblings are applying to (single-gender schools, schools with not all levels, among others). Moreover, the number of seats available may not be enough to allocate all students with sibling priority, as it is the case in non-entry levels. Finally, we observe that the percentage of partially successful family applications is around 3% in all four scenarios.

Our simulations show that family lotteries and updating siblings' priorities (step 2 in Section 4.6) play a different role. While updating siblings' priorities substantially increases the ranking of a young student in school c once her older sibling gets assigned to c, family lotteries correlate the siblings' rankings throughout all schools where both apply (as shown in Appendix A.2).

Table 7 Percentage of sucessfull family applications - Simulations

Scenario	Not updating preferences	Updating preferences
Lotteries by student	53%	62%
Lotteries by family	57%	66%

6. Conclusions

Centralized procedures to assign students to schools are becoming the norm in many countries. In this paper, we describe the design and implementation of the new school choice system in Chile, which expands previous applications in three main areas.

First, we introduce, analyze and evaluate the impact of features of our design intended to favor siblings in getting assigned to the same school. Concretely, we propose the use of two lotteries, one to order families and the other to break ties among siblings. In addition, our mechanism updates students' preferences to prioritize siblings getting assigned to the same school if they are part of a family application. Our results show that these features improve the fraction of siblings assigned in the same school by 13% compared to the standard approach of breaking ties at the student level. Second, we propose a multi-level mechanism that allows students to have a secured enrollment in their current school. This feature of the system eliminates the risk of ending up unassigned when trying to move to a new school. Finally, we implement a mechanism with multiple quotas and priority groups. We show via simulations that having quotas for disadvantaged students is not very effective when the majority of students are eligible and the quota is relatively small. Hence, other approaches should be considered if the goal is to really benefit this group.

The experience of implementing a large-scale nationwide system stresses the importance of having a continuous collaboration with policy makers, and the need of implementing changes in small steps. Having a gradual implementation not only allows to learn from the experience and continuously improve the system, but also gives time to the general public—and final users of the system—to get information, learn and understand the benefits of the new system. Overall, we will continue working to improve the system, increasing its efficiency and fairness to give equal opportunities to all students, regardless of their background.

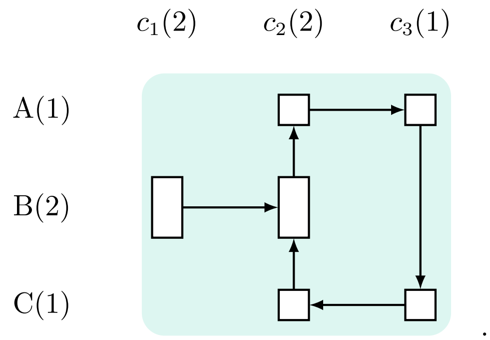

Figure 8 Example of instance where no stable matching exists. Each row is a family and each column is a school. In parenthesis is the size of the family or the number of available seats

Appendix A: Model

A.1. Family application

Regarding stability with family application, in Figure A.1 we show an instance with one level, three families and three schools, where no stable matching exist. Each row represents a family and each column represents a school, and in parentheses are the size of the family and the number of available seats. The arcs represent the preferences or priorities, pointing from the less preferred to the most preferred. In this case, each family is the most preferred for one school, so in any stable matching all students are matched. This also means that the two kids of family B must be assigned together. It is easy to check that none of the 2^3 possible combinations is a non-wasteful matching without justified envy.

A.2. Family lotteries vs. student lotteries

In this section we formally state and prove Proposition 2. Consider a family with two children, Alice (a) and Bob (b) applying to different levels, which we call H and L. As usual, the set of schools is C ($|C| = m$). School $c \in C$ has q_c^H, q_c^L available seats in levels H and L, respectively. For simplicity we will assume there are no specific quotas or priorities.[3] Also there are sets S_H, S_L of students applying to levels H and L, respectively. In particular $a \in S_H$ and $b \in S_L$.

Each student $s \in S_H \cup S_L$ has a preference profile over a subset $C_s \subseteq C$ denoted by $\prec_s$. For ease of presentation, the priorities of a student s in schools in C is given by a vector $u_s = (u_{s,c})_{c \in C} \in [0,1]^C$ so that the higher $u_{s,c}$, the higher the priority of student s in school c (in the random priority model these numbers can be thought to be i.i.d. $U[0,1]$ random variables, and in the following analysis we can obviate the null set where two students applying to the same level get equal lottery numbers in the same school).

The following result concerns only level H and is related to Lemma 4 of Abdulkadiroğlu et al. (2015).

LEMMA 1. *Given $\prec_s$ and u_s for all $s \in S_H \setminus \{a\}$ there exists a vector of cutoffs $(\tau_c)_{c \in C}$ such that for all preference profiles $\prec_a$ and all vectors u_a, if $\mu(a)$ denotes the assigned school of a in the DA mechanism, then $\mu(a) = c$ if and only if $u_{a,c'} < \tau_{c'}$ for all $c \prec_a c'$ and $u_{a,c} > \tau_c$.*

Proof. Recall that as proved by Dubins and Freedman (1981) the DA mechanism is truthful in the following sense: given the preferences of all students but a and all priorities, for all pairs of preference profiles $\prec_a$ and $\prec'_a$, if we denote as μ and μ' the assignments when student a declares $\prec_a$ and $\prec'_a$ respectively, then $\mu'(a) \preceq_a \mu(a)$.

As is also known from the standard literature on Deferred Acceptance, the target student-optimal assignment is unique and therefore independent from the order in which the student proposals are processed. Thus we may assume that an initial stable assignment has been reached without the participation of a, and then she is assigned her corresponding lottery number and inserted to run the process to completion.

For each $c \in C$ we define τ_c as the minimum value that $u_{a,c}$ can take to get a accepted to c if she were to apply to it as her first preference (note that they are well defined since the acceptance or rejection to c as a first preference does not depend on the next ones). In this case it is clear that any value of $u_{a,c}$ higher than τ_c would also result in acceptance to c, and a priority number lower than τ_c would result in rejection by construction.

We will now show that this same vector $(\tau_c)_{c \in C}$ also works for an arbitrary preference profile $\prec_a$.

First we claim that if $u_{a,c} < \tau_c$, then a cannot be accepted to c. Indeed, suppose by contradiction that $u_{a,c} < \tau_c$ and a is accepted to c. Noting that the definitions of the τ_c's do not depend on $\prec_a$ we can assume that the altered profile $\prec'_a$, given by restricting $\prec_a$ to start from school c, was the real preference profile and that $\prec_a$ is a deviation from the truth. By definition of τ_c, a will be rejected from c if she applies with profile $\prec'_a$, but accepted with profile $\prec_a$ by hypothesis, which contradicts the truthfulness of the mechanism.

Returning to the proof of the lemma, to prove the right implication suppose that a is assigned to c. The inequality $u_{c,a} > \tau_c$ follows from the previous claim. If $u_{c',a} > \tau_{c'}$ for some $c \prec_a c'$, then once again a could alter her preference profile to start from c' and by definition be accepted to her more preferred option c', contradicting the truthfulness of the mechanism.

For the left implication suppose by contradiction that the inequalities hold and there is a school $c' \neq c$ such that a would be assigned to c' instead. From the claim and the inequalities $u_{a,c'} < \tau_{c'}$ we get that it is not possible that $c \prec_a c'$. Also, if $c' \prec_a c$ we can once again consider the restricted preference profile starting from c and the inequality $u_{a,c} > \tau_c$ to contradict the truthfulness of the mechanism. $\square$

With this lemma at hand we want to compare the probability that a and b get assigned to the same school if on the one hand we draw u_a and u_b as vectors of i.i.d. uniform random variables $U[0,1]$, or on the other hand we draw u_a as a vector of i.i.d. random variables $U[0,1]$ and set $u_{b,c} = u_{a,c}$. To this end we call $\mathbb{P}_S$ the probability measure induced by the former situation (student lottery) and by $\mathbb{P}_F$ the one for the latter situation (family lottery).

Proposition 2. *(Formal statement) Given $\prec_s$ and u_s for all $s \in S_H \cup S_L \setminus \{a,b\}$ then $\mathbb{P}_S(\mu(a) = \mu(b)) \le \mathbb{P}_F(\mu(a) = \mu(b))$.*

Proof. We proceed by partitioning the event $\mu(a) = \mu(b)$ over the possible common school assignment $c \in C$. From Lemma 1 we know that the event $\mu(a) = c$ is equivalent to $u_{a,c'} < \tau_{c'}$ for all $c \prec_a c'$ and $u_{a,c} > \tau_c$. Therefore, since u_a is a vector of uniform i.i.d. random variables in $[0,1]$, conditional on the event $\mu(a) = c$, we have that u_a is a vector of independent random variables but with $u_{a,c} \sim U[\tau_c, 1]$, $u_{a,c'} \sim U[0, \tau_{c'}]$ for $c' \succ_a c$, and $u_{a,c'} \sim U[0,1]$ for $c' \prec_a c$.

If we apply Lemma 1 to level L, we get certain cutoffs $(\bar{\tau}_c)_{c \in C}$ such that $\mu(b) = c$ if and only if $u_{b,c'} < \bar{\tau}_{c'}$ for all $c' \prec_b c$ and $u_{b,c} > \bar{\tau}_c$. Now, since under family lotteries $u_{a,c'} = u_{b,c'}$ for all $c' \in C$, we have that $\mathbb{P}_F(u_{b,c'} < \bar{\tau}_{c'} | \mu(a) = c) \ge \mathbb{P}_S(u_{b,c'} < \bar{\tau}_{c'} | \mu(a) = c) = \mathbb{P}_S(u_{b,c'} < \bar{\tau}_{c'})$ for all $c' \neq c$ and $\mathbb{P}_F(u_{b,c} > \bar{\tau}_c | \mu(a) = c) \ge \mathbb{P}_S(u_{b,c} > \bar{\tau}_c | \mu(a) = c) = \mathbb{P}_S(u_{b,c} < \bar{\tau}_c)$. Then, because the variables in the vector u_b are independent, we can multiply the inequalities and get the following.

$$\mathbb{P}_F(\mu(b) = c | \mu(a) = c) \ge \mathbb{P}_S(\mu(b) = c).$$

Note that for a given school $c \in C$, the marginal probabilities $\mathbb{P}_F(\mu(a) = c)$ and $\mathbb{P}_S(\mu(a) = c)$ are equal since they concern level H only. Hence, we can multiply by $\mathbb{P}_F(\mu(a) = c)$ on both sides of the previous inequality and sum over all $c \in C$ to obtain that

$$\sum_{c \in C} \mathbb{P}_F(\mu(b) = c, \mu(a) = c) \ge \sum_{c \in C} \mathbb{P}_S(\mu(b) = c, \mu(a) = c),$$

and therefore, $\mathbb{P}_F(\mu(a) = \mu(b)) \ge \mathbb{P}(\mu(a) = \mu(b))$. $\square$

Appendix B: Results

B.1. Other Quotas

Recall that students can belong to three quotas: (1) special needs, (2) high-achieving, and (3) disadvantaged. Students are indifferent to being assigned by any quota or non of them and schools only declare their total available seats and the mechanism calculates seats for the different types of quotas that are allowed by the system. In Table 8 we show the distribution of the 524,178 declared seats for the 2018 process.

Table 8 Total seats declared by schools - Main Round

Quota	# seats	% of total
Special needs	15,324	2.9%
Disadvantaged	43,336	8.3%
High-achieving	2,591	0.5%
Regular	462,927	88.3%

Figure 9 Results by quota - Main Round

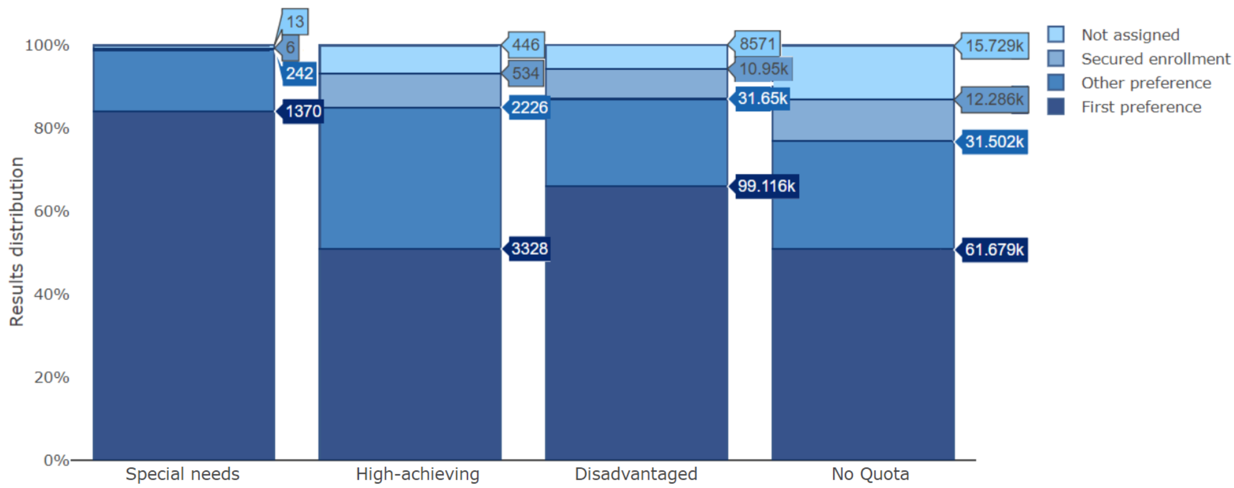

Figure 9 shows the results for students belonging to different quotas (recall a student may belong to more than one quota). It is clear to see that students belonging to special needs and disadvantaged quotas outperform students belonging to high-achieving quota or students that do not belong to any quota in both percentage of students assigned to their first choice and percentage of unassigned students. The low performance of the high-achieving quota compared to the other two quotas could be explained by the fact that high-achieving students apply to a subset of very over-demanded schools.

B.2. Heterogeneity Simulations

From the point of view of schools, MINEDUC seeks to have balanced and heterogeneous schools with respect to the socioeconomic composition. In this sense, we analyze in both scenarios (with and without the quota) the balance of disadvantaged students in schools. For this aim, we consider the following measure of heterogeneity: among all the students of the first round that get an assignment, we pick a student uniformly at random and count the fraction of disadvantaged students that are assigned to her course. This defines a random variable that depends both on the lottery used for the tie-breaking rule and on the selected student.

Let S_{dis} be the set of disadvantaged students that participate in the first round. Given an assignment μ, let $S(\mu) := \{s \in S : \mu(s) \neq \emptyset\}$ be the set of all students that get an assignment in μ, and similarly, let $S_{\text{dis}}(\mu) := S(\mu) \cap S_{\text{dis}}$ be the set of all the disadvantaged students that get an assignment in μ.

For a fixed lottery, let μ^{lottery} be the assignment obtained from its induced tie-breaking and s be a student chosen at random among all the students that get an assignment in μ^{lottery}. For a school $c \in C$ with $\mu(c) \neq \emptyset$, let $f^{\text{lottery}}(c) := \frac{|\mu^{\text{lottery}}(c) \cap S_{\text{dis}}|}{|\mu^{\text{lottery}}(c)|}$ be the fraction of disadvantaged students assigned to c in μ. Then, our random variable can be expressed as $f^{\text{lottery}}(\mu^{\text{lottery}}(s))$. Its conditional expectation given the lottery turns out to be the ratio of all the disadvantaged students assigned in μ^{lottery} to all the students assigned in μ^{lottery}, since

$$\begin{aligned}
\mathbb{E}\left[f^{\text{lottery}}(\mu^{\text{lottery}}(s)) \,|\, \text{lottery}\right] &= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{t \in S(\mu^{\text{lottery}})} f^{\text{lottery}}(\mu^{\text{lottery}}(t)) \\
&= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{c \in C : \mu^{\text{lottery}}(c) \neq \emptyset} f^{\text{lottery}}(c) |\mu^{\text{lottery}}(c)| \\
&= \frac{|S_{\text{dis}}(\mu^{\text{lottery}})|}{|S(\mu^{\text{lottery}})|}.
\end{aligned}$$

Its second moment, on the other hand, is given by

$$\begin{aligned}
\mathbb{E}\left[\left(f^{\text{lottery}}(\mu^{\text{lottery}}(s))\right)^2 \,|\, \text{lottery}\right] &= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{t \in S(\mu^{\text{lottery}})} (\left(f^{\text{lottery}}(\mu^{\text{lottery}}(t))\right)^2 \\
&= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{c \in C : \mu^{\text{lottery}}(c) \neq \emptyset} \left(f^{\text{lottery}}(c)\right)^2 |\mu^{\text{lottery}}(c)| \\
&= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{c \in C : \mu^{\text{lottery}}(c) \neq \emptyset} \frac{|\mu^{\text{lottery}}(c) \cap S_{\text{dis}}|^2}{|\mu^{\text{lottery}}(c)|}.
\end{aligned}$$

We estimate $\mathbb{E}[f^{\text{lottery}}(\mu^{\text{lottery}}(s))]$ by computing the average of $\mathbb{E}[f^{\text{lottery}}(\mu^{\text{lottery}}(s)) \,|\, \text{lottery}]$ over the results of the 10,000 simulations. Similarly, we estimate $\text{Var}[f^{\text{lottery}}(\mu^{\text{lottery}}(s))] = \mathbb{E}\left[\left(f^{\text{lottery}}(\mu^{\text{lottery}}(s))\right)^2\right] - \mathbb{E}[f^{\text{lottery}}(\mu^{\text{lottery}}(s))]^2$ by averaging $\mathbb{E}\left[\left(f^{\text{lottery}}(\mu^{\text{lottery}}(s))\right)^2 \,|\, \text{lottery}\right]$ over the 10,000 simulations and then subtracting the square of the estimator for $\mathbb{E}[f^{\text{lottery}}(\mu^{\text{lottery}}(s))]$. Finally, we calculate the standard deviation as the square root of the variance.

B.3. Family Application

We measure the success of family applications of size two as a function of the number of schools their students declare in common in their preference lists. Table 9 shows, as expected, that the success rate increases with the number of common preferences, having its greatest increment when the number of common schools grows from one to two. Furthermore, in both rounds, blocks (families) of size two of the same level were more successful in percentage than those of different levels.

Table 9 Results of family applications for blocks of size 2 by number of schools in common

	Main round		Complementary round	
# schools in common	# blocks	% of success	# blocks	% of success
1	1,291	38.5%	497	67.7%
2	3,216	69.8%	2,245	76.6%
3	2,441	71.7%	1,750	77.8%
≥4	2,603	72.6%	1,889	80.5%
Total	9,551		1,421	

Indeed, the main round has 2,832 blocks of size two of the same level and 6,719 of different levels, with success rates of 77,8% and 62.2% respectively. For the complementary round, there are 362 such blocks of the same level, with a success rate of 82.3%, and 1,059 of different levels, with a success rate of 70.7%.

Endnotes

1. The Law also radically changed the way in which families apply and are assigned to schools, which made the transmission of information essential to the implementation. The key to these challenges was gradualism. The system was first implemented in 2016 in the least populous of the sixteen regions in Chile. This allowed to gain practical experience to improve the system as more regions were subsequently added. The system will be fully in place by 2020.
2. This 3% corresponds to 307 partially successful family applications. However, only 750 FAs were of size 3 or more, therefore this represents 41% of the possibly successful FAs.
3. This is actually w.l.o.g.

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Improving the Chilean College Admissions System

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In this paper we present the design and implementation of a new system to solve the Chilean college admissions problem. We develop an algorithm that obtains all stable allocations when preferences are not strict and when all tied students in the last seat of a program (if any) must be allocated. We use this algorithm to determine which mechanism was used to perform the allocation, and we propose a new method to incorporate the affirmative action that is part of the system and correct the inefficiencies that arise from having double-assigned students. By unifying the regular admission with the affirmative action, we have improved the allocation of approximately 3% of students every year since 2016. From a theoretical standpoint we show that some desired properties, such as strategy-proofness and monotonicity, cannot be guaranteed under flexible quotas. Nevertheless, we show that the mechanism is strategy-proof in the large, and therefore truthful reporting is approximately optimal.

Key words: college admissions, stable assignment, flexible quotas, non-strict preferences.

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1. Introduction

Centralized admission systems have been increasingly used in recent years to carry out the assignment of students to schools and colleges. A variety of mechanisms have been studied, including the celebrated Deferred Acceptance (DA) algorithm (Gale and Shapley (1962)), the Immediate Acceptance (Boston) algorithm (Abdulkadiroglu et al. (2005) and Ergin and Somnez (2006)) and the Top-Trading Cycles algorithm (Shapley and Scarf (1974)). An important part of the literature in market design has been devoted to characterize these mechanisms, mostly focusing on canonical examples that illustrate their properties. Another important body of the literature studies real life applications by combining the aforementioned mechanisms with specific rules such as restrictions in the length of preferences, tie breaking rules, affirmative actions, among many others. In this paper we try to contribute to both by studying the Chilean college admissions problem.

A centralized mechanism to match students to programs[1] has been used in Chile since the late 1960's by the *Departamento de Evaluación, Medición y Registro Educacional* (DEMRE), the analogue of the American College Board. Every year more than 250,000 students participate in the system, which includes more than 1,400 programs in 41 universities. This system has two main[2] components: a regular admission track, where all students that graduated from high-school can participate; and an affirmative action policy, that aims to benefit underrepresented groups by offering them reserved seats and economic support. More specifically, to be considered for the reserved seats and the scholarship—called *"Beca de Excelencia Academica"*, or simply BEA—a student must belong to the top 10% of his class, must graduate from a public/voucher school and his family income must be among the lowest four quintiles.

When the affirmative action was introduced in 2007, the procedure to match students to programs relied on a black-box software that could not be updated to incorporate this new feature. Hence, the authorities decided that the admission of BEA students would be run after the admission of Regular students. Since BEA students can apply to both regular and reserved seats,[3] running the process sequentially introduces inefficiencies. For instance, a BEA student can be assigned to two different programs, and the seat that this student decides not to take cannot be re-allocated to another student. Due to this problem more than 1,000 vacancies were not filled every year, mainly affecting students from under-represented groups.

In this paper we provide a "reverse-engineering" approach to correct these inefficiencies. The reason why we start from the current system and we don't simply propose a complete re-design is that DEMRE wanted to keep the current rules and incorporate the reserved seats keeping the system as close as possible to its current state. Hence, there were two practical questions to be answered: (1) what was the mechanism that was currently being used, and (2) how could this mechanism be modified to unify the admission tracks. To answer these questions and address the aforementioned inefficiencies, our first goal was to identify the mechanism "inside the black-box". Based on the rules of the system, we had enough evidence to think that the desired outcome was a stable matching. In addition, we realized that unlike other systems all students tied in the last seat had to be admitted, so quotas had to be *flexible* in order to allocate them. With these features in mind, we implemented an algorithm based on Baïou and Balinski (2004) that obtains all stable allocations satisfying the rules of the system. By comparing the results of our algorithm with the actual assignment of past years we found that the mechanism used was a university-proposing deferred acceptance, with the special feature of flexible quotas to allocate all tied students in the last seat. Furthermore, we show that, unlike the case with strict preferences, the Chilean mechanism is not strategy-proof nor monotone. Nevertheless, we argue that flexible quotas do not introduce a major strategic concern as the mechanism is strategy-proof in the large.

After identifying the algorithm being used, our next goal was to integrate both systems in order to maximize the utilization of vacancies. To solve this problem we introduce a new approach where each type of seat (regular or reserved) is assumed to belong to a different program with its own capacity and requirements, and students benefited by the affirmative action can apply to both.

This research is the outcome of an ongoing multiyear collaboration with DEMRE (2012-2019), aiming to improve the Chilean college admissions system. All the solutions described in this paper were adopted and implemented starting in 2014 with a pilot phase. In 2015 the system switched to a student-optimal mechanism with flexible quotas, and in 2016 the unified allocation was finally adopted. Based on simulations in 2014-2015 and actual data in 2016, we find that our implementation has improved the allocation of approximately 3% of the students participating each year. Furthermore, our "white box" implementation made the admission process fully transparent and reduced the execution time from over 5 hours to a couple of minutes. This improvement in transparency and performance has allowed the evaluation and introduction of different policies (e.g. the inclusion of the high-school class rank as admission factor; see Larroucau et al. (2015), among others) that otherwise could not have been included.

The reminder of the paper is organized as follows. Section 2 provides a background on the Chilean tertiary education system and the college admissions process. In Section 3 we discuss the closest related literature. In Section 4 we develop a model that formalizes the problem, we describe the mechanisms and present their properties. We discuss the implementation in Section 5. Finally, we provide concluding remarks in Section 6.

2. The Chilean College Admissions System

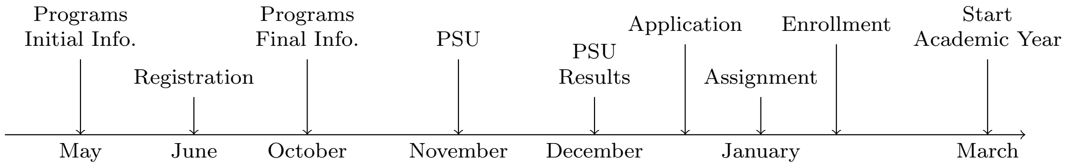
Tertiary education in Chile is offered by 149 institutions,[4] which can be classified in three types: (i) Universities (60), which have the exclusive right to award academic degrees—Bachelor, Master and Doctorate—and offer academic programs that require a previous degree, such as Medicine and Law; (ii) Professional Institutes (IP) (43), which offer professional/technical programs that lead to a professional/technician qualification; and (iii) Technical Schooling Centers (CFT) (46), which exclusively offer vocational programs leading to a technician qualification. These institutions not only differ in the type of programs they offer, but also in their programs' duration—IP and CFT programs tend to be shorter—and their application requirements. In particular, IP and CFT only require a secondary education license for admission and some Professional Institutes may select their students based on their grades during high-school. In contrast, most universities require students to take a series of standardized tests (*Prueba de Selección Universitaria* or PSU). These tests include Math, Language, and a choice between Science or History, providing a score for each of them. The performance of students during high-school gives two additional scores, one obtained from the average grade during high-school (*Notas de Enseñanza Media* or NEM) and a second that depends on the relative position of the student among his/her cohort (*Ranking de Notas* or Rank).[5]

The admissions process to these institutions is semi-centralized, with the most selective universities having their own centralized system and the remaining institutions carrying their admission processes independently. In the centralized system, which is organized by CRUCH,[6] students submit a single application list and a centralized algorithm simultaneously performs the allocation to all programs that are part of it. To participate in this system universities must (i) certify their quality, (ii) guarantee that their controllers are non-profit organizations, and (iii) agree with the terms and conditions, such as publishing their requirements for admission, the number of seats offered for each program, among others. On the other hand, IP, CFT and the universities that are not part of the centralized system run their admissions independently.[7]

In this paper we focus on the centralized part of the system, whose timeline is summarized in Figure 1. The process starts in May, when each program defines the specific requirements that must be met by applicants to be acceptable, such as minimum application score or minimum tests scores. In addition, each program freely[8] defines the weights assigned to each score and also the number of seats offered for (i) the Regular process, where all students compete, and for (ii) the special admission track related to the affirmative action policy (BEA process/track). Programs have until October to update this information. In June students must register to take the PSU, which takes place at the end of November. Scores are published by the end of December and, right after this, the application process starts. Students have 5 days to submit their list of preferences, which can contain at most 10 different programs. These programs must be listed in strict order of preference.

Each program's preference list is defined by first filtering all applicants that do not meet the specific requirements. Then, students are ordered in terms of their application scores, which are

Figure 1 Timeline of the Centralized Process



computed as the weighted sum of the applicants' scores and the weights pre-defined by each program. Note that two candidates can obtain the same application score, and therefore programs' preferences are not necessarily strict.

Considering the preference lists of applicants and programs, as well as the number of seats offered in both admission tracks, DEMRE runs an assignment algorithm to match students and programs. Specifically, the Regular process is solved first considering all applications and the Regular seats. Once the Regular process is done, the BEA process is solved considering the reserved seats, the students shortlisted for the scholarship (BEA students), and their applications to programs that (i) are more preferred (according to their preference list) than the program they were assigned (if any) in the Regular process, and that (ii) wait-listed them in the Regular process. As a result, BEA students can result *double-assigned*, i.e. they can get assigned to a program in the regular process and to another—strictly preferred—program in the BEA process. DEMRE reports both allocations, and double-assigned students are allowed to enroll in any of their two allocations.

DEMRE performs the matching for both processes using a black-box software for which no information is available regarding the specific algorithm used. Instead, the following description is provided:[9]

> *"SORTING OF APPLICANTS PER PROGRAM AND ELIMINATION OF MULTIPLE ALLOCATIONS:*
>
> *(a) Once the final application score is computed, candidates will be ordered in strict decreasing order based on their scores in each program.*
>
> *(b) Programs complete their vacancies starting with the applicant that is first in the list of candidates, and continue in order of precedence until seats are full.*
>
> *(c) If an applicant is selected in his first choice, then he is erased from the lists of his 2nd, 3rd, 4th, until his last preference. If he is not selected in his first choice, he is wait-listed and moves on to compete for his 2nd preference. If he is selected in this preference, he is dropped from the list of his 3rd to this 10th choice, and so on. In this way, it is possible that a student is selected in his 6th preference*

and wait-listed in his top five preferences; however, he will be dropped from the lists of his preferences 7th to 10th.

(d) This procedure to select candidates is the result of an agreement between the universities to have a unified and integrated process, so that no student is admitted by more than one program. Nevertheless, a student can be wait-listed in more than one program if his score is not enough to be admitted.

(e) All candidates that apply and satisfy the requirements of the corresponding program and institution will be wait-listed.

THEREFORE, IT IS FUNDAMENTAL THAT APPLICANTS SELECT THEIR PROGRAMS IN THE SAME ORDER AS THEIR PREFERENCES".

This description suggests that the final allocation must be stable, in the sense that there is no pair student/program who simultaneously prefer to be matched together rather than to their matches in the proposed assignment. Indeed, as the results are public and students can easily check if their application scores are higher than that of the last student admitted in a program they prefer, legal problems may arise if the resulting matching was unstable. However, it is unclear from the description which specific stable assignment is implemented. Moreover, by analyzing the results of previous assignments we realized that, in case of a tie in the last seat of a program, the number of seats were increased in order to include all tied applicants. This feature of the system was confirmed by DEMRE, and applies to both admission tracks (Regular and BEA). From now on we refer to this as *flexible quotas*.

The results of the assignment process are released by mid-January, and at this point the enrollment process starts. In its first stage, which lasts for three days, students can enroll in the programs they resulted assigned (either in the Regular or in the BEA process). In its second stage, which lasts for one week, programs with seats left after the first stage can call students in their wait-lists and offer them the chance to enroll. This must be done in strict order of preference given by the application scores, and students must decline their enrollments in the first stage to enroll in a new program.[10] However, programs can decide not to call students if they have already filled a minimum number of Regular seats,[11] and they are not forced to re-allocate unassigned BEA seats. By the rules of the system, all seats left after the second stage of enrollment are lost, including those seats not taken by students with double-assignments.[12] This is the main source of inefficiency that we address in this paper.

3. Literature review

This paper is related to several strands of the literature. The most closely related is Biró and Kiselgof (2015), which analyzes the college admissions system in Hungary, where all students tied in the lowest rank group of a program are rejected if their admission would exceed the quota. This mechanism is opposed to the Chilean case, where the quota is increased just enough so that all tied students are admitted. Biró and Kiselgof (2015) formalize these ideas by introducing the concepts of H-stability and L-stability, that correspond to the rules in Hungary and Chile respectively. They also provide a natural adaptation of the Deferred Acceptance algorithm to compute H-stable and L-stable based on ascending score limits, and provide an alternative proof of the manipulability of H-stable and L-stable mechanisms. In a recent paper, Kamiyama (2017) presents a polynomial time algorithm to check whether a student can manipulate his preferences to obtain a better allocation. Our paper contributes to this strand of the literature by independently introducing the notion of L-stability, providing an algorithm based on Baïou and Balinski (2004) to find all L-stable matchings, and implementing it to solve a real, large and relevant problem.

Our paper is also related to the literature on affirmative action. Most of the research in this strand has focused on proposing mechanisms to solve the college admissions problem with diversity constraints and deriving properties such as stability, strategy-proofness and Pareto optimality. From a theoretical perspective, Echenique and Yenmez (2012) point out that the main tension between diversity concerns and stability is the existence of complementarities, although the theory requires substitutability for colleges' choices. Abdulkadiroğlu (2007) explores the Deferred Acceptance algorithm under type-specific quotas, finding that the student-proposing DA is strategy proof for students if colleges' preferences satisfy responsiveness. Kojima (2012) shows that majority quotas may actually hurt minority students. Consequently, Hafalir et al. (2013) propose the use of minority reserves to overcome this problem, showing that the deferred acceptance algorithm with minority reserves Pareto dominates the one with majority quotas. Ehlers et al. (2014) extend the previous model to account for multiple disjoint types, and propose extensions of the Deferred-Acceptance algorithm to incorporate soft and hard bounds. Other types of constraints are considered by Kamada and Kojima (2015), who study problems with distributional constraints motivated by the Japanese Medical Residency. The authors propose a mechanism that respects these constraints while satisfying other desirable properties such as stability, efficiency and incentives.

Some authors have recently analyzed the impact of the order in which reserves are processed. Dur et al. (2016a) analyze the Boston school system and show that the precedence order in which seats are filled has important quantitative effects on distributional objectives. This paper formalizes our idea that processing reserved seats in a lower precedence order benefits BEA students. In a follow-up paper, Dur et al. (2016b) characterize optimal policies when there are multiple reserve groups, and analyze their impact using Chicago's system data.

Finally, our paper also contributes to the literature on designing large-scale clearinghouses. Institutional details and special requirements oftentimes forbid the use of tools directly taken from the theory, and other engineering aspects become relevant in the design process (Roth 2002). Roth and Peranson (2002) report the design of a new clearinghouse to organize the labor market for new physicians in the United States. Since the new algorithm was finally adopted by the National Resident Matching Program (NRMP) in 1997, more than 20,000 doctors have been matched to entry level positions every year, and other labor markets have adopted the Roth-Peranson design, including Dental, Pharmacy and Medical Residencies (see Roth (2002) for other examples). In the school-choice context, Abdulkadiroğlu et al. (2005) describe the design of a new mechanism to match entering students to public high-schools in New York. The new algorithm helped to dramatically reduce the number of students assigned to schools for which they had expressed no preference, and has motivated other school districts to implement centralized clearinghouses (e.g. Boston, Amsterdam, New Orleans, Chicago, among others). Closer to our setting, a recent paper by Baswana et al. (2018) describe the design and implementation of a clearinghouse to perform the allocation of students to technical universities in India. Their heuristic approach, which also allocates all tied students at the cutoffs and extends DA to accommodate the multiple types of seat reservations for affirmative action, has been successfully running since 2015.

4. Model

The following framework is assumed hereafter. Consider two finite sets of agents: programs $C = \{c_1, \ldots, c_m\}$ and applicants $A = \{a_1, \ldots, a_n\}$. Let $V \subset C \times A$ be the set of *feasible pairs*, with $(c, a) \in V$ meaning that student a has submitted an application to program c and meets the specific requirements to be admissible in that program. A *feasible assignment* is any subset $\mu \subseteq V$. We denote by $\mu(a) = \{c \in C : (c, a) \in \mu\}$ the set of programs assigned to a and $\mu(c) = \{a \in A : (c, a) \in \mu\}$ the set of students assigned to program c. Each program c has a quota $q_c \in \mathbb{N}$ that limits the number of students that can be admitted. Moreover, program $c \in C$ ranks applicants according to a *total pre-order* $\leq_c$, namely a transitive relation in which all pairs of students are comparable. The indifference $a \sim_c a'$ denotes as usual the fact that we simultaneously have $a \leq_c a'$ and $a' \leq_c a$, and we write $a <_c a'$ when $a \leq_c a'$ but not $a \sim_c a'$. On the other side of the market, each applicant $a \in A$ ranks programs according to a *strict total order* $<_a$, i.e. for any programs c, c' such that $c >_a \emptyset$ and $c' >_a \emptyset$ (i.e. c, c' are acceptable to student a), we have either $c <_a c'$ or $c' <_a c$.

A *matching* is an assignment $\mu \subseteq V$ such that for each applicant a the set of assigned programs $\mu(a)$ has at most one element, while for each program c the set of assigned students $\mu(c)$ has at most q_c elements. A matching μ is *stable* if for all pairs $(c, a) \in V \setminus \mu$ we have that either the set $\mu(a)$ has an element preferred over c in the strict order $<_a$, or the set $\mu(c)$ contains q_c elements preferred over a in the strict order $<_c$. In the first case the applicant a likes the match proposed by μ better than c, while in the second case the program has all its vacancies filled with students strictly preferred than a. If both conditions fail simultaneously a and c would be better off by being matched together rather than accepting the assignment μ, in which case (c, a) forms a *blocking pair*. In other words, a matching is stable if it has no blocking pairs.

In their seminal paper Gale and Shapley (1962) introduced the Deferred Acceptance (DA) algorithm, which returns the stable matching that is most preferred by agents on the proposing side. Hence, by changing the proposing side DA allows to find two extreme stable matchings: the student-optimal and the university-optimal. However, there are many reasons why the clearinghouse may want a stable outcome that is different from the extreme ones. For instance, the clearinghouse may be concerned about fairness (e.g. Teo and Sethuraman (1998) and Schwarz and Yenmez (2011)), and may prefer to benefit some specific agents in the market. In a recent paper, Dworczak (2018) introduces the concept of Deferred Acceptance with Compensation Chains (DACC), which generalizes DA by allowing both sides of the market to propose. The author shows that a matching is stable if and only if it can be obtained through a DACC algorithm, and provides an algorithm that finds the stable matching given a sequence of proposers.

The aforementioned approach could be used to obtain all stable matchings by considering different sequences of proposers. However, this would require running the algorithm for each potential sequence, which is inefficient specially when the core of stable outcomes is relatively small. Therefore, we adopt an alternative approach and extend the algorithm introduced by Baïou and Balinski (2004), which uses a graph representation of the admissions problem. Following their approach, an *instance* of the college admissions problem can be fully described in terms of a pair $\Gamma = (G, q)$, where $G = (V, E)$ is an admission graph consisting of a set of feasible nodes V on a grid $C \times A$ and a set of directed arcs $E \subseteq V \times V$ that represent programs and applicants preferences; and q is a vector of quotas. Each row in the grid represents a program $c \in C$, and each column represents an applicant $a \in A$. The preferences of program c are encoded by horizontal arcs from (c, a) to (c, a') whenever $a \leq_c a'$, and those of student a by vertical arcs from (c, a) to (c', a) representing $c <_a c'$. For simplicity, the arcs that can be inferred by transitivity are omitted.

By exploiting this graph representation, the algorithm proposed by Baïou and Balinski (2004) recursively eliminates pairs $(c, a) \in V$ that are strictly dominated and thus cannot belong to any stable matching. More precisely, (c, a) is *a-dominated* if there are q_c or more applicants that have c as their top choice and dominate a in the strict preference $<_c$. In this case, program c is guaranteed to fill its quota with applicants above a so that a has no chance to be assigned to c, and the pair (c, a) can be eliminated from further consideration.

Similarly, (c, a) is *c-dominated* if there is a program $\tilde{c}$ that places a among the top $q_{\tilde{c}}$ applicants (*i.e.* less than $q_{\tilde{c}}$ applicants are ranked above a) and that is preferred by a over c, i.e. $\tilde{c} >_a c$. In this case, student a is guaranteed to be assigned to a program ranked at least as high as program $\tilde{c}$ in his preference list, so the pair (c,a) cannot belong to any stable assignment.

As a result, the algorithm returns a *domination-free subgraph* $G^* = (V^*, E^*)$, with node set $V^* \subseteq V$ in which all dominated nodes have been removed. The domination free equivalent subgraph G^* contains all possible stable allocations, including the two most interesting (and extreme) cases: the *student-optimal* matching μ^*_A, that assigns each applicant $a \in A$ to its best remaining choice in G^* (if any); and the *university-optimal* matching μ^*_C that assigns to each program $c \in C$ its q_c top choices in G^*. In this way, the algorithm by Baïou and Balinski (2004) returns the allocations that could be obtained using DA but also the nodes that could potentially be in other stable outcomes, allowing to find other non-extreme stable assignments.

In order to apply this mechanism to the Chilean case, we extend it to incorporate two special features: (1) the existence of ties and flexible quotas, and (2) the affirmative action. The next two sections describe how we incorporate these elements into the mechanism.

4.1. Ties and Flexible Quotas: FQ-matchings

Suppose now that programs' preferences may not be strict, and that programs are required to adjust their quotas to include all applicants tied in the last seat. More precisely, a program c may exceed its quota q_c only if the last group of students admitted are in a tie and upon rejecting all these students c results with unassigned seats. We also impose a non-discrimination condition: an applicant a' who is tied with a student a admitted to a program c must himself be granted admission to c or better. The following definitions state these conditions formally.

Definition 1. We say that μ satisfies *quotas-up-to-ties* if for each program c and $a \in \mu(c)$ the set of strictly preferred students assigned to c satisfies $|\{a' \in \mu(c) : a' >_c a\}| < q_c$.

Definition 2. We say that μ satisfies *non-discrimination* if whenever $a \in \mu(c)$ and $a' \sim_c a$ with $(c,a') \in V$ then $a' \in \mu(c')$ for some program $c' \geq_{a'} c$.

With these preliminary definitions we introduce our notion of matching, which requires in addition that each applicant is assigned to at most one program.

DEFINITION 3. A *matching with flexible quotas* (FQ-matching) is an assignment $\mu \subseteq V$ that satisfies quotas-up-to-ties and non-discrimination, and for which every applicant $a \in A$ is assigned to at most one program so that $\mu(a)$ has at most one element.

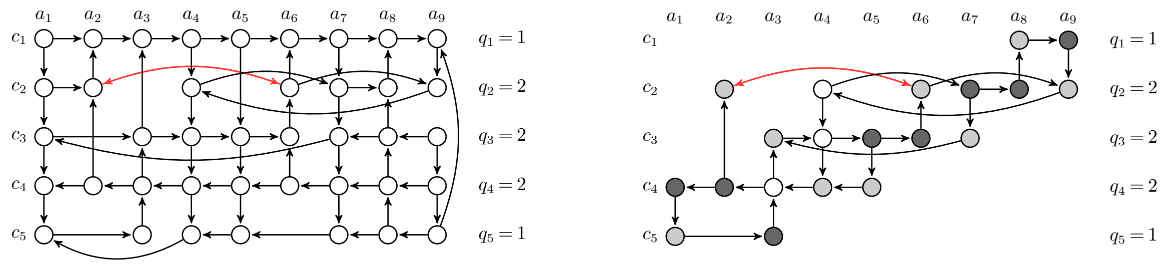
By construction, FQ-matchings are stable. In Appendix A we provide a formal proof of stability, and in Appendix C we describe how our definition of FQ-matching relates to other notions of stability, such as weak, strong and super-stability. In addition, notice that if there are no ties in the preferences of programs then non-discrimination holds trivially, while quotas-up-to-ties reduces to $|\mu(c)| \leq q_c$; hence, FQ-matching coincides with the standard notion of stable matching.

To compute an FQ-matching we propose the following procedure. As in the algorithm by Baïou and Balinski (2004), start by recursively removing all strictly dominated nodes, ensuring that students tied in the last place of a program are kept. In Appendix A we show that all along this elimination process we preserve exactly the same stable FQ-matchings as in the original instance Γ. Hence, the resulting domination-free subgraph G^* contains exactly the same set of FQ-matchings as G. Finally, starting from G^* we can obtain an FQ-matching by assigning each student to a program. In particular, the two extreme allocations can be obtained by greedily assigning each student to his top preference (student-optimal), or each program to its most desired q_c students including those tied in the last place (university-optimal).[13] In Appendix F we describe the algorithm that was finally implemented (in 2016), which is a faster version as it only computes the student-optimal FQ-matching. To accomplish this, the algorithm recursively eliminates all nodes that are a-dominated, and later assigns each student to his top choice in the resulting sub-graph. This is a good alternative to other algorithms, such as DA or score-limit (see Biró and Kiselgof (2015)), to compute the student-optimal assignment when quotas-up-to-ties and non-discrimination are required.

Applying this procedure to the example in Figure 2a we obtain the reduced graph G^* presented in Figure 2b and the extreme assignments: μ_A^* (light gray) and μ_C^* (gray). This example shows that the inclusion of a single tie may considerably change the outcome.[14]

In Appendix B we show that the two extreme FQ-matchings are optimal, but they lack two important properties: monotonicity and strategy-proofness (SP). The lack of strategy-proofness

Figure 2 Non-strict preferences



(a) Admission graph

(b) Optimal matchings

can be troublesome because it may induce agents to misreport their preferences strategically, giving an unfair advantage to more sophisticated students. However, we argue that the mechanism is *strategy-proof in the large* (SP-L), which means that students find approximately optimal to submit their true preferences in a large market for any full support i.i.d. distribution of students' reports (see Azevedo and Budish (2018)). In fact, as Azevedo and Budish (2018) argue, the relevant distinction for practice in a large market is whether a mechanism is "SP-L vs not SP-L" and not "SP vs not SP", since students in a large market do not know what are the realized reports of every other student, so imposing optimality of truthful reporting against every report realization (as in SP) is too strong. Thus, the lack of strategy-proofness is not a problem in our setting.

Overall, the implementation of an FQ-matching involves a trade-off between potentially exceeding capacities and obtaining a better allocation for students in the Pareto sense. If universities don't want to arbitrarily discriminate students and their marginal cost of increasing their capacity is low enough, allowing for ties and flexible quotas can be a sensible policy because it translates to a Pareto improvement for students[15] and eliminates any fairness concerns that can arise due to tie breaking rules. We discuss this in more detail in Section 5.1.1.

4.2. Unifying Admission Tracks

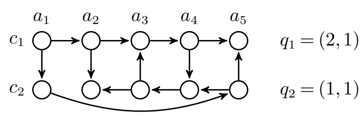
Using the model described in the previous section we can directly include the affirmative action and solve both admission tracks (Regular and BEA) simultaneously.

To accomplish this, we consider a unified admission instance $\Gamma^U = (G^U, q^U)$ where each program $c \in C$ is split into two virtual programs c^R and c^B that represent the Regular and the BEA processes, with quotas q_c^R and q_c^B respectively. The preferences of students that are not shortlisted for the scholarship remain unchanged. In contrast, each program in the preference list of a BEA student is also divided into the two virtual programs, giving a higher position in the preference list to the Regular process, i.e. for any two programs with $c_1 >_a c_2$, the new preference order is $c_1^R >_a c_1^B >_a c_2^R >_a c_2^B$. We decided to use this order because DEMRE wanted to prioritize BEA students. Then, by applying to the regular seats first, BEA students with good scores can be admitted in regular seats, reducing the competition for reserved seats and therefore weakly increasing the total number of BEA students admitted in the system. This idea is formalized in Dur et al. (2013), and recently extended to more reserve groups in Dur et al. (2016b). In Appendix A we show that every student is weakly better off compared to the sequential solution.

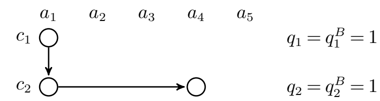
Example 1. Consider the admission graph in Figure 3a and suppose that students a_1 and a_4 are shortlisted for the scholarship. In the sequential case the Regular process is run first considering admission graph G^R in Figure 3a and quotas $q_c = q_c^R$. As a result we obtain the allocation[16] $\mu(\Gamma^R) = \{(c_1, a_4), (c_1, a_5), (c_2, a_2)\}$. Then, the BEA process $\Gamma^B = (G^B, q^B)$ is built considering only the shortlisted students and the preferences where they were wait-listed in the Regular process. Figure 3b illustrates the corresponding graph G^B. The resulting allocation for the BEA process is $\mu(\Gamma^B) = \{(c_1, a_1), (c_2, a_4)\}$, and therefore student a_4 is assigned to c_1 in the Regular process and to c_2 in the BEA process, while a_3 remains unassigned. Independently of which option is taken by a_4, a seat that could have been otherwise assigned to a_3 will be lost.

The unified graph of this problem is shown in Figure 3c. In this case we observe that there is a unique FQ-matching given by $\mu(\Gamma^U) = \{(c_1^R, a_3), (c_1^R, a_5), (c_1^B, a_1), (c_2^R, a_2), (c_2^B, a_4)\}$. In this case all applicants are assigned and no seats are lost. More importantly, every student is indifferent or better off compared to the sequential assignment.

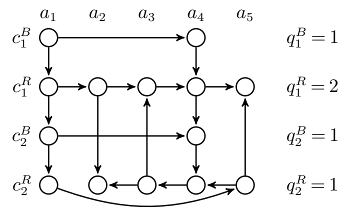
Figure 3 Unified process



(a) Original admission graph G^R.



(b) BEA admission graph G^B.



(c) Unified admission graph G^U.

5. Implementation

In this section we report the results on the implementation of this project. We start providing a general description of the Chilean college admissions problem. Then, we describe the results of our first goal, which was to find the algorithm that has been used in Chile to perform the allocation. Finally, we close this section with the results of unifying the admission tracks and other additional side effects of this project.

5.1. General Description

In Table 1 we present general descriptives on the programs that are part of the centralized admission system. We observe that, between 2014 and 2016, the number of universities did not changed, while the number of programs slightly increased. However, we see that the number of seats available decreased over the years.

To describe the other side of the market, in Table 2 we present the number of Regular and BEA students at each stage of the admission process. A *participant* is a student that registered to participate in the standardized national exam and has at least one valid score. Once the results of the national exam are published, participants have five days to submit their applications to the centralized clearinghouse. We refer to the students that apply to at least one program that is

Table 1 General description — Programs

	2014	2015	2016
Universities	33	33	33
Programs	1,419	1,423	1,436
Regular seats	110,380	105,516	105,513
Reserve seats	4,394	4,422	4,295

Table 2 General description — students

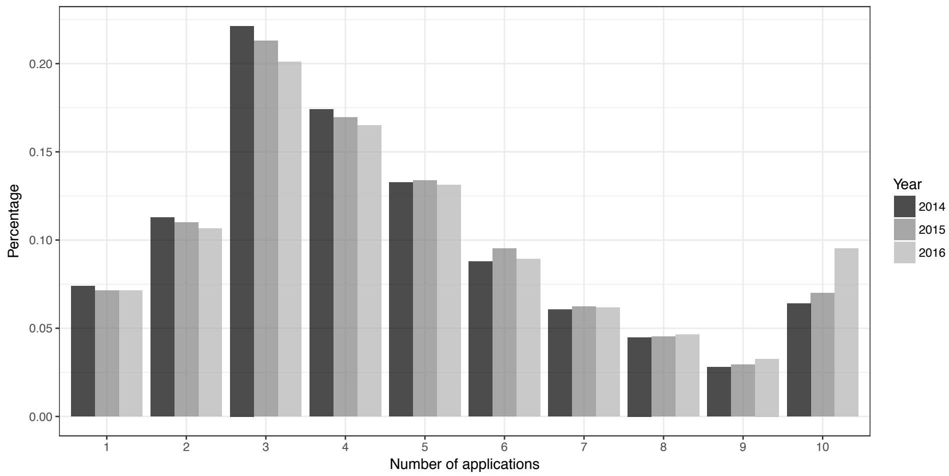
		Regular			BEA		
		2014	2015	2016	2014	2015	2016
Participants		228,318	241,873	250,320	15,990	16,710	16,911
Applicants		108,144	113,900	129,896	11,017	11,688	12,010
Assigned	Regular seats	86,048	87,466	90,741	9,520	10,154	8,886
	Reserve seats	-	-	-	1,325	1,404	1,345

part of the centralized system as *applicants*. Finally, we refer to students that were admitted by a program that is part of the centralized system as *assigned*.

First, we observe that the total number of participants has increased over the years, reaching a total of 267,231 participants in 2016. Second, comparing the number of participants and the number of applicants we observe that close to a half of the students that registered for the national exam applied to programs that are part of the system. The main reason for this is that CRUCH sets a minimum threshold of 450 points[17] for students to be eligible by any program that is part of the system, and since tests are standardized to have mean 500, roughly half of the students will not satisfy this criteria.

Also related to the application process, in Figure 4 we show the distribution of applications per student for each year.

Figure 4 Distribution of applications per student



The median number of applications is 4 and the share of each number of applications stays roughly constant across years. As students are restricted to submit a list with no more than 10 programs, we observe that between 5% and 10% of applicants submit a full list of 10 applications.

Notice that some universities further restrict the number of programs to which a student can apply, and also the position that an application can take in the applicant's list.[18] Theoretically, any restriction on the length of the application list will break strategy-proofness. Nevertheless, whether these constraints are binding or not in practice, and what the strategic implications are for students, are questions for future research.

Even though the number of participants and applicants have increased over the years, Table 2 shows that the total number of admitted students has decreased, in line with the reduction of seats that we see in Table 1. In fact, comparing applicants and assigned students we find that close to 80% get assigned to some program in 2014, 78% in 2015 and 70% in 2016.

In Figures 5a and 5b we show the distribution of the preference of assignment for Regular and BEA students respectively. We see that close to 50% of students get assigned to their first reported preference, and close to 90% get assigned to one of their first three preferences. Although both Regular and BEA students exhibit the same pattern of assignment, notice that the latter get assigned consistently more to their first preference than Regular students.

Finally, in Table 3 we present demographic characteristics of the students that are assigned.

Figure 5 Preference of assignment

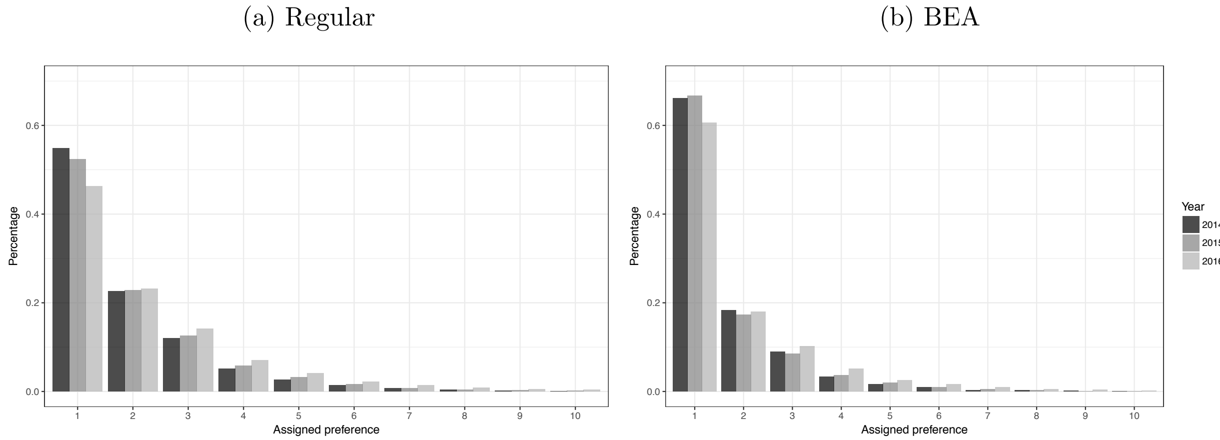


Table 3 General description — Assigned

		Regular			BEA		
		2014	2015	2016	2014	2015	2016
Assigned	Total	86,048	87,466	90,741	9,745	10,378	10,231
Gender	Female	49.5%	49.5%	50.2%	57.6%	58.4%	59.5%
Average Scores	Math/Verbal[1]	588	589.3	588.7	591.1	595.9	593.4
	NEM[2]	586	588.4	592	696.8	696.7	700.5
	Rank[3]	608.7	614.4	615.5	770.6	776.9	774.5
Income[4]	[0, $288]	28.8%	26.4%	23.5%	46%	42.6%	40.9%
	($288, $576]	26.7%	27.5%	29%	34.6%	37.1%	38.6%
	($576, $1,584]	26.8%	27.5%	28.9%	18.7%	19.5%	19.1%
	> $1,584	17.8%	18.6%	18.7%	0.7%	0.8%	1.4%
High-School	Private	23.4%	23.4%	22.5%	0%	0%	0%
	Voucher[5]	52.4%	52.9%	53.3%	60.7%	61%	61.4%
	Public	24.2%	23.7%	24.1%	39.3%	39%	38.6%

[1] Score constructed with the average Math score and Verbal score. For students using scores from previous year, we considered the maximum of both averages.
[2] Score constructed with the average grade along high-school.
[3] Score constructed with the relative position of the student among his/her classmates.
[4] Gross Family monthly income in thousands Chilean pesos (nominal).
[5] Partially Subsidized schools.

The fact that around 23% of admitted students graduated from a private school is surprising, considering that they represent only 12% of the total number of participants in the admission process. Similarly, students from the highest income group only represent 9% of the total number of participants, but they account for 18% of admitted students. These numbers shed some light on the huge inequalities in opportunities that characterize the Chilean college admission process.

The point of having reserve seats is to alleviate these inequalities and favor underrepresented groups. The comparison of the left and right columns in Table 3 illustrates this. Compared to Regular students, in the BEA group the fraction of female students is higher, the average scores in all the exams are also higher, the fraction of students with higher income levels is smaller, and all students come from public/voucher schools.

5.1.1. Effect of Flexible Quotas One of the most distinctive features of the Chilean case is the use of flexible quotas. This approach belongs to the general category of *equal treatment policies* (Biró and Kiselgof 2015), where all tied students whose admission would exceed a program's capacity are either accepted or rejected, as it is the case in Chile and Hungary respectively. An alternative approach is to break ties using a combination of randomness and administrative rules, as it is the case in most school districts and in many college admissions settings, such as in Spain, Turkey, Germany, France, among others. A special case of these tie-breaking rules are those that exclusively depend on randomization, such as the *single tie-breaking* (STB) and *multiple tie-breaking* (MTB).[19] Which approach to use is a relevant design decision, and it heavily depends on the characteristics of the problem.

We identify three dimensions that could help in guiding this decision. First, the nature of agents' preferences plays a critical role. Indeed, if preferences are fine enough so that the number of ties is relatively small, the benefits of having flexible quotas—non-discrimination and better allocation for students in the Pareto sense—may outweigh its costs—exceeding capacities. This would be the case in most college admissions settings, where preferences are built based on scores from exams and/or grades. In contrast, when preferences are rather coarse (e.g. in school choice settings with a limited number of priority groups) having flexible quotas may lead to large violations of initial capacities, which could make its implementation unfeasible. Second, the level of heterogeneity in students' preferences is also relevant, as it prevents that a small number of programs concentrate most of the ties. A last element to consider is whether the allocation depends on factors that are perceived as relevant to the process. For example, waiting times for public housing, the condition of a patient for organ transplants, or exam scores in college admissions are generally considered as fair factors, so using random tie-breakers may be considered arbitrary and discriminatory. In fact, the use of random orders to break ties in the admission process to high-schools in Chile (see Aramayo et al. (2019)) has generated a huge debate. Their opponents argue that some measure of academic achievement should be considered instead.[20]

To illustrate the effect of having flexible quotas, we compare the official assignments to the results that would be obtained if ties were handled with other approaches. To our knowledge this is the first paper to compare equal treatment policies with random tie-breakers, and thus contributes to the literature that compares STB and MTB empirically (see Abdulkadiroğlu et al. (2009) and de Haan et al. (2015)) and theoretically (see Ashlagi et al. (2015) and Arnosti (2015)).

In Table 4 we report the number of extra seats required as a result of flexible quotas, and how these are distributed across programs. We observe that the number of additional seats created is small (maximum of 83 seats in 2015), which represents less than 0.1% of the total number of seats for each year. In addition, we observe that these seats are evenly spread across programs, as the maximum number of seats created by a given program is 3. Hence, we conclude that having flexible quotas does not involve a large cost for programs.

Table 4 also reports the number of students that benefit from having flexible quotas as opposed to an H-stable mechanism (Biró and Kiselgof 2015), i.e. one that rejects all tied students whose admission would exceed the program's capacity. The first group—*Improvements*—includes those students that improve their assignment, while the second—*New Assignments*—considers students who are assigned to some program in the official assignment (with flexible quotas) and would not

Table 4 Impact of Flexible Quotas vs. H-stability

	2014	2015	2016
Extra Seats	58	83	66
Programs with flexible quotas	50	73	58
Maximum number of flexible quotas	2	3	3
Benefits compared to H-stability			
Improvements	122	309	203
New Assigned	75	137	109
Total	197	446	312

Table 5 Impact of Flexible Quotas vs. Random Tie-Breaking

Benefits from Flexible Quotas	STB			MTB		
	2014	2015	2016	2014	2015	2016
Improvements	88.7	159.2	107.3	87.8	161.5	107.5
	(4.5)	(7.8)	(6.0)	(4.6)	(6.5)	(7.0)
New Assigned	35.1	63.0	56.3	35.6	63.1	56.5
	(1.9)	(2.2)	(2.2)	(2.0)	(2.3)	(2.0)
Total	123.8	222.2	163.6	123.4	224.6	164.0
	(3.9)	(7.1)	(6.0)	(3.9)	(6.4)	(7.1)

be assigned under the alternative mechanism. Similarly, in Table 5 we summarize the benefits from having flexible quotas compared to breaking ties randomly using STB and MTB. These results are obtained from 100 simulations for each tie-breaking rule, and as before we separate the students that benefit in two groups: improvements and new assignments.

As expected, all students weakly prefer their allocation under flexible quotas, and a significant number of students strictly prefers it compared to H-stability, STB and MTB. In addition, we observe that the average number of students that benefit from flexible quotas largely exceeds the number of extra seats created, with almost 3 students benefiting from each extra seat. Among these, we find that roughly 2/3 are students who improve their assignment, while 1/3 are students who would not be assigned if a random tie-breaker were used. All these results suggest that having flexible quotas benefits an important number of students without generating a big cost for programs, and we expect to find similar patterns in other college admissions systems that are similar to the Chilean case, such as those in Hungary, Turkey and Spain.

5.2. Identifying the Current Mechanism

Our first goal was to identify which mechanism has been used to solve the Chilean college admissions problem. After implementing the algorithms previously described above and including all the constraints that are part of the system, we solved the admission instances from 2012 to 2014, comparing the FQ-student-optimal and FQ-university-optimal allocations with the official results obtained using DEMRE's black box. Based on this comparison, the rules of the system and evidence provided by DEMRE we concluded that the algorithm used is equivalent to the FQ-university-optimal matching, as the results are exactly the same for all the years considered.

Given that our algorithm returns all stable allocations for each instance, by comparing the two extreme assignments (FQ-student and FQ-university optimal) we find that the number of differences between these allocations has been at most 10 since 2012. This suggests that the size of the core of stable assignments in the Chilean case is rather small, supporting the theoretical results in Roth and Peranson (2002) and Ashlagi et al. (2017).

Even though the number of differences is small, we proposed DEMRE to adopt an FQ-student optimal matching because it benefits some students but, more importantly, because it is a message for students that the mechanism aims to give them the best possible allocation. DEMRE agreed with this view, and after a pilot version in 2014 they adopted our FQ-student optimal algorithm to perform the allocation in 2015.

5.3. Integrating Admission Tracks

Having identified the algorithm that is used to perform the allocation, our second goal was to integrate the admission tracks in order to alleviate the aforementioned inefficiencies. To accomplish

Table 6 Impact of Unified Assignment 2014-2016

	2014	2015	2016
Double Assigned	1,100	1,180	1,127
Improvements	1,737	1,915	1,749
New Assigned	568	672	777

this, we implemented the framework described in Section 4.2 and we ran it in parallel during the admission processes of 2014 and 2015. Based on the results, we convinced DEMRE to adopt our unified FQ-student optimal allocation in 2016, and this allocation has been the official mechanism used since then. In this section we report the results from our simulations (2014 and 2015), and the actual impact of our implementation in 2016.

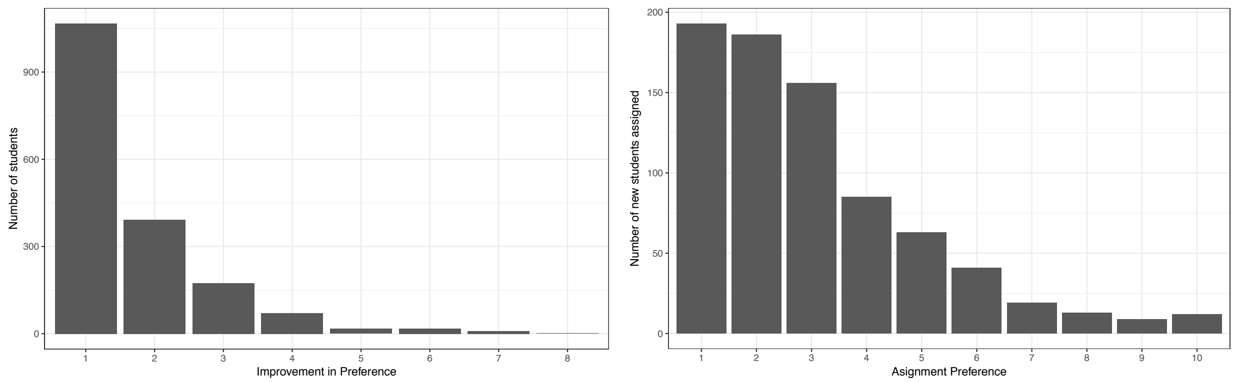
In Table 6 we present a summary of the results. The first row presents the number of students that would have been double-assigned under the old system. The second row presents the number of students that improved their assignment compared to the old system. Finally, the third row shows the number of students who are assigned to some program under the new system and weren't assigned in the old system.

We observe that the number of students that benefit from unifying the admission tracks is larger than the number of seats lost due to double-assignments. The reason is that a student that directly benefits releases a seat that can be used by another student, who in turn allows another student to take his old seat, and so on. This chain of improvements eventually ends either because there are no students wait-listed in that program, or because they reach a student who was unassigned and therefore does not release another seat. Overall, we observe that around 3% of students who decide to apply to a program in the system benefited from our implementation, and this number is relatively constant across years.

Improving the assignment of students is relevant because the probability of enrollment is increasing in the preference of assignment[21], and most programs in the system have positive and high expected returns, which are measured in terms of the net present value of future earnings over

Figure 6 Benefits of unifying the admission tracks - 2016

(a) Improvement in assigned preference (b) Preference assigned of new assigned students



the life cycle after graduation (see Lara et al. (2017)). Moreover, there is evidence that students that were assigned in low listed preferences have a higher probability of future dropout from their programs (see Canales and de los Ríos (2007)).

In Figure 6a we present the distribution of improvements in allocated preferences in the actual implementation of our new approach (2016). We observe that most students improve in their allocation by getting assigned to the program listed immediately above the program they were previously assigned (improvement equal to 1). In Figure 6b we show the preference of assignment for those students who were not assigned under the old system and thanks to the new system are allocated. Most of the students that were unassigned under the sequential allocation benefit from the unified allocation by getting assigned to their top choice. A potential reason for this is that an important fraction of these students apply to less than three programs.

We provide more details on those students that benefit from unifying the admission tracks in Table 7. We first observe that most of the students that benefit from unifying the admission tracks are Regular students. The reason is that seats that were dropped by a BEA student with double assignment are now used by other students, and this generates improvement chains that reach other (mostly Regular) students. In addition, comparing the characteristics of those who improve (*Improvements*) with those who get assigned and would not under the old system (*New Assigned*) we find that the latter group has lower scores, and a larger fraction of students who come from

Table 7 General description — impact of unifying admission tracks

		Improvements						New Assigned					
		Regular			BEA			Regular			BEA		
		2014	2015	2016	2014	2015	2016	2014	2015	2016	2014	2015	2016
Assigned	Total	1,592	1,791	1,640	145	124	109	548	647	765	20	25	12
Gender	Female	47.2%	44.4%	45.7%	66.9%	60.5%	62.4%	52.2%	47.9%	51.6%	65%	68%	58.3%
Average Scores	Math/Verbal[1]	584.2	582	579.7	579.8	573.7	584.1	560.8	551.2	548.2	564.8	561	558
	NEM[2]	577.8	576.9	575	686	675.7	691.1	540	538.9	538.4	677.1	658	689.2
	Rank[3]	600.2	600.6	596.1	755.8	752.9	770	554.8	556.4	553.5	753.2	734	764.1
Income[4]	[$0, $288]	28.1%	27.4%	25.2%	48.3%	41.9%	48.6%	30.1%	32.3%	28.8%	60%	36%	75%
	($288, $576]	30.2%	29%	31.3%	29.7%	39.5%	37.6%	34.5%	32.9%	34%	25%	32%	8.3%
	($576, $1,584]	27.4%	29.3%	28.5%	20%	18.5%	13.8%	26.1%	24.3%	28.8%	15%	32%	16.7%
	> $1,584	14.3%	14.3%	14.9%	2.1%	0%	0%	9.3%	10.5%	8.5%	0%	0%	0%
High-School	Private	18.4%	18.6%	16.2%	0%	0%	0%	13.8%	14.5%	13.4%	0%	0%	0%
	Voucher[5]	56.7%	57.3%	56.7%	63.4%	61.3%	65.1%	59.5%	61.4%	61.5%	60%	72%	58.3%
	Public	26.7%	24.9%	24.2%	27.1%	36.6%	38.7%	34.9%	24.1%	25%	40%	28%	41.7%

[1] Score constructed with the average Math score and Verbal score. For students using scores from previous year, we considered the maximum of both averages.
[2] Score constructed with the average grade along high-school.
[3] Score constructed with the relative position of the student among his/her classmates.
[4] Gross Family monthly income in thousands Chilean pesos (nominal).
[5] Partially Subsidized schools.

lower income families and public schools. The reason for this is that students who improved were also assigned under the old system, while those from the *New Assigned* group were not. Therefore, students from the *Improvements* group have on average higher scores, and these are positively correlated with family income.

The differences in terms of scores and demographics are also present if we compare these groups with the overall group of assigned students described in Table 3. Indeed, previously assigned students have on average higher scores and higher family income than students that were benefited by the unified assignment. For instance, the share of assigned students coming from private high-schools was about 24% for Regular students, while it was close to 18% and 14% for students in the groups of *Improvements* and *New Assigned* respectively.

Another interesting result is that most of BEA students are assigned to regular seats, and more than half of the reserve seats remain unfilled (before the enrollment process begins), and this pattern continues even after the implementation of the unified system (see Tables 1 and 2). Indeed, we proposed DEMRE to use the unfilled reserve seats with students from the Regular process, but they declined because some universities "were not open to this option".[22]

5.4. Additional Side Effects

In terms of running times, our implementation considerably outperforms the algorithm used previously by DEMRE to solve the admissions problem. In fact, their black-box software takes up to 5 hours to return the final assignment, while our implementation solves the problem in less than 2 minutes on a standard laptop. This time reduction has had a significant impact since it allows to evaluate different policy changes in the system, such as the inclusion of new admission criteria, the impact of new instruments, and the redesign of affirmative action policies. In particular, the new algorithm was used to evaluate the effect of including the high-school class rank as an admission factor through simulations changing the conditions in which this new instrument is included (Larroucau et al. (2015)). Furthermore, the efficiency gains have opened other directions for future research, involving the evaluations of policies that could stress the system in the future. For instance, the impact of free-of-charge access, the inclusion of professional and vocational institutions to the admission process, and the implementation of admission quotas for underrepresented groups. This type of evaluations was not possible in the past due to the computational time involved.

6. Conclusions

We investigate how the Chilean college admissions system works. There are two main features that make the Chilean system different from the classic college admissions problem: (i) preferences of colleges are not strict, and all students tied for the last seat of a program must be assigned; and (ii) the system considers an affirmative action that is solved sequentially after the Regular process.

Then, students who benefit from the affirmative action can be double-assigned, introducing a series of inefficiencies in the assignment and enrollment processes. Even though the authorities were aware of this problem, they couldn't solve it because they relied in a black-box software that couldn't be updated to incorporate the affirmative action.

To identify which mechanism was used, we develop an algorithm that finds all stable allocations satisfying the rules of the system, i.e. flexible quotas and non-discrimination of tied students. We also introduce the notion of FQ-matching to account for these features, and we characterize its main properties. We show that this mechanism leads to the optimal stable allocations satisfying flexible quotas and non-discrimination, but it lacks monotonicity and strategy-proofness. Nevertheless, we also show that our mechanism is SP-L, and since the Chilean college admissions problem is large, the lack of SP is not a major concern.

By comparing the results of our algorithm with historical data we find that the algorithm that has been used is a variation of the university-optimal stable assignment that satisfies flexible quotas and non-discrimination. Even though the number of differences is small, we convinced DEMRE to switch to the FQ-student optimal mechanism, which was finally adopted in 2015 after a pilot version in 2014.

Having identified the algorithm, we propose a new method to incorporate the affirmative action that is based on treating regular and reserve seats as different programs. The unified approach to solve the problem was adopted and implemented by DEMRE in 2016, after two years of analyzing its potential impact. The results of the implementation in 2016, as well as the pilot results in 2014 and 2015, show that around 3% of the total number of students that are admitted each year benefit from the unified assignment. Among the students who actually benefited in 2016, 30.8% would not have been assigned to any program under the old system, and 69.2% are students who improved compared to what they would get under the old system. The benefited students have, on average, lower scores and lower family incomes compared to the students that would have been assigned under the old system.

In addition to its direct impact on students, the efficiency of our algorithm reduced the running time by two orders of magnitude relative to the old system, enabling the realization of simulations to evaluate different policies oriented to make the admission process more fair and inclusive. Finally, our method helped to improve the transparency of the system, and allowed other changes to be implemented on top of it.

Certainly there many directions for future work. While working on this project we realized that many students skip applying to programs where their chances of admission are too low, even though the constraint on the length of their preference list is not binding. Hence, considering their reports as truthful would lead to serious biases in the estimation of preferences, leading to wrong evaluations of policies. We are currently working on a model of preferences that takes this fact into account (see Larroucau and Rios (2019)). Another direction that emerged from this project is on trying to understand why some students apply to programs where they have no chance of getting admitted as they don't satisfy the requirements to be eligible. We are currently working on understanding why this is the case and designing changes to the application process aiming to reduce mistaken reports. Finally, another research question that arose from this project is how universities decide whether or not to join a centralized system, and when it is optimal for them to do so.

Overall, we hope that the current results encourage the Chilean authorities to keep improving the system and other college systems to evaluate and adopt flexible quotas, as this would increase the overall efficiency of the processes and improve the welfare of students.

Appendix/Electronic Companion

Appendix A: Proofs

To ease exposition we introduce some more notation. For any feasible pair $(c,a) \in V$ we write $c \rhd_a \mu(a)$ if a is either unassigned or strictly prefers c to his assigned program in μ, while $a \rhd_c \mu(c)$ means that c has not completed its vacancies or strictly prefers a to its worst assigned student. Formally, we define

- $c \rhd_a \mu(a) \Leftrightarrow$ the set $\mu(a)$ is either empty or it contains an element $c' <_a c$,
- $a \rhd_c \mu(c) \Leftrightarrow$ the set $\mu(c)$ has fewer than q_c elements or it contains an element $a' <_c a$

as well as the analog notions with non-strict preferences

- $c \unrhd_a \mu(a) \Leftrightarrow$ the set $\mu(a)$ is either empty or it contains an element $c' \leq_a c$,
- $a \unrhd_c \mu(c) \Leftrightarrow$ the set $\mu(c)$ has fewer than q_c elements or it contains an element $a' \leq_c a$.

In addition, let μ_A be a function that receives an instance $\Gamma = (G,q)$ and returns a matching where each student is greedily assigned to his/her top preferences in G. Similarly, let μ_C be the function that given an instance returns the matching that greedily assigns each program c to its top q_c applicants, including those tied in the last place. The next theorem shows that quota violations in the greedy assignments do not occur when G has no strictly dominated nodes.

Property A.1. Given an instance $\Gamma = (G,q)$ with $G = (V,E)$, if $(c,a) \in V$ is either a-dominated or c-dominated then $(c,a) \notin \mu$ for any stable FQ-matching μ, and the instance $\Gamma' = (G \setminus (c,a), q)$ is equivalent to Γ.

Proof: Let μ be an FQ-matching in Γ. Clearly the conditions for FQ-matching are preserved when we remove a node not in μ. Hence, if we prove the first assertion $(c,a) \notin \mu$ it will also follow that μ is an FQ-matching for Γ'. Now, if (c,a) is c-dominated there is a program $\tilde{c} >_a c$ for which a is top $q_{\tilde{c}}$, and therefore (c,a) cannot belong to μ since otherwise $(\tilde{c},a)$ would be a blocking pair. Similarly, if (c,a) is a-dominated, then there are q_c or more applicants $a' >_c a$ with c as their top choice. If $(c,a) \in \mu$, then all the pairs (c,a') must also be in μ since otherwise we would have a blocking pair, but this contradicts quotas-up-to-ties so that $(c,a) \in \mu$ is impossible in this case too.

Let us prove conversely that any assignment μ that is an FQ-matching for Γ' is also an FQ-matching for Γ. The properties of quotas-up-to-ties and $|\mu(a)| \leq 1$ involve only the nodes in μ and are not affected by the addition of the node (c,a). Hence, it suffices to establish non-discrimination and stability.

Non-discrimination. Since this property already holds for all the nodes in $V \setminus \{(c,a)\}$ we must only prove that it also holds for (c,a). Indeed, suppose that a is tied with some $a' \in \mu(c)$. Recall that (c,a) is strictly dominated, however it cannot be a-dominated since otherwise the same would occur for a' and it could not have been assigned to c. Hence, it must be the case that (c,a) is c-dominated which means that a is among the top $q_{\tilde{c}}$ applicants for some program $\tilde{c} >_a c$. But then a must be assigned to an even better choice $c' \geq_a \tilde{c}$, since otherwise $(\tilde{c},a)$ would provide a blocking pair in Γ'. Then $a \in \mu(c')$ for some $c' >_a c$ and non-discrimination holds for (c,a) as claimed.

Stability. We already know that there are no blocking pairs in Γ'. Let us prove that (c,a) is not a blocking pair in Γ. Suppose first that (c,a) is c-dominated so that a is top $q_{\tilde{c}}$ on some program $\tilde{c} >_a c$. In this case μ must assign a to some $c' \geq_a \tilde{c}$ since otherwise $(\tilde{c},a)$ would be a blocking pair in Γ', and therefore we do not have $c \rhd_a \mu(a)$. Similarly, if (c,a) is a-dominated there are q_c or more applicants a' ranked strictly above a that have c as their top choice. All the nodes (c,a') are in Γ' so that $\mu(c)$ must fill the quota q_c with applicants at least as good as the lowest ranked of these a'. Since this is still above a we do not have $a \rhd_c \mu(c)$. Combining both cases, we cannot have $c \rhd_a \mu(a)$ and $a \rhd_c \mu(c)$, proving that (c,a) is not a blocking pair.

Property A.2.

a) If the instance Γ has no a-dominated nodes then $\mu_A(\Gamma)$ is a stable FQ-matching.

b) If the instance Γ has no c-dominated nodes then $\mu_C(\Gamma)$ is a stable FQ-matching.

Proof: *a)* Since μ_A assigns each student a to its most preferred program it is obvious that it satisfies non-discrimination as well as stability, and $\mu_A(a)$ contains at most one element. It remains to show that it satisfies quotas-up-to-ties, which follows directly from the definition of μ_A and the fact that there are no a-dominated nodes. Indeed, for each program c and $a \in \mu_A(c)$ the node $(c,a) \in V$ is not a-dominated so that the set $\{a' \in \mu_A(c) : a' >_c a\}$ cannot contain q_c or more elements.

b) We already observed that in general μ_C is stable and satisfies quotas-up-to-ties. Also no student a can be simultaneously among the top q_c applicants for two different programs: otherwise the one which is less preferred by a would be c-dominated. Hence, $\mu_C(a)$ contains at most one element. It remains to show that μ_C satisfies non-discrimination. Consider a student $a \in \mu_C(c)$ and a node $(c,a') \in V$ with $a' \sim_c a$. By definition of μ_C we have that a is among the top q_c applicants for c, and then the same holds for a' so that $a' \in \mu_C(c)$.

A.1. Unified Process

Let μ_A^S be the student-optimal FQ-matching obtained by applying the sequential process, and let $\mu_A(\Gamma)$ be the student-optimal FQ-matching for an instance $\Gamma = (G,q)$. In addition, let Γ^R and Γ^B be the Regular and BEA instances respectively. Then, the *sequential student-optimal FQ-matching* μ_A^S is defined as

$$\mu_A^s(a) = \begin{cases} \mu_A(\Gamma^B)(a) \text{ if } \mu_A(\Gamma^B)(a) \neq \emptyset \\ \mu_A(\Gamma^R)(a) \text{ otherwise.} \end{cases} \tag{1}$$

PROPERTY A.3. The unified student-optimal FQ-matching $\mu_A(\Gamma^U)$ dominates the sequential student-optimal assignment μ_A^S, that is to say, $\mu_A(\Gamma^U)(a) \geq_a \mu_A^S(a)$ for all $a \in A$.

Proof: We first observe that the Regular matching $\mu_A(\Gamma^R)$ is the same as the one obtained from the unified graph G^U by setting the BEA quotas to 0, that is $q_c^B = 0$ for all $c \in C$. Since increasing quotas only benefits students in the student-optimal allocation we have that $\mu_A(\Gamma^R)(a) \leq_a \mu_A(\Gamma^U)(a)$ for all applicants $a \in A$. This already shows that all non-BEA students are not worse off in $\mu_A(\Gamma^U)$ as compared to μ_A^s.

Let us consider next the second stage in which BEA students compete for the programs in which they were not admitted in the Regular process. Consider the residual graph $\hat{G}$ obtained after running the Regular process on the unified instance Γ^U. We note that all the nodes (c, a) where a is shortlisted for the BEA scholarship and $c <_a \mu_A(\Gamma^R)$ are c-dominated and can be removed from $\hat{G}$. This further reduces the graph to $\bar{G}$ in which every BEA student keeps only the programs in which he/she was not admitted in the Regular process. Then, the second stage process can be seen as equivalent to running the matching over the residual graph $\bar{G}$, this time with the regular quotas set to 0, $q_c^R = 0$ for all $c \in C$. Following a similar argument as in the previous case, we know that $\mu_A(\Gamma^B)(a) \leq_a \mu_A(\Gamma^U)(a)$ for all applicants $a \in A$. This shows that all BEA students are not worse off in $\mu_A(\Gamma^U)$ compared to μ_A^s.

Appendix B: Properties of FQ-matchings

B.1. Optimality.

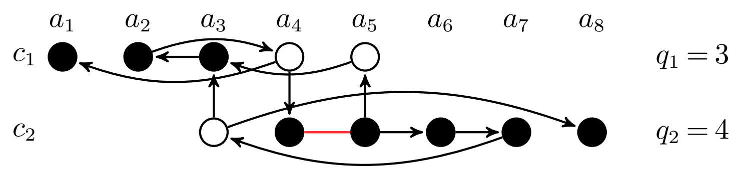
Ensuring that a given matching is optimal among the set of stable matchings guarantees that no agent (student or program) can be better off without harming another agent. This is formalized in the next theorem.

THEOREM 1. *The assignments μ_A and μ_C obtained from our procedure are FQ-matchings and are optimal for students and programs respectively among the set of FQ-matchings. Moreover,*

- *the allocation obtained from greedily assigning each students his top choice in G_A^* is equivalent to μ_A,*
- *the allocation obtained from greedily assigning each program its favorite students upon completing capacity (up to ties) in G_C^* is equivalent to μ_C.*

Proof (optimality): Let $\mathcal{M}$ be the set of nodes that belong to any FQ-matching. From Property A.1 we know that the domination-free subgraph G^* constructed by deleting both a-dominated and c-dominated nodes contains all FQ-matchings, and therefore $\mathcal{M} \subseteq G^*$. Since we also know that $\mu_A = \mu_A(G^*, q)$ is a FQ-matching we get $\mu_A \subseteq \mathcal{M} \subseteq G^*$. Finally, since $\mu_A(G^*, q)$ assigns each student his top choice in G^*, we conclude that the top program for each applicant a is the same

Figure 7 Admission graph with student-optimal FQ-matching.



in $\mathcal{M}$ and G^*. Similarly, $\mu_C(G^*, q) = \mu_C \subseteq \mathcal{M} \subseteq G^*$, and since $\mu_C(G^*, q)$ assigns each program c its top q_c students, we have that the top q_c choices for any program c are the same in $\mathcal{M}$ and G^*. This yields in particular that μ_C is an FQ-matching, concluding our proof.

The arguments for the other algorithms are essentially the same. Each of these algorithms computes a reduced subgraph G^* with $\mathcal{M} \subseteq G^*$ and the resulting student-optimal and/or university-optimal assignments are stable FQ-matchings so that they are contained in $\mathcal{M}$. Hence the top choices for applicants and/or programs are the same in $\mathcal{M}$ and G^*.

B.2. Lack of Monotonicity.

Monotonicity is a relevant feature since it ensures that any improvement in the scores of an agent cannot harm his assignment. The next examples show that neither the student-optimal nor the university-optimal FQ-matchings are monotone for students. This lack of monotonicity implies that a student could improve his outcome by strategically under-performing in the tests. However, to accomplish this he would have to know beforehand the preferences and scores of all other students, which is not possible since the results are announced to all students at the same time. Thus, in practice the lack of monotonicity of μ_A and μ_C is not a serious concern.

B.2.1. Student-optimal FQ-matching is not monotone Consider the admission graph of Figure 7 with program c_2 indifferent between a_4 and a_5. Note that (c_1, a_5) is strictly c-dominated and can be dropped. This gives G^* from which we obtain the matching μ_A (black nodes). As both a_4 and a_5 are tied in the last vacant, the student-optimal matching μ_A with flexible quotas assigns both of them to program c_2 exceeding the quota by one unit.

Suppose now that a_5 improves its ranking for program c_1 so that $a_5 >_{c_1} a_3$, while everything else remains the same as shown in Figure 8. In this case node (c_1, a_3) is applicant dominated as there are 3 better ranked applicants whose first preference is c_1, and it is then removed. Nodes (c_2, a_4) and (c_2, a_5) are also applicant dominated as now the only remaining choice for a_3 is c_2. After cleaning all dominated nodes and assigning each student to his top remaining preference we obtain μ_A depicted by the black nodes in Figure 8. Thus, a_5 improved its ranking in c_1 but moved from being assigned in c_2 to be unassigned. This shows that the student-optimal mechanism μ_A is not monotone.

Figure 8 Non-monotone student-optimal FQ-matching.

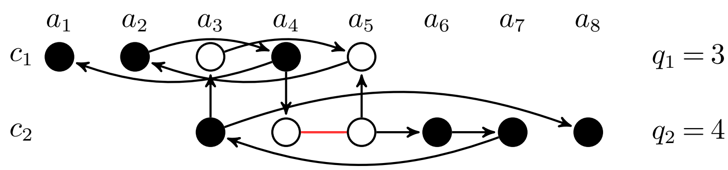


Figure 9 Admission graph with university-optimal FQ-matching.

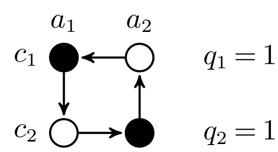
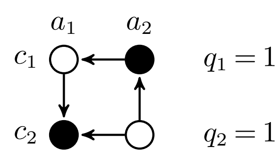


Figure 10 Non-monotone university-optimal FQ-matching.



B.2.2. University-optimal FQ-matching is not monotone Consider the admission graph of Figure 9, where the black nodes represent the university-optimal assignment.

Suppose that a_2 decreases its ranking in c_2 so that $a_2 <_{c_2} a_1$. The new admission graph and the resulting university-optimal assignment are shown in Figure 10. Comparing both results we observe that a_2 moves from being assigned in c_2 (his second preference) to be matched in c_1 (his top preference). Thus, being worst ranked by c_2 helped him to improve his assignment, and therefore the university-optimal assignment is not applicant-monotone.

B.3. Lack of Strategy-Proofness.

A strategy-proof (SP) mechanism ensures that no student can be assigned to a more preferred program by misreporting their true preferences. In our context we only focus on strategy-proofness for students since applicants have to state their preferences after universities and when they already know their scores. Moreover, we will assume that the reported weights of each program reflect the true "preference orders" over students, although the underlying preferences of programs could be different than just a rank over students. For instance, programs or the universities they belong to could also have preferences over sets of students, and the quota policy could be seen as a strategy to reach some distributional concern. In the next examples we show that neither the student-optimal FQ-matching nor the university-optimal FQ-matching are strategy-proof.

B.3.1. Student-optimal FQ-matching is not strategy-proof In the admission graph of Figure 8 applicant a_5 was unassigned under μ_A. Suppose that this applicant lies when he states his

Figure 11 Student-optimal FQ-matching is not strategy-proof.

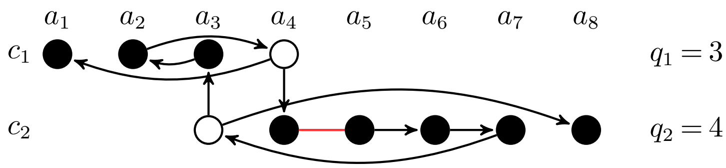
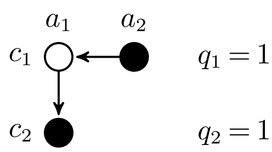


Figure 12 University-optimal FQ-matching is not strategy-proof.



preferences, applying only to program c_2. The resulting admission graph and μ_A are presented in Figure 11. We observe that in this case each student is assigned to his top preference under μ_A. In particular, by lying about his true preferences, a_5 moves from being unassigned to be matched in c_2, which is his second true preference. This shows that the student-optimal matching with flexible quotas is not strategy-proof.

B.3.2. University-optimal FQ-matching is not strategy-proof Consider the university-optimal matching μ_C in Figure 9 where both students are assigned to their second preference. If a_2 lies and only applies to c_1, the results returned by μ_A and μ_C are the same since node (c_1, a_1) is program dominated, and both students are assigned to their top choice. Thus c_2 can improve his assignment in μ_C by not revealing his true preferences.

B.4. Strategy-Proofness in the Large.

According to Azevedo and Budish (2018), a mechanism is strategy-proof in the large (SP-L) if, for any full-support i.i.d. distribution of students' reports, being truthful is approximately optimal in large markets. SP-L does not require truthful reporting to be optimal for any market size, and only requires it to be approximately optimal in large markets.

To use the framework in Azevedo and Budish (2018) we need some definitions. A mechanism is *semi-anonymous* if agents are divided into a finite set of groups, and an agent's outcome depends only on her own action, her group, and the distribution of actions within each group. In addition, a *semi-anonymous* mechanism is *envy-free* if no agent prefers the assignment of another agent in the same group. Define then a *semi-anonymous* mechanism as SP-L if no agent wants to misreport as a different type within the same group.

As Azevedo and Budish (2018) argue, in a *semi-anonymous* mechanism, the gain of player i from misreporting as player j (from the same group), can be decomposed as the sum of the gain from receiving j's allocation, holding fixed the aggregate distribution of reports, plus the gain from affecting the aggregate distribution of reports. *Envy-freeness* implies that the first term is non-positive, and their Lemma A.1 shows that the second component becomes negligible in large markets. Then Theorem 1 in Azevedo and Budish (2018) shows that a sufficient condition for a *semi-anonymous* mechanism to be SP-L is *envy-freeness*.

It is not immediate that we can directly apply Lemma A.1 to our setting. The reason is that Azevedo and Budish (2018)'s results apply only to finite sets of groups and rely on an approximation where there are many students per group[23]. For the Chilean setting, students' groups are given by their vector of scores and whether they are Regular or BEA. So, even though scores are discrete, the set of groups is quite large, and thus the number of students per group may not be large enough. Nevertheless, we argue that Theorem 1 in Azevedo and Budish (2018) still holds for the Chilean college admissions problem.

The reason for this is that application scores for admitted students tend to be dense, so a single student, who takes the societal distribution of play as exogenous, cannot have a large discontinuous influence on the cutoffs that determine the allocation. Given this, to show that our mechanism is SP-L it is enough to show that it satisfies *envy-freeness*. The proof of this is almost direct. Conditional on students' groups, we have already shown that both *university-optimal* and *student-optimal* FQ-matchings are stable and satisfy *non-discrimination*. This implies that both FQ-matchings satisfy *envy-freeness*, and therefore we conclude that our mechanism is strategy-proof in the large.

Appendix C: Notions of stability

As observed in Irving (1994) and Irving et al. (2000), when agents have non-strict preferences the notions of blocking pair and stability admit three natural extensions: weak, strong and super stability. For the case with no ties, it was shown that weakly stable matchings always exist but not necessarily the others.

Formally, a matching μ is

(a) *weakly stable* if there is no pair $(c,a) \in V \setminus \mu$ such that $c \rhd_a \mu(a)$ and $a \rhd_c \mu(c)$.

(b) *strongly stable* if there is no pair $(c,a) \in V \setminus \mu$ such that $c \unrhd_a \mu(a)$ and $a \unrhd_c \mu(c)$ with one of these preferences in the strict sense.

(c) *super stable* if there is no pair $(c,a) \in V \setminus \mu$ such that $c \unrhd_a \mu(a)$ and $a \unrhd_c \mu(c)$.

Clearly super stability implies strong stability which in turn implies weak stability, and the three concepts collapse to stability when preferences are strict. We show next that for FQ-matchings these three notions coincide.

THEOREM 2. *If an FQ-matching μ is weakly stable then it is super stable.*

Proof: Let μ be a weakly FQ-matching and suppose by contradiction that it is not super stable: there exists $(c,a) \in V \setminus \mu$ with $c \unrhd_a \mu(a)$ and $a \unrhd_c \mu(c)$. Since by assumption $\leq_a$ is a total order and

$c \notin \mu(a)$, any $c' \in \mu(a)$ with $c' \leq_a c$ satisfies also $c' <_a c$ so that $c \rhd_a \mu(a)$. From weak stability it follows that $a \rhd_c \mu(c)$ cannot hold, so that $|\mu(c)| \geq q_c$ and there exists $a' \in \mu(c)$ such that $a' \sim_c a$. By non-discrimination it follows that a should have been assigned to a program at least as good as c, which contradicts $c \rhd_a \mu(a)$.

Appendix D: Equivalence between FQ-matchings and L-stable score limits

FQ-matchings turn out to be the same as the allocations obtained from L-stable score limits introduced in Biró and Kiselgof (2015). In their setting an applicant $a \in A$ is characterized by a set of integer scores s_c^a that determine the preferences of the programs. Note that in a college admission instance (G, q) we may define the score s_c^a as the rank[24] of student a in the preference list of the program c so that both settings are basically equivalent.

Given a set of score limits $l = (l_c)_{c \in C}$, Biró and Kiselgof (2015) define a corresponding assignment μ^l by letting $(c, a) \in \mu^l$ if c is the most preferred program of student a for which he attains the score limit, that is to say, $s_c^a \geq l_c$ and $s_{c'}^a < l_{c'}$ for all $c' >_a c$. Let $x_c^l = |\mu^l(c)|$ be the number of students assigned to program c. A score limit l is called *L-feasible* if for each program c with $|\mu^l(c)| \geq q_c$ we have $|\{(c, a) \in \mu^l : s_c^a > l_c\}| < q_c$, and is called *L-stable* if moreover any reduction of a score limit $l_c > 0$ leads to infeasibility.

L-feasibility is the analog of quotas-up-to-ties: a program may exceed its quota q_c but only to the extent that the last group of admitted students are tied with score equal to l_c. Since μ^l always satisfies non-discrimination and each student is assigned at most once, it follows that l is L-feasible if and only if μ^l is an FQ-matching. The following result establishes the connection between FQ-matchings and L-stable score limits.

THEOREM 3. *If l is an L-stable score limit then μ^l is an FQ-matching. Conversely, any FQ-matching can be expressed as $\mu = \mu^l$ for an L-stable score limit l.*

Proof: Let l be L-stable and suppose that μ^l has a blocking pair $(c, a) \notin \mu^l$. This means that c prefers a over some of its currently matched students $b \in \mu^l(c)$ so that $s_c^a > s_c^b \geq l_c$. Hence, a attains the score limit l_c and must have been assigned to c or better in μ^l, so that (c, a) cannot be a blocking pair. Therefore, μ^l is an FQ-matching.

Now let μ be an FQ-matching and let l_c be the rank s_c^a of the q_c-th student $a \in \mu(c)$, setting $l_c = 0$ if $|\mu(c)| < q_c$. We claim that $\mu = \mu^l$. Indeed, for each $(c, a) \in \mu$ we have that $s_c^a \geq l_c$ by definition of l_c, whereas stability implies $s_{c'}^a < l_{c'}$ for all $c' >_a c$, so that $(c, a) \in \mu^l$ and therefore $\mu \subseteq \mu^l$. Conversely, let $(c, a) \in \mu^l$. Then $s_c^a \geq l_c$ by non-discrimination a must be admitted to c or better. Since, moreover, $s_{c'}^a < l_{c'}$ for all $c' >_a c$ it must be the case that a is precisely matched with c and $(c, a) \in \mu$. This establishes the equality $\mu = \mu^l$. In particular μ^l is an FQ-matching which, as noted before, is equivalent to the fact that l is L-feasible. In order to show that l is L-stable let us consider a program with $l_c > 0$ and denote $\tilde{\mu}^l$ the assignment obtained after reducing l_c by one unit. The property $l_c > 0$ implies that the program had all its positions filled, namely $|\mu(c)| \geq q_c$ and $|\{(c,a) \in \mu^l : s_c^a > l_c\}| < q_c$. After reducing the score limit to $l_c - 1$ the set of students assigned to program c increases and L-feasibility fails since $|\{(c,a) \in \tilde{\mu}^l : s_c^a > l_c - 1\}| = |\mu(c)| \geq q_c$. This proves that l is L-stable completing the proof.

Appendix E: Implementation and complexity of the algorithms

The admission graph can be built in $O(|V|)$ operations, while removing a node can be done in constant time as it suffices to reassign the pointers and flags for its 4 adjacent nodes. We also need the following procedures for detecting strictly dominated nodes:

a-DOMINATION: For each program c scan the corresponding row in the graph from the top ranked student downwards, counting the students that place c at the top. When this counter reaches q_c continue scanning the row removing all nodes (c, a') which are strictly below in the order $<_c$.

c-DOMINATION: For each applicant a scan the corresponding column in the graph from the top program downward and stop as soon as a program c is found for which a is among the top q_c candidates. Then continue scanning the column removing all subsequent nodes (c', a) with $c' <_a c$.

Concerning the algorithm's complexity we already noted that initializing the admission graph takes $O(|V|)$. Since removing a node changes the top preferences for students and programs, the cycle must be repeated as long as strictly dominated nodes are found. However, each successful cycle removes at least one node so there are at most $|V|$ cycles, and in every iteration the procedures for detecting strict dominations run in $O(|V|)$ so that the overall complexity for the repeat is $O(|V|^2)$. When these procedures do not find any strictly dominated node we have the domination-free subgraph G^* and we proceed to compute μ_A and μ_C by greedily assigning the top remaining preferences. This final step takes $O(|V|)$ operations. We summarize this discussion in the following theorem.

THEOREM 4. *Our procedure computes FQ-matchings μ_A and μ_C in time $O(|V|^2)$.*

Note that the overall time spent in removing nodes is bounded by $O(|V|)$, and therefore the quadratic complexity comes from the search of these dominated nodes. In fact there are instances where this search takes indeed $O(|V|^2)$ operations. In AppendixF we present an alternative algorithm that improves this complexity.

In Appendix B (Theorem 1) we show that there is no need to drop all dominated nodes to obtain μ_A and μ_C. In fact, μ_A can also be obtained by greedily assigning each student to his top choice in the admission graph G_A^*, which is obtained by recursively dropping all a-dominated nodes. Similarly, μ_C can be obtained from G_C^*, which is obtained by erasing all c-dominated nodes. This can reduce the computation time if only one of these assignments is needed.

Appendix F: Faster algorithm for FQ-matchings

An observation that might be exploited to improve the previous algorithms is that not all dominated nodes need to be removed. For instance, the greedy assignment μ_C computed from the original graph in Figure 2 assigns at most one program to each student and therefore it gives already an FQ-matching, without removing any of the c-dominated nodes in G. Also, for the student-optimal assignment μ_A not all a-dominated nodes will induce violations of the quotas-up-to-ties. A natural approach would then consist in dropping only those nodes that are causing these violations. This is similar to the strategy used in the deferred-acceptance algorithm of Gale and Shapley (1962).

We describe the idea for the student-optimal assignment μ_A. For each program c we set up an ordered list L_c in which we will sequentially add and remove applicants, with a counter $c.size$ to record the size of the list. Initially these lists are empty with $c.size = 0$. For each $a \in A$ we set a pointer $a.top$ to its most preferred program and, if this pointer is not null, we push a into a stack S that contains the applicants with no program assigned yet.

We iterate as follows. We pop a student a from the stack S and insert it in the ordered list L_c of the most preferred program c, increasing $c.size$ by one unit. If this counter exceeds q_c we check quotas-up-to-ties and eventually remove the last group of students in L_c to ensure that this property holds, by using the following procedure.

CHECK-QUOTAS-UP-TO-TIES: Find the set T_c of applicants tied in the last position in L_c. If $c.size \leq q_c + |T_c|$ we keep the list as it is, otherwise each node (c,b) for $b \in T_c$ is a-dominated so we remove b from L_c and update $b.top$ to its next most preferred program $c' <_b c$. If such c' exists we push b back into the stack S and otherwise we leave b unassigned. If the tie T_c is removed we update $c.size \leftarrow c.size - |T_c|$.

Algorithm 1 Fast student-optimal FQ-matching

read instance and build admission graph
initialize admission lists L_c and stack S
while (S is non-empty) **do**
  $a \leftarrow \text{pop}(S)$
  $c \leftarrow a.top$
  insert a into L_c and increase $c.size$ by one
  if ($c.size > q_c$) **then**
    CHECK-QUOTAS-UP-TO-TIES
  end if
end while
return assignment μ_A represented by the final lists L_c

To present our next result we let r_c be the largest size of a tie in the preorder $\leq_c$ so that $|\mu(c)| \leq q_c + r_c$ in any assignment satisfying quotas-up-to-ties. We denote $\bar{q}$ the maximum of the quantities $q_c + r_c$ and $\bar{r}$ the maximum of r_c.

THEOREM 5. *Algorithm 1 computes an FQ-matching μ_A in time $O((\bar{r} + \log \bar{q})|V|)$.*

Proof: To prove that the algorithm is finite we observe that each **while** cycle inserts a student into a new program, in decreasing order, so that the number of times a student may be reassigned is bounded by the number of nodes in its corresponding column in Γ. It follows that the algorithm terminates after at most $|V|$ cycles. Upon termination we have each student assigned to its most preferred program in the instance $\tilde{\Gamma}$ that contains all the nodes not removed during the execution, so that μ_A is the corresponding student-optimal assignment. Now, by construction the lists L_c satisfy quotas-up-to-ties throughout the algorithm, so that the computed μ_A is a FQ-matching for $\tilde{\Gamma}$. Since the nodes removed by the algorithm were all a-dominated, Theorem 1 guarantees that μ_A is also a FQ-matching for Γ.

In order to estimate the worst case complexity let us compute the number of basic operations per cycle. The insertion operation can be executed in time $O(\log |L_c|)$ which is bounded by $O(\log \bar{q})$ since L_c satisfies quotas-up-to-ties. The rest of the cycle deals with T_c which can have up to r_c elements, and therefore the number of operations involved is $O(\bar{r})$. Since there are at most $|V|$ cycles this yields the announced worst case complexity.

Appendix G: Additional Examples

G.1. Example 1: Effect of allowing indifference in preferences

Building on the example illustrated in Figure2, Table 8 compares the program assigned to each applicant for the case of strict preferences (assuming that $a_6 >_{c_2} a_2$) and when there is a single tie (i.e. $a_6 \sim_{c_2} a_2$) and flexible quotas (last two columns). In parenthesis we show the rank of the assigned program in the applicant's preference list.

Table 8 Matchings with strict vs unstrict preferences.

	Strict		Unstrict	
Applicant	μ_A	μ_C	μ_A	μ_C
a_1	$c_5(1)$	$c_4(2)$	$c_5(1)$	$c_4(2)$
a_2	$c_4(3)$	$c_4(3)$	$c_2(2)$	$c_4(3)$
a_3	$c_4(3)$	$c_5(4)$	$c_3(2)$	$c_5(4)$
a_4	-	-	$c_4(2)$	-
a_5	$c_3(2)$	$c_3(2)$	$c_4(1)$	$c_3(2)$
a_6	$c_3(3)$	$c_3(3)$	$c_2(2)$	$c_3(3)$
a_7	$c_2(4)$	$c_2(4)$	$c_3(3)$	$c_2(4)$
a_8	$c_2(2)$	$c_2(2)$	$c_1(1)$	$c_2(2)$
a_9	$c_1(2)$	$c_1(2)$	$c_2(1)$	$c_1(2)$

The assignment μ_A allowing for ties and flexible quotas benefits considerably the students: 7 applicants improve the order of the preference on which they are assigned, one moves from being unassigned to be selected in his second most desired program, and only one remains in the same program. This result is in line with Balinski and Sönmez (1999), who observed that increasing the quotas (in the fixed quota model) can never hurt a student under a student-optimal matching. In contrast, in this example the assignment μ_C does not change when we move from strict to non-strict preferences. This is not always the case and changes may occur if the university-optimal matching happens to have a tie in the last vacancy of some program, so that a student may shift to a different alternative inducing further shifts in a domino effect.

Appendix H: Enrollment

As we have discussed before, there exists the possibility that the inefficiencies generated by the double assignment could be resolved in the subsequent enrollment process. The intuition behind this is that after the assignment is announced, students must decide whether or not to enroll in the program they were assigned, and BEA students with double-assignment must choose only one program to enroll. Students who decide to enroll have three days to complete the process, which is called "First Period of Enrollment" (FPE). If a student decides not to enroll (either because the student has double assignment and/or opted for an outside option), his seat can be used by a wait-listed student. This can happen during the "Second Period of Enrollment" (SPE), which takes place right after the FPE. However, the SPE only lasts for two days, and therefore programs don't have much time to call wait-listed students and offer them admission. In addition, to speed up the enrollment process, universities announce more vacancies (normal vacancies plus overcrowd vacancies) than the number of students they want to enroll (normal vacancies). Every student has the right to enroll into their assigned program, but programs won't call wait-listed students unless the number of enrolled students is less than the normal vacancies. Hence, students who are not initially assigned are not guaranteed to be admitted into a program even if they are in the first place of the wait list and assigned students decide not to enroll during the FPE.

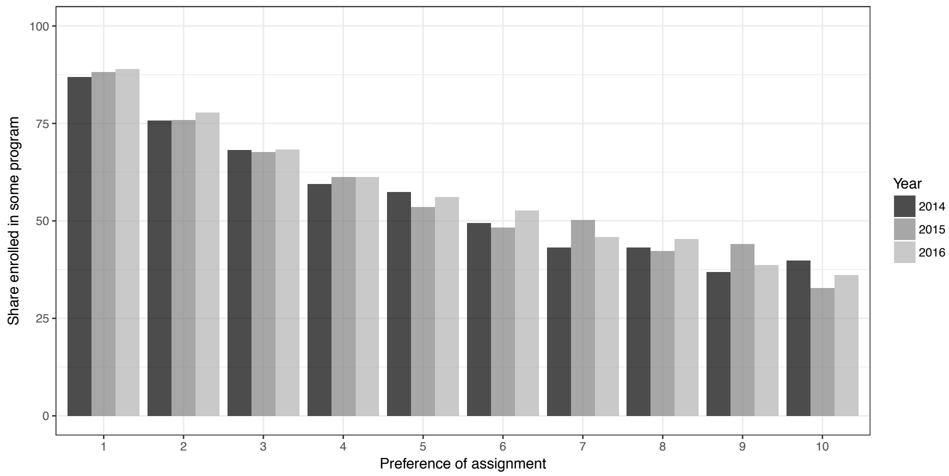
In Table 9 we show assignment and enrollment statistics for the years considered in our study (2014-2016). We observe that around 75% of those students who are admitted end up enrolling in a program that is part of the centralized system. Among those who enroll, we find that most of them do it in the program where they were assigned by the clearinghouse. However, around 3.5% of the students enroll in a program they prefer compared to the one assigned by the clearinghouse, mostly thanks to the SPE. Finally, we find that less than 1% enroll in a less preferred program. We don't have a clear explanation for this behavior.

Table 9 Enrollment - General

		2014	2015	2016
Assigned	Total	95,793	97,844	100,972
Enrolled	As assigned	72,006	74,030	75,909
	Better	2,664	2,706	2,676
	Worst	1,031	708	435
	Total	75,701	77,444	79,020

In Figure 13 we show how the probability of enrollment in any program (number of Enrolled Total over Assigned Total) decreases with the preference where the student was admitted. This illustrates that being assigned to a higher preference increases the probability of enrolling in a program that is part of the system, which in turn justifies the relevance of admitting students in a preference that is as good as possible for them.

Figure 13 Share of students enrolled by assignment preference



In terms of the impact of the unified assignment, in Table 10 we compare the results of the unified allocation (simulated in 2014-2015 and actual in 2016) with the actual enrollment decisions of students. The idea is to assess whether the enrollment process leads to the same results as the ones obtained with the unified allocation. We find that, among the students who could have improved in their assignment if our policy were in place in 2014 and 2015, only a third of them get a better allocation in the enrollment process. In contrast, almost all of them get enrolled in a better assignment in 2016 (when our policy was in place). This suggests that the inefficiencies generated by the double assignment are not completely addressed in the enrollment process[25].

Table 10 Enrollment

		2014	2015	2016
Improved	Admitted - Total	1,737	1,915	1,749
	Enrolled - As assigned	431	469	1,392
	Enrolled - Better	44	38	34
	Enrolled - Worst	802	909	13
	Enrolled - Total	1,277	1,416	1,439
New admitted	Admitted - Total	568	672	777
	Enrolled - As assigned	146	172	446
	Enrolled - Better	20	38	17
	Enrolled - Worst	10	15	0
	Enrolled - Total	191	249	467

Endnotes

1. In Chile, students apply directly to a major in a given university, such as Medicine in the University of Chile. We refer to program as a pair major-university.

2. In addition to what we describe in this paper, each university has special admission programs such as for athletes, racial minorities, among others. In addition, there are other centralized admission tracks that were added to the system in 2017 that we don't address in this paper for simplicity.

3. BEA students are indifferent between regular and reserved seats because they obtain the scholarship regardless of how they were admitted, and there are no differences between these types of seats.

4. We don't include military and police academies (7).

5. Some programs such as music, arts and acting, may require additional aptitude tests.

6. The *Consejo de Rectores de las Universidades Chilenas* (CRUCH) is the institution that gathers these universities and is responsible to drive the admission process, while DEMRE is the organism in charge of applying the admission tests and carrying out the assignment of students to programs.

7. Many of these institutions run two admission processes: the first, and most significant in terms of vacancies, is simultaneous to the centralized process, while the second takes place in late July/early August and grants admission for the second semester.

8. Respecting some basic criteria defined by CRUCH.

9. This was directly translated from the document "Normas, Inscripción y Aspectos Importantes del Proceso de Admisión, 2013" CRUCH (2013), page 8.

10. Students get a full-refund of the enrollment fees if they decide to decline their enrollment in in the first stage to enroll in a new program.

11. To be more precise, the system differentiates between normal and overcrowd seats. During the enrollment process, if an admitted student does not enroll then wait-listed students are offered admission only up to the normal vacancies. Hence, only those students who were admitted before the enrollment process can use overcrowd seats.

12. In Appendix H we compare our results using enrollment data and we show that the inefficiencies introduced by the double assignment are not addressed in the enrollment process.

13. In Appendix E we show that the complexity of this procedure is $O(|V|^2)$.

14. Further details regarding this example are provided in Appendix G.

15. As long as we consider applications as fixed, allowing for ties and flexible quotas will weakly increase the number of seats per program, resulting in a Pareto improvement for students.

16. In this example the student-optimal and the university-optimal algorithms return the same allocation.

17. Average between Math and Language.

18. For example, University of Chile requires applicants to apply to at most 4 of its programs, and these applications must be listed within the top 4 positions in the applicant's list.

19. Under STB every program uses the same random ordering to break ties, while under MTB each program uses its own random order.

20. See Plaza Pública Cadem - Encuesta N 262 - 21 Enero 2019.

21. In Figure 13 (see Appendix H) we show that the share of students that enroll after being assigned in one of their 10 listed preferences is decreasing in the number of assigned preference.

22. As an anonymous referee pointed out, this is not necessarily a source of inefficiency. It could be the case that some universities are just willing to enroll more BEA students if they get more applicants than anticipated, but they are not willing to fill that capacity with extra regular students.

23. Their scaling regime considers a fixed number of schools and capacities going to infinity.

24. The students in the least preferred group have rank 1, the next group is ranked 2, and so on.

25. To be able to fully measure the impact of our policy change in the enrollment process, we would have to structurally model the application and enrollment behavior of students. However, the previous statistics are strong evidence that the enrollment process does not fully address the inefficiencies generated by the double assignment.

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