School Choice in Chile

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Centralized school admission mechanisms are an attractive way of improving social welfare and fairness in large educational systems. In this paper we report the design and implementation of the newly established school choice mechanism in Chile, where over 274,000 students applied to more than 6,400 schools. The Chilean system presents unprecedented design challenges that make it unique. On the one hand, it is a simultaneous nationwide system, making it one of the largest school admission problems worldwide. On the other hand, the system runs at all school levels, from Pre-K to 12th grade, raising at least two issues of outmost importance; namely, the system needs to guarantee their current seat to students applying for a school change, and the system has to favor the assignment of siblings to the same school. As in other systems around the world, we develop a model based on the celebrated Deferred Acceptance algorithm. The algorithm deals not only with the aforementioned issues, but also with further practical features such as soft-bounds and overlapping types. In this context we analyze new stability definitions, present the results of its implementation and conduct simulations showing the benefits of the innovations of the implemented system.

CCS Concepts: • **Theory of computation → Algorithmic mechanism design**; • **Applied computing → Economics**.

Additional Key Words and Phrases: school choice; matching; two-sided market

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1 INTRODUCTION

According to the Duncan Index of segregation, Chilean schools are extremely socially segregated [35]. Several authors have shown that the costs of school segregation are important, including low social cohesion and lack of equal opportunities and social mobility [14, 25, 34, 36, 37]. While the drivers of school segregation include societal aspects well beyond the school system, social movements and politicians were provably right at blaming features of the admission system.

The School Inclusion Law marks a breaking point in the organization and functioning of the school system. The Law, promulgated in 2015, changed the old admission process drastically by (i) eliminating co-payments in publicly subsidized schools, (ii) forbidding publicly subsidized schools

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from selecting their students based on social, religious, economic, or academic criteria, and (iii) defining priorities that must be used to assign students to schools.

In this paper we report the results of an ongoing collaboration with the Chilean Ministry of Education (MINEDUC) addressing the practical challenges of implementing the School Inclusion Law. To this end, we designed and implemented a centralized system that (i) provides information about schools—seats available, mission, values, educational project, among others—to help parents and students in building their preferences; (ii) collects families' preferences through an online platform, reducing the time and cost that visiting each school implied in the past; and (iii) assigns students to schools using a transparent, fair and efficient procedure.

One of the distinctive features of the new school admission system in Chile is its broadness, as it runs nationwide and throughout all school levels. Being nationwide, the school admission system accommodates the needs of both urban and rural families, and covering all levels highlights the relevance of accounting for preferences in the joint allocation of siblings. The Law also radically changed the way in which families apply and are assigned to schools, which made the transmission of information essential to the implementation. The key to these challenges was gradualism. The system was first implemented in 2016 in the least populous of the sixteen regions in Chile. This allowed to gain practical experience to improve the system as more regions were subsequently added. The system will be fully in place by 2020.

At the core of the system is the assignment algorithm, which adapts the celebrated Deferred Acceptance (DA) algorithm—introduced in the seminal paper by Gale and Shapley [21]—to incorporate all the elements required by law and by MINEDUC. In particular, the system considers a set of priority groups—students with siblings in the school, students with parents that work in the school and former students of the school—that are served in strict order of priority. The system also includes quotas for students (i) from disadvantageous environments, (ii) with special educational needs and disabilities, and (iii) with high academic performance. Within each priority group, ties are broken randomly.

From a computational perspective, our implementation of the Deferred Acceptance algorithm relies in an adaptation of the approach of directed graphs proposed by Baïou and Balinski [12], which is efficient and allows us to find an assignment in a few seconds. The admission system runs throughout all educational levels from the highest (12th grade) to the lowest (Pre-K). This makes patent the need to secure their current enrollment to students who want to change to another school. Additionally, some families may have two or more of their children simultaneously applying to schools and may naturally want their children to attend the same school. Two features of our implementation favor the assignment of siblings to the same school. First, the tie-breaking lotteries are run over families (rather than over students). Second, families can express their willingness to have their children assigned to the same school by filling a *family application* (FA). A family application ensures that once the oldest child is assigned to some school, the application of the younger are modified to put that school as their most preferred one.

In summary, our design presents at least three innovative features:

- It allows each student currently enrolled in a given school to apply to a different school while guaranteeing him/her a seat in his/her current school.
- It runs tie-breaking lotteries over families, significantly increasing the fraction of siblings that end up assigned to the same school.
- It adds a family application heuristic that improves the chances of having siblings assigned to the same school.

The results reported in this paper consider the current state of the system, which includes all regions, except the Metropolitan area of Santiago, reaching 274,990 students and 6,421 schools in the main round. In the current admission process—for students who started their academic year in March, 2019—students applied to 3.4 schools on average, and 59.2% of students were assigned to their top preference. Moreover, 82.5% of students were assigned to one of the schools in their application list, 8.6% were assigned by secured enrollment to their current schools, and only 8.9% resulted unassigned. In addition, there were 10,301 family applications involving 21,424 students and 65.3% of these were successful, i.e. siblings got assigned to the same school, while 3% were partially successful, i.e. only a subset of siblings got assigned together.[1] We also provide simulations evaluating different elements of our design.

Designing, implementing and improving the Chilean school choice system has resulted in many lessons that could be useful for other practitioners designing large-scale clearinghouses. From a theoretical standpoint, we contribute to the existing literature by introducing the notion of family applications. We show that a stable matching may not exist, and we provide heuristics that are successful at increasing the fraction of siblings assigned to the same school. Finally, our results show that having lotteries over families (considering students applying to a given school at all levels simultaneously) significantly increases the fraction of siblings assigned to the same school. From a practical standpoint, a key lesson is that having a continuous communication and collaboration with policy-makers is essential, as many aspects evolve over time and must be incorporated in the design. In addition, fragmenting the implementation in a given number of steps allowed us to gain experience, solve unexpected problems and continuously improve the system. As centralized procedures to assign students to schools are becoming the norm in many countries, we expect that the lessons and solutions offered in this work are deemed useful in other implementations.

The reminder of the paper is organized as follows. In Section 2 we describe the school choice problem in Chile, with the main features requested by law and by MINEDUC. In Section 3 we discuss how this paper relates to several strands of the literature. In Section 4 we present our model and describe its implementation. In Section 5 we present the results, focusing on the admission process of 2018. In addition, we evaluate the effects of (i) quotas for disadvantaged students and (ii) family applications via simulations. Finally, in Section 6 we conclude and provide directions for future work.

2 THE PROBLEM IN CHILE

The Chilean school choice system considers fourteen levels, ranging from pre-kindergarten to 12th grade. Among these there are five entry levels: pre-kindergarten, kindergarten, 1st, 7th and 9th grade, which are the levels where a school can start. Depending on their type of funding, schools can be classified in three types: (1) private, for those schools that are independent and privately funded; (2) voucher, where families make co-payments to complement state subsidies; and (3) public, for those schools that are fully funded and operated by local governments.[2] Voucher and public schools, which are the focus of this paper, account for more than 90.3% of the total number of students in primary and secondary education [30].

Before the introduction of the School Inclusion Law, schools ran their admission processes independently, often selecting their students based on arbitrary rules, such as interviews with the students and their parents, results of unofficial admission exams, past academic records, among many others. Since the admission processes were not coordinated, in many cases parents were

[1] This 3% corresponds to 307 partially successful family applications. However, only 750 FAs were of size 3 or more, therefore this represents 41% of the possibly successful FAs.

[2] The requirements for a school to be partially funded are relatively minimal. The most relevant ones are: (1) have a school building that is certified and allowed to be used as a school, (2) have at least 15% of disadvantaged students if there is enough demand, (3) have a set of rules of procedure that define the relation between students and teachers and (4) have all salaries paid at the moment of applying to government funds.

forced to strategically decide whether to accept an offer or to reject it and wait until other schools released their admission offers. Moreover, many schools used "first-come first-served" rules to prioritize students, resulting in many parents waiting in long overnight queues to secure a seat for their children.[3] Overall, the freedom of schools to choose their students and the existence of voucher schools are considered among the main reasons that explain the polarization and segregation of the Chilean school system [33].

To address these problems, the School Inclusion Law forbids any sort of discrimination in the admission processes of schools that receive (partial or full) government funding, and mandates schools to use a centralized system that collects families' and students' preferences and returns a fair allocation. In this system, students and families can access a platform where they can collect information—number of open seats, number of students per classroom and level, educational project, rules and values, co-payments required, among others—to build their preferences, and later they can use it to apply to as many schools as they want by submitting a strict order of preferences. The system collects all these applications and runs a mechanism that aims to assign each student to their top preference provided that there are enough seats available. More specifically, if the number of applicants is less than the number of open seats, the law requires that all students applying to that school are admitted, unless they can be allocated in a school they prefer. On the other hand, for schools that are over-demanded the law defines a set of priority groups that are used to order students. In particular, there are three priority groups, which are processed in strict order of priority:

(1) *Siblings.* For students that have a sibling already enrolled at the school.
(2) *Working parent.* For students that have a parent working at the school.
(3) *Former students.* For students that were enrolled at the school in the past and were not expelled from it.

Within each priority group students are randomly ordered and each school uses a different random tie-breaker. In addition to these priorities, the law specifies three different types of quotas:

(1) *Special needs.* This quota serves students with disabilities. It reserves at most two seats per classroom per school and it is processed before any other priority group or quota. The quota only applies to schools who have a validated special program.
(2) *High-achieving.* This quota applies to students with high academic performance. It is processed right after the special needs quota, and considers between 30% to 85% of the total number of seats depending on the school. Only a subset of pre-selected schools can implement this quota in 7th and 9th grade, taking an exam to the applicants to rank them.
(3) *Disadvantaged.* This quota prioritizes the most vulnerable students (bottom third in terms of income).[4] In each level of all schools 15% of seats are reserved for disadvantaged students, and this group of students must be processed right after the first priority group, i.e. students with siblings.

There are two additional features that are relevant in the design of the system. First, students that are currently enrolled at a school and apply in the system to change their allocation have a *secured enrollment* in their current school, i.e. in case of not being assigned to a school of their preference they can keep their current assignment. Second, families having two or more children that participate in the centralized system can choose to *apply as a family*, which means that they prioritize having their children assigned to the same school over a better school in their reported preferences.

[3] Some recent news articles illustrating the problem are [1], [16], [32].
[4] According to the Social Registry of Homes

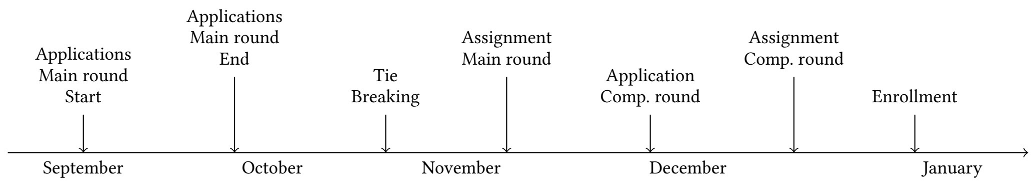

Fig. 1. Timeline of the Admission Process

The timing of the admission process is summarized in Figure 1. After collecting information, families submit their application between September and October. After all applications are received, the centralized mechanism generates the lotteries that will be used to order students in over-demanded schools and executes the main round of the process. For those students that result unassigned or did not apply in the main round, there is a complementary process where students submit a new application list that includes schools with available seats. Finally, students that result unassigned in the complementary round are assigned to the closest school with available seats that does not charge a co-payment. We refer to this as *distance assignment*. In case that there are no schools with seats available within 17km, students remain unassigned and MINEDUC gives them a solution. The process ends in late December with the enrollment.

3 LITERATURE

This paper is related to four strands of the literature: school choice, affirmative action, assignment of families and tie-breaking.

School Choice. In the last two decades, and starting from the theoretical formalization of the school choice problem by Abdulkadiroğlu and Sönmez [8], there have been reforms to the school choice system of many places around the world. The first major reform was introduced in New York, where a variation of the Deferred Acceptance (DA) algorithm with restricted lists was implemented [5]. In 2005, the Boston Public School system decided to switch from the so called *Boston Mechanism* (BM), also known as *Immediate Acceptance* (IA) mechanism, to DA in order to address the strategic incentives introduced by the former algorithm [7]. Despite its lack of strategy-proofness, BM has been revisited in the last few years since it better captures cardinal preferences and therefore can lead to higher social welfare [4]. Since then, other systems such as Barcelona [15], Amsterdam [22], New Orleans [3], among others, have implemented centralized school choice systems using some variant of DA, BM or *top-trading cycles* (TTC). This paper contributes to this literature by adding a new case study with some additional features that have not been explored in previous literature, such as the admission of siblings in different levels and the secured enrollment problem. In addition, this is one of the first papers that describes the implementation of a system at a country level.

Priorities and Affirmative Action. Many school choice systems include affirmative action policies to promote diversity in the classrooms. Abdulkadiroğlu [2] explores DA under type-specific quotas, finding that the student-proposing DA is strategy-proof for students if schools' preferences satisfy responsiveness. Kojima [27] studies the implementation of majority quotas and shows that this may actually hurt minority students. Consequently, Hafalir et al. [23] propose the use of minority reserves to overcome this problem, showing that DA with minority reserves Pareto dominates the one with majority quotas. Ehlers et al. [20] extend the previous model to account for multiple disjoint types, and propose extensions of DA to incorporate soft and hard bounds. Other types of constraints are considered by Kamada and Kojima [26], who study problems with distributional constraints motivated by the Japanese Medical Residency. Dur et al. [18] analyze the Boston's school system and Dur et al. [19] analyze the impact of these policies using Chicago's system data. However, these models consider disjoint types of students. Kurata et al. [29] study the overlapping types problem and show that, even in the soft-bound minority quotas scenario, a stable matching might not exist. As a solution, they propose a model in which stability is recovered by counting each student towards the seat type he was assigned to. Our contribution to this literature is by implementing a new case study and analyzing the effect of different minority quotas in the implementation. Moreover, we show that quotas depend on students truly being a minority and that there can also be some auto-segregation of minorities.

Assignment of families. When designing a school choice mechanism, families are in general considered. We will analyze two aspects related to families in our system: (i) family application for two or more siblings who apply to the same schools and (ii) the tie-breaking rule used. The family application can be thought both as the existence of coalitions in the system, or as a many-to-many matching between families and schools. A similar structure can be found in the labor market of medical residents, where couples want to work in the same city. Studying that market, Roth [31] shows that one cannot guarantee the existence of a matching without justified envy when couples have arbitrary preferences over pairs of hospitals. Kojima et al. [28] show that a stable matching exists if the number of couples is relatively small and preference lists are sufficiently short relative to the size of the market. Another positive result is presented by Ashlagi et al. [10], who introduce a new algorithm that finds a stable matching with high probability (in large matching markets) and where truth-telling becomes an approximate equilibrium for the induced game. We contribute to this literature by showing that a stable matching may not exist when there are family applications, and by introducing a new heuristic that can solve this problem.

Tie-breaking. A common approach to deal with ties in school choice is to use random tie-breaking rules, such as *single tie-breaking* (STB)—all schools use the same ordering for breaking ties—and *multiple tie-breaking* (MTB)—each school uses its own random order. Abdulkadiroğlu et al. [6] are the first to empirically compare these tie-breaking rules, and they find that there is no stochastic dominance in NYC. A similar pattern is found in Amsterdam, as described by De Haan et al. [17]. These findings are in line with the theoretical results by Ashlagi et al. [11], who find that when there is low competition there is no stochastic dominance between random assignments. However, they also show that when there is a shortage of seats STB almost dominates MTB, and also leads to a lower variance in students' rankings. Moreover, Arnosti [9] shows that STB can lead to more matches when preferences are short and random. We contribute to this literature by introducing and studying the effect of family applications and breaking ties between families instead of students.

4 MODEL AND IMPLEMENTATION

The Chilean school choice problem can be formalized as follows. Let K be the set of all levels, including pre-kindergarten and kindergarten in preschool, 1st to 8th grade in primary school and 9th to 12th grade in secondary school. Each school offers some or all of these levels, and students can only apply to schools that offer their level. For simplicity, we will first focus on a fixed level to define the basic setting, and later (in Section 4.3) we will introduce how levels interact with each other through the family application.

At a fixed level, let $S = \{s_1, ..., s_n\}$ be the set of students and $C = \{c_1, c_2, ..., c_m\}$ be the set of schools. Each school c has a capacity $q_c \in \mathbb{N}$ that accounts for the number of available seats. Students have a strict preference profile $\succ_S = (\succ_{s_1}, ..., \succ_{s_n})$ over schools, where $c \succ_s c'$ means that student s strictly prefers school c to c'. Students who rank a subset of schools implicitly declare that they prefer to be unassigned than to be assigned to a school that is not in their preference list.

Schools, on the other hand, have a weak priority profile $\succsim_C = (\succsim_{c_1}, \ldots, \succsim_{c_m})$ over students, where $s \succsim_c s'$ means that at school c, student s has a higher or equal priority ranking than student s'. Students within a priority group are randomly ordered, creating a strict priority profile $\succ_C = (\succ_{c_1}, \ldots, \succ_{c_m})$ over students.

A **matching** μ is a function from the set $C \cup S$ to the subsets of $C \cup S$ such that:

(i) $\mu(s) \in C$ or $\mu(s) = \emptyset$ for every student $s \in S$.
(ii) $\mu(c) \subseteq S$ and $|\mu(c)| \leq q_c$ for every school $c \in C$.
(iii) $c \in \mu(s)$ if and only if $s \in \mu(c)$ for every student $s \in S$ and school $c \in C$.

This definition formalizes that a feasible matching cannot assign more students to a school than its capacity, and it cannot assign a student to more than one school.

In what follows, we extend this model to address the particular features of the Chilean problem, namely: (1) secured enrollment, for students already enrolled at a school; (2) quotas for students of different types; (3) family applications; and (4) the tie breaking rule.

4.1 Secured enrollment

The system allows students of any level to apply to schools of their choice, provided that those schools offer their level. In particular, some students may want to change to another school they prefer, but are better off at their current school than remaining unassigned or assigned to other schools they prefer less. The School Inclusion Law gives students the right to keep a seat at their current school in case they do not get a better assignment.

To address this requirement, we add to each student that participates in the system and seeks to change to another school a preference over the current school at the bottom of their preference list. In addition, we consider the secured enrollment of a student as a priority criterion with a higher priority than any other priority group. With this extra criterion, the complete list of priority groups is given by:

$$\text{Secured Enrollment} \succ_c \text{Siblings} \succ_c \text{Working Parent} \succ_c \text{Former Students} \quad \forall c \in C. \quad (1)$$

4.2 Quotas

In order to promote diversity within schools, the School Inclusion Law includes affirmative action policies for financially disadvantaged students as well as children with special needs. Furthermore, a limited number of schools are allowed to reserve seats for students with high-achieving records.

Let $T = \{\textit{Special needs}, \textit{High-achieving}, \textit{Disadvantaged}, \textit{Regular}\}$ be the set of all possible types that students may belong to. In general, for each student, these types are school-dependent[5] and may overlap. Each student belongs to at least one type, being *Regular* the default (i.e., *Regular* encodes the absence of type). We define a mapping $\tau : S \times C \rightarrow 2^T$ that maps students to their types on each school.

A function $p : C \times T \rightarrow \mathbb{N}$ defines type-specific quotas for each school, where p_{ct} represents a soft lower bound for school c, i.e. a flexible limit that regulates school c's priorities dynamically, giving higher priority to students of type t up to filling p_{ct} seats. Furthermore, we assume that in each school quotas can be met without violating its capacity, i.e. $\sum_{t \in T} p_{ct} \leq q_c$ for every school $c \in C$.

As shown by Kurata et al. [29], when student types can overlap the general concepts of stability for a matching with soft lower bounds proposed in literature [20, 23] are insufficient to guarantee the existence of a stable matching. To overcome this difficulty, they propose a new model based on the framework of matching with contracts due to Hatfield and Milgrom [24]. In this new model,

[5]As an exception, the disadvantaged type is school-independent.

schools provide distinct reserved seats for each student type, and assignments are interpreted as contracts that explicitly state that a student is assigned to a particular reserved seat at a school, in contrast to previous models where a student accounts for seats of multiple types.

We adhere to Kurata's setting by extending our model as follows. Every student s has now strict preferences $\succ_s$ over contracts of the form $(c,t) \in C \times T$ and every school c has now a weak priority profile $\gtrsim_c$ over contracts of the form $(s,t) \in S \times T$. Then, a **matching** μ is a function from $(S \cup C) \times T$ to the subsets of $(S \cup C) \times T$ such that:

(i) $\mu(s) \in C \times T$ or $\mu(s) = \emptyset$ for every student $s \in S$.
(ii) $\mu(c) \subseteq S \times T$ and $|\mu(c)| \leq q_c$ for every school $c \in C$.
(iii) $\mu(s) = (c,t)$ if and only if $(s,t) \in \mu(c)$ for every student $s \in S$, school $c \in C$ and type $t \in T$.

In other words, a student s is either unassigned or assigned to a seat of type t in school c, $\mu(c)$ is the set of students assigned at school c, each one to a type-specific seat, and student s is assigned to a seat of type t in school c if and only if school c's assignment contains s assigned to a seat of type t. Note that this definition does not require that type t students must be matched to seats of type t.

Let $\mu_t(c) := \{s \in S : \mu(s) \in \{c\} \times T \text{ and } t \in \tau(s,c)\}$ be the set of students of type t assigned to school c. Two well-known and desirable properties of a matching studied in the literature of school choice problems are to be fair or justified envy-free and non-wasteful. In our setting, a student s has **justified envy** towards a student s' with assignment $\mu(s') = (c', t')$ in matching μ if there exists a type $t \in T$ such that:

(i) $(c', t) \succ_s \mu(s)$,
(ii) $(s,t) \succ_{c'} (s', t')$,
(iii) and either $t' = t$ or $|\mu_{t'}(c')| > q_{c't'}$.

That is, s has justified envy towards s' assigned to school c' in a seat of type t' if either s prefers (c', t') to his assignment and is preferred by the school on that seat, or for some type $t \neq t'$, s prefers (c', t) to his assignment, school c' has exceeded the quota of type t' students and prefers s in a seat of type t to s' in a seat of type t'.

A student s **claims an empty seat** of a school c in matching μ if there exists $t \in T$ such that $(c,t) \succ_s \mu(s)$ and one of the following conditions hold:

(i) $|\mu(c)| < q_c$ or
(ii) $\mu(s) = (c, t')$ for some type $t' \in T$, $(s,t) \succ_c (s,t')$ and $|\mu_{t'}(c)| > q_{ct'}$.

A student s **claims an empty seat by type** in school c in matching μ if there exists $t \in T$ such that $(c,t) \succ_s \mu(s)$ and the following condition holds:

(iii) $|\mu_t(c)| < q_{ct}$.

Namely, s claims an empty seat of type t at c if s prefers that contract to her assignment and either c has empty seats, or s is assigned to c in a seat that exceeded its quota and the school prefers having s in a seat of type t, or the quota of type t students has not been met at c.

When no student claims an empty seat or claims an empty seat by type at any school, we say the matching is **non-wasteful**. Finally, a matching is **stable** if it is non-wasteful and eliminates justified envy for all students. This notion of stability matches the one proposed by Kurata et al. [29], even when we assume some students might remain unassigned.

In Table 1 we describe the schools' weak preference profiles over contracts $(s,t) \in S \times T$ for each fixed type $t \in T$. At every school c, students currently enrolled at the school have the highest priority in all types of seats. Then, for the special needs and high-achieving seats, students of the corresponding type are given the second highest priority, and the rest of the students are given priorities according to (1). As required by law, students that have siblings currently enrolled at the school have higher priority than disadvantaged students, even in seats reserved for that type.

Table 1. Weak priorities by type-specific seats. Lower numbers indicate higher priority.

Priority	$\succsim_{c,\text{special needs}}$	$\succsim_{c,\text{high-achieving}}$	$\succsim_{c,\text{disadvantaged}}$	$\succsim_{c,\text{regular}}$
1	Secured enrollment	Secured enrollment	Secured enrollment	Secured enrollment
2	Special needs	High-achieving	Siblings	Siblings
3	Siblings	Siblings	Disadvantaged	Working parent
4	Working parent	Working parent	Working parent	Former students
5	Former students	Former students	Former students	No priority
6	No priority	No priority	No priority	

Priorities of schools over contracts $(s, t) \in S \times T$ for each fixed student and preferences of students over contracts $(c, t) \in C \times T$ also need to be defined to fully state our model, even though neither schools nor students have real preferences over the type of seat defined by the contract, i.e. schools rank students for each type of seat and students rank schools. The way to break ties is not straightforward: as shown by Dur et al. [19], different tie-breaking rules might favor some type of students. To reduce over-representation of quotas, we break ties in a way that students and schools favor assignments of students to seats of one of their corresponding types.

4.3 Family application

Families that have two or more children participating in the system are given the option to prioritize assigning them to the same school of their preference list to the detriment of better schools reported in each individual preference list. This is called *family application*.

It is important to emphasize the different roles that siblings play in the system. On the one hand, there is a sibling's priority that gives special priority to students applying to schools where they have siblings already enrolled at. On the other hand, there is the family application, which addresses siblings (either in the same or in a different level) that are all participating in the admission process.

We implement family applications by defining an equivalence relation on S that captures the sibling relationship among students that are participating in the system. Thereby, families correspond to equivalence classes of size two or more induced by this relation. We only consider families that have at least one school in common in their members' preference lists.

Similar to the matching with couples problem, a stable matching might not exist in the school admission problem with family applications. To explain why, we must first define stability in this context. Consider the simplest version of the school choice problem: without families, types or quotas. A stable matching can be described as a matching μ where there is no pair of student-school (s, c) such that $c \succ_s \mu(s)$ and $|\{s' \in \mu(c) : s' \succ_c s\}| \leq q_c - 1$. Now, a school c in level k has capacity q_c^k and a family A has preferences over the possible assignments of its members. In particular, we consider that a family A has one strict order of preferences $\succ_A$ over a set of acceptable schools, and that they prefer the most assignments where $A \subseteq \mu(c)$ for some c and the rest of comparisons are given by the Pareto partial ordering induced by $\succ_A$. Then, we say that a matching μ is stable if (i) there are no triplets of student, school and level s, c, k where s is not matched to the same school as the rest of the family $A \ni s$, such that $c \succ_s \mu(s)$ and $|\{s' \in \mu(c, k) : s' \succ_c s\}| \leq q_c^k - 1$; and (ii) there are no pairs A, c, such that $|\{s' \in \mu(c, k) : s' \succ_c A\}| \leq q_c^k - |A_k|$ for all $k \in K$ and either $c \succ_A \mu(A)$ if $|\mu(A)| = 1$ or c is just acceptable for A if $|\mu(A)| > 1$.

Proposition 4.1. *If there are family applications, a stable matching might not exist. This is true even in the case where the families have at most two siblings, the preferences of schools are over families, and students of the same family have the same preferences.*

In Figure 2 we present an instance with two levels, four families $\{A, B, C, D\}$ and four schools $\{c_1, c_2, c_3, c_4\}$. In this example, each school has one seat in each level, families A and C have only one child (in levels 1 and 2 respectively), and families B and D have two children, one in each level.

For each level we represent the instance as a graph, where each row corresponds to a family and each column to a school, and preferences are captured through arrows pointing towards more preferred options. Notice that each family is the most preferred one for some school, so no student can result unassigned and siblings must be assigned in the same school in any stable matching. However, we claim that there is no stable matching. To see this, suppose that there is a stable matching μ. Then, both children from family B must be assigned in the same school $c \in \{c_1, c_2\}$, which we denote by $B \in \mu(c)$. Hence, there are two cases:

(1) if $B \in \mu(c_1)$, then family C will be assigned to c_2 (its top choice) in level 2, family D will take its favorite school—c_3—in both levels, and family A will take its favorite school—c_4—in level 1. As a result, no family with higher priority than B is assigned to c_2, and since c_2 is family B's top choice in both levels we conclude that μ is not stable.
(2) if $B \in \mu(c_2)$, then family C is assigned to school c_3 in level 2, family D can only be accommodated in school c_4 (as both children must be assigned to the same school in any stable matching), and thus family A results unassigned. This contradicts the fact that all families should be allocated in any stable matching, and thus we conclude that there is no stable matching.

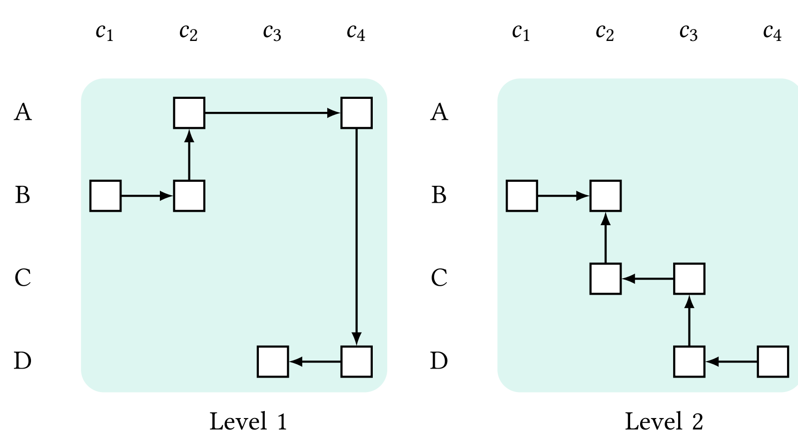

Fig. 2. Instance with two levels where no stable matching exist because of the family applications.

Due to this result, we implement the following heuristic to find a feasible solution to the problem with family applications.

(1) Start from level $k = 12$, i.e. the highest level.
(2) Obtain an assignment for level k. Call it μ_k.
(3) Using μ_k, update the preferences of students whose siblings where assigned in Step (2) and that applied as a family, so that their top choice becomes the school where their elder sibling was assigned.
(4) Update $k = k - 1$, and go back to Step (2). If there are no levels left, stop.

4.4 Tie breaking rule

In order to run the matching algorithm, ties between students need to be broken. By law, schools are allowed to have their own lotteries, so a multiple tie-breaking rule must be used.

As explained before, the system seeks to give siblings a higher chance of being assigned together. In order to do so, a second feature we implement is a tie-breaking rule by family, where each family can account for one or more students. More specifically, all families—independent of the number of students, and considering all levels—that apply to a given school are first (randomly) ordered to break ties. Then, for those families having more than one student, a second random order is used to break ties among siblings. This random order over students is used to break ties in each level of the school. In case that a student has a priority criterion or belongs to a quota, the school updates priorities in each seat type and each level independently.

To illustrate the idea behind the family lottery and how this can help to get more siblings assigned to the same school consider the following example.

Example 4.2. Consider two schools, c and c', that have a single seat in levels k_1 and k_2, and suppose that level k_1 is processed first. In addition, suppose that there are two families, f and f', with one child applying to each level, i.e. $f = \{s_1, s_2\}$ and $f' = \{s'_1, s'_2\}$ (s_1 and s'_1 apply to k_1). Finally, we assume that all students prefer c over c'.

Using our approach, families are first ordered according to a random order. As a result, we may get that $f \succ_c f'$. Then, students within each family are randomly ordered, which may result in the order $s_1 \succ_c s_2 \succ_c s'_1 \succ_c s'_2$. As a result, students s_1 and s_2 are assigned to school c and students s'_1 and s'_2 are assigned to school c'.

To illustrate how this helps families, suppose that a lottery by student is used instead. One possible result is to obtain $s_1 \succ_c s'_1 \succ_c s'_2 \succ_c s_2$, and therefore s_1 is assigned to school c in level k_1, but his sibling s_2 gets assigned to c'. Similarly, students s'_1 and s'_2 end up assigned to schools c' and c respectively, and thus they are also assigned to different schools.

4.5 Implementation

Finally, we present the algorithm that determines the matching of students to schools:

For each level $k \in K$, from the highest (12th grade) down to the lowest (Pre-K) do:

(1) Update preferences of students in family application
(2) Create the weak priority list for each school, and subsequently randomly break the ties between and within families.
(3) Create the strict priority list for each type of seats at each school.
(4) Create student preferences over seats in each school.
(5) Run the student-optimal *Deferred Acceptance* algorithm for overlapping types on all students and schools that belong to level k.

Our implementation of the Deferred Acceptance algorithm is based in the approach of directed graphs first proposed by Balinski and Ratier [13] for one-to-one stable matchings and later extended by Baïou and Balinski [12] for many-to-one matchings. At each level, we encode the instance as a graph in which every node represents a student applying to a type-specific seat in a school, and every directed edge connects either two prefences or two priorities, from the least preferred (or prioritized) one to the most. Then, the algorithm eliminates nodes that do not belong to any stable matching, until no further eliminations are possible. Finally, in order to find the student-optimal matching, we pick the top preference of each student that has at least one preference remaining in the graph. This algorithm allows us to solve all levels of the nationwide instance of the school admission problem in just a few seconds.

Table 2. Evolution of the System - Main Round

	2016	2017	2018
Regions	1	5	15
Schools	63	2,174	6,421
Students	3,436	76,821	274,990
% assigned 1st preference	58.0	56.2	59.2
% assigned any preference	86.4	83.0	82.5
% unassigned	9.0	8.7	8.9

5 RESULTS

In this section we report part of the implementation results. We start by describing how the system evolved from 2016 to 2018. Then, we focus on the current admission process and report the results of the main and complementary rounds in Sections 5.1 and 5.2 respectively. In Section 5.3 we analyze the effect of the quota for disadvantaged students. Finally, in Section 5.4 we study the impact of the family application.

In Table 2 we summarize how the admission system evolved. In 2016 we considered only the entry levels of the Magallanes region, located in the extreme south of the country. In 2017, the system was extended to all levels in Magallanes, and to four more regions considering only their entry levels. For the 2018 process all the levels of the aforementioned regions were added, and all the remaining regions (except for the Metropolitan area) were included at their entry levels. By 2020, the plan is to implement the system in the entire country and considering all levels, i.e. from Pre-K to 12th grade. As the table shows, most of the relevant performance metrics of the main round—fraction of students assigned to their top choice and unassigned—have remained stable over time.

5.1 Main Round

In 2018, 274,990 students and 6,421 schools—divided in 32,198 sections, i.e. school-level pairs—participated in the system, with a total of 522,859 available seats (average of 16.2 seats per section). In Table 3 we classify students based on (1) their gender, (2) whether they have any type of priority in the schools they applied to, and (3) whether they were eligible for any quota in the schools of their choice.[6] Notice that the percentage of disadvantaged students exceeds 50% of the total number of applicants. As the quota for this group is only 15%, an interesting design question is whether having a quota has any impact when the targeted population is relatively large. We analyze this in Section 5.3.

Analyzing the submitted preferences we observe that students apply on average to 3.4 schools. Among these applications, 73.1% are to public schools and 26.9% to voucher schools, although only 11% of the total seats available are of the latter type. Out of the 485,905 applications made by disadvantaged students, 22.0% were made to voucher schools, which is significantly less than the general population. These differences are not surprising considering that disadvantaged students have less resources, and therefore their willingness to pay is probably lower.

In Figure 3 we present the distribution of assignments by preference. We observe that 59.2% and 12.8% of the applicants are assigned to their first and second preference respectively. In addition,

[6]Priorities and quotas may depend on the school (e.g. being the child of a school employee is only valid for that school), and are not mutually exclusive.

Table 3. Characterization of Applicants - Main Round

		# applicants	% of total applicants
Gender	Female	134,973	49.1%
	Male	140,016	50.9%
Priority	Siblings	66,743	24.3%
	Working parent	3,700	1.3%
	Former students	9,165	3.3%
Quota	Special needs	1,631	0.6%
	Academic excellence	6,534	2.4%
	Disadvantaged	150,287	54.7%

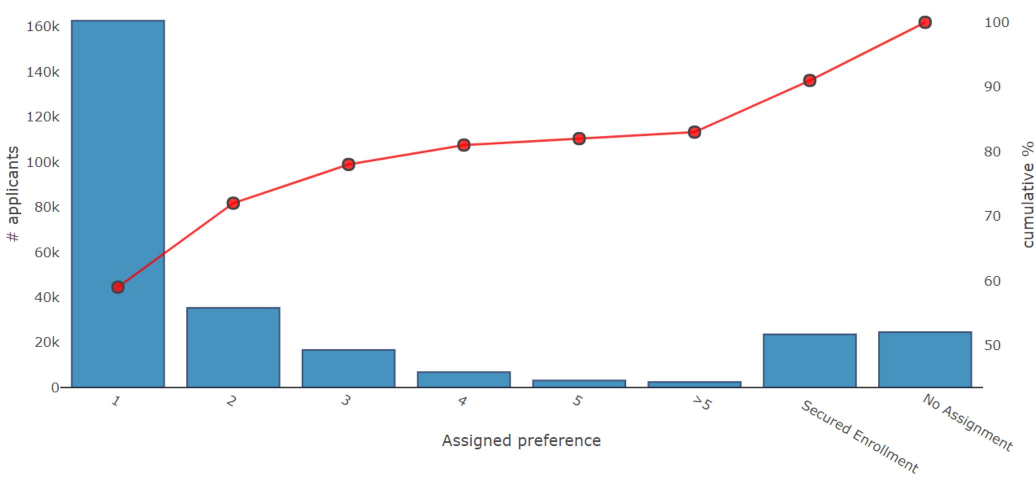

Fig. 3. Number of assigned students according to their preferences and the cumulative percentage it represents, including students assigned by secured enrollment and students that were not assigned - Main Round

8.6% are assigned to their current school via secured enrollment, and 8.9% are left unassigned[7]. Results of the assignment by type of level and by the priority criteria of students are presented in the full version of the paper.

Figure 4 shows the fraction of students that (1) are assigned to one of their preferences, (2) are assigned to their current school by secured enrollment, and (3) result unassigned, conditional on the number of reported preferences. We observe that when the number of declared preferences increases so does the probability of being assigned, but the average preference of assignment also increases. Moreover, we find that students who result unassigned apply on average to fewer schools (3.36, with std. dev. 1.49) than those who result assigned (3.42, with std. dev. 1.83). Applicants assigned by secured enrollment usually submit even less preferences (3.05, with std. 1.49), which is expected as they have a secured option.

[7]Recall that these students —the unassigned— have the chance to participate in the complementary process, whose results are described in Section 5.2.

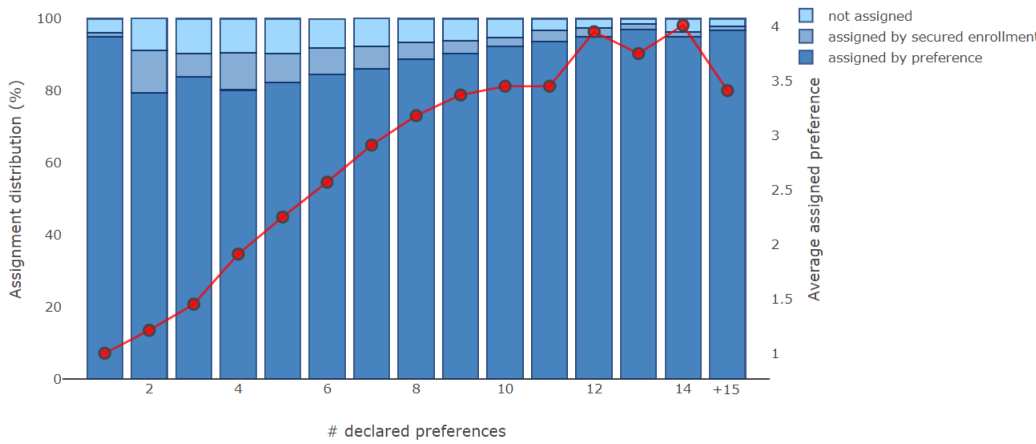

Fig. 4. Assignment distribution and average rank distribution by number of declared preferences - Main Round

Table 4. Characterization of Applicants - Complementary Round

		# applicants	% of total applicants
Gender	Female	23,063	49.4%
	Male	23,635	50.6%
Priority	Siblings	5,443	11.7%
	Working parent	328	0.7%
	Former students	2,441	5.2%
Quota	Disadvantaged	23,414	50.1%

5.2 Complementary Round

A total of 46,698 students participated in the complementary round including unassigned students from the main round and new applicants. In Table 4 we characterize these students based on their gender, priority type and eligibility for disadvantaged quota—the other quotas are not considered in the complementary round. In general we observe that there are no significant differences relative to the main round.

In Figure 5 we present the distribution of preference of assignment in the complementary round. We observe that the results are not as good as in the main round, as 47% are assigned to their top choice, 28% are assigned by distance, and 3.6% resulted unassigned. This result can be explained by the number of submitted preferences, as students that get assigned apply to 3.49 (std. 1.83) schools on average, compared to 2.57 (std. 0.98) for students with secured enrollment, 3.28 (std. 1.47) for students assigned by distance, and 2.79 (std. 2.08) for unassigned students.

Recall that those students who are not assigned to any of their preferences in the complementary round may be allocated to the nearest public school with remaining open seats within 17km, i.e. distance assignment. Indeed, 13,064 students were assigned by distance, and the average distance for these students was 2.17km, compared to 2.19km for those students who were assigned to one of their preferences in the complementary process and 3.35km for those assigned by secured

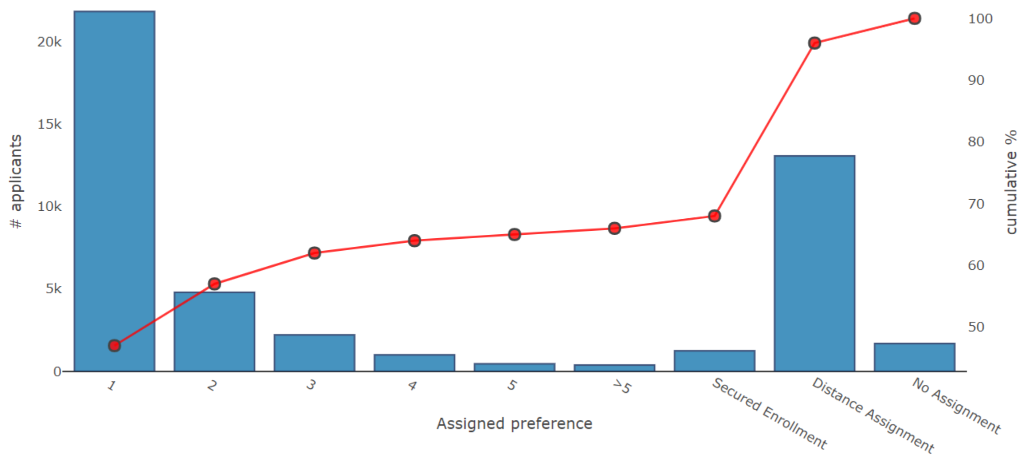

Fig. 5. Number of assigned students according to their preferences and the cumulative percentage it represents, including students assigned by secured enrollment, distance and students that were not assigned - Complementary Round

Table 5. Results for disadvantaged and non-disadvantaged students - Main Round

	Disadvantaged	Non disadvantaged
Number of applicants	150,287	124,703
1st preference	66.0%	51.0%
Other preference	21.1%	26.1%
Secured enrollment	7.3%	10.1%
Not assigned	5.7%	12.8%
Average rank	1.53	1.82
Average applications	3.02	3.37

enrollment. Finally, only 1,691 students—0.6% of the total number of applicants considering both rounds—resulted unassigned and were manually allocated by MINEDUC.

5.3 Quotas

An important design question is whether quotas for disadvantaged students should be considered, especially when these students are the majority (54.7%) of those who participate in the system and when random numbers are used to break ties. In Table 5 we compare how well these students perform compared to those who are not eligible for this quota.[8] As expected, students eligible for the quota perform better than those who are not, with more disadvantaged students being assigned to their top choice and less of them resulting unassigned.

In addition to helping students from disadvantageous environments, a second goal of the quota is to reduce segregation and achieve a higher level of socioeconomic heterogeneity in all the

[8]The results for the other quotas, i.e. for students with special needs and for high-achieving students, are reported in the full version of the paper.

Table 6. Results with and without socioeconomic quota - Simulations

	With quota		Without quota	
	Disadvantaged	Non-disadv.	Disadvantaged	Non-disadv.
1st preference	66.1%	50.1%	65.8%	51.3%
Other preference	21.1%	26.1%	21.0%	26.1%
Secured enrollment	7.2%	10.0%	7.4%	10.0%
Not assigned	5.6%	12.8%	5.8%	12.6%
Heterogeneity of schools				
	Mean	Std. Dev.	Mean	Std. Dev.
Fraction of disadv. classmates	0.5661	0.2173	0.5650	0.2198

classrooms. An analysis on the heterogeneity of schools in pre-kindergarten can be found in the full version of the paper.

To complement this analysis, we conducted a simulation study to better understand the effect of having this quota. In particular, we performed 20,000 simulations, where half of them were done considering the quota of 15% and the other half were done eliminating this quota. In the latter case, seats were assigned regularly by priority groups.

In Table 6 we show the results of the simulations. As expected, disadvantaged students perform better when there is a quota, but the differences are not significant compared to the case with no quota. The fact that there is almost no change in performance when removing the quota could be due to auto-segregation, i.e. disadvantaged students apply to different schools than non disadvantaged students. Table 6 also shows the mean and standard deviation of the fraction of disadvantaged classmates that each student has.[9] We observe that the results are practically the same with and without quota, and therefore we conclude that having the quota has no major impact in the heterogeneity of schools.

5.4 Family Application

Another distinctive feature of the Chilean school choice problem is the family application, which aims to increase the number of siblings that get assigned to the same school. In 2018, a total of 21,424 students were part of 10,301 family applications in the main round, with 2,869 (27.9%) formed by students that belong to the same level,[10] and 7,432 (72.1%) having at least two students of different levels. For concreteness, we focus on the results for the main round.

We say that a family application is successful if all of its members are assigned to the same school. Similarly, for families that include three or more students, we say that a family application is partially successful if at least two of its students, but not all of them, are assigned to the same school. Overall we observe that 6,725 (65.3%) family applications were successful, while 307 (3%) were partially successful. Figure 6 shows the success of family applications by size. As expected,

[9]Refer to the full version of the paper for details on the computation.

[10]The overall number of siblings in the same level is less than 2%. However, family applications with siblings applying to the same level are over-represented mostly because (i) families can decide whether or not to apply as a family, and (ii) only PK, K, 1st, 7th and 9th grade are considered in the system, making more likely to have siblings in the same level.

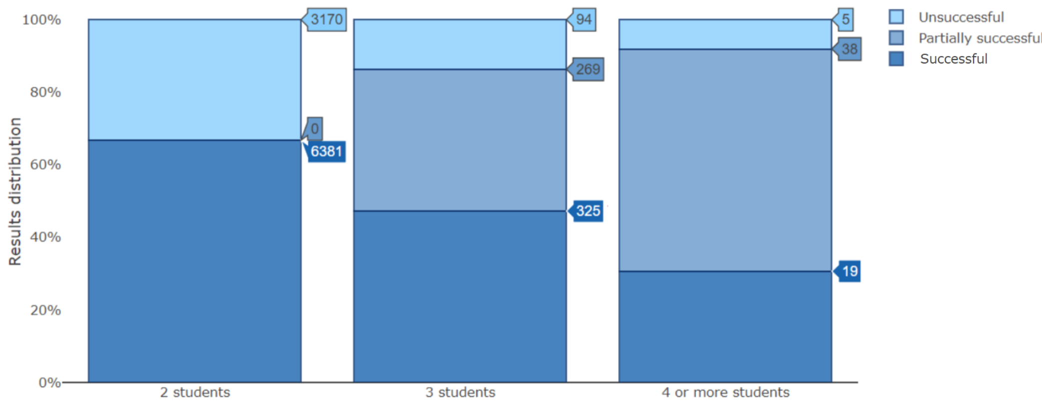

Fig. 6. Number of successful and partially successful families by size - Main Round

larger families are less likely to be successful, as they require more students being allocated to the same school.[11]

As described in Section 4.3, to implement applications we consider lotteries by families and update students' preferences. However, there may be other approaches that could lead to more successful families. To explore the benefits of our approach, we conduct a simulation study to compare our current approach with three other alternatives: (i) lotteries by student in each school and updating preferences (ii) lotteries by families without updating preferences, and (iii) lotteries by student without updating preferences. Notice that the latter is the standard approach used in other school choice settings.

In Table 7 we report the results of 10,000 simulations for each approach. We observe that the fraction of successful families is larger when both components—lottery per family and updating preferences—are implemented.[12] In addition, comparing the results of the current mechanism with alternatives (i) and (ii) we observe that updating preferences seems to be more relevant than having a lottery per family, as the former explains 9% of successful families, while the latter only explains 4%. These results suggest that updating preferences should be prioritized to improve the chances of siblings of getting assigned together.

Table 7. Family application - Simulations

Approach	Successful families	Partially successful families
Current	6,747 (66%)	303 (3%)
(i)	6,342 (62%)	309 (3%)
(ii)	5,906 (57%)	344 (3%)
(iii)	5,446 (53%)	351 (3%)

[11]We refer to the full version of the paper for results on family applications depending on the number of common schools a family declares.

[12]The mechanism cannot guarantee that all family applications will be successful. For example, siblings may apply to different schools, and younger siblings may not be eligible in some schools where their elder siblings are applying to (single-gender schools, schools with not all levels, among others). Moreover, the number of seats available may not be enough to allocate all students with sibling priority, as it is the case in non-entry levels.

6 CONCLUSIONS

Centralized procedures to assign students to schools are becoming the norm in many countries. In this paper we describe the design and implementation of the new school choice system in Chile, which expands previous applications in three main areas.

First, we introduce, analyze and evaluate the impact of features of our design intended to favor siblings in getting assigned to the same school. Concretely, we propose the use of two lotteries, one to order families and the other to break ties among siblings. In addition, our mechanism updates students' preferences to prioritize siblings getting assigned to the same school if they are part of a family application. Our results show that these features improve the fraction of siblings assigned in the same school by 13% compared to the standard approach of breaking ties at the student level. Second, we propose a multi-level mechanism that allows students to have a secured enrollment in their current school. This feature of the system eliminates the risk of ending up unassigned when trying to move to a new school. Finally, we implement a mechanism with multiple quotas and priority groups. We show via simulations that having quotas for disadvantaged students is not very effective when the majority of students are eligible and the quota is relatively small. Hence, other approaches should be considered if the goal is to really benefit this group.

The experience of implementing a large-scale nationwide system stresses the importance of having a continuous collaboration with policy makers, and the need of implementing changes in small steps. Having a gradual implementation not only allows to learn from the experience and continuously improve the system, but also gives time to the general public—and final users of the system—to get information, learn and understand the benefits of the new system. Overall, we will continue working to improve the system, increasing its efficiency and fairness to give equal opportunities to all students, regardless of their background.

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REFERENCES

[1] 24Horas. 2017. Apoderados hacen largas filas para postular a colegio en Santiago. https://www.24horas.cl/nacional/apoderados-hacen-largas-filas-para-postular-a-colegio-en-santiago-2483007

[2] Atila Abdulkadiroğlu. 2007. Controlled School Choice. *New York* 2006, April (2007), 1–44. http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.169.4192

[3] Atila Abdulkadiroglu, Yeon-Koo Che, Parag A. Pathak, Alvin E. Roth, and Olivier Tercieux. 2017. *Minimizing Justified Envy in School Choice: The Design of New Orleans' OneApp.* Technical Report. National Bureau of Economic Research.

[4] Atila Abdulkadiroğlu, Yeon-Koo Che, and Yosuke Yasuda. 2011. Resolving conflicting preferences in school choice: The" boston mechanism" reconsidered. *American Economic Review* 101, 1 (2011), 399–410.

[5] Atila Abdulkadiroğlu, Parag A. Pathak, and Alvin E. Roth. 2005. The New York City high school match. *American Economic Review* 95 (2005), 364–367.

[6] Atila Abdulkadiroğlu, Parag A. Pathak, and Alvin E. Roth. 2009. Strategy-proofness versus efficiency in matching with indifferences: redesigning the New York City high school match. *American Economic Review* 99 (2009), 1954–1978.

[7] Atila Abdulkadiroğlu, Parag A. Pathak, Alvin E. Roth, and Tayfun Sönmez. 2005. The Boston public school match. *American Economic Review* 95 (2005), 368–371.

[8] Atila Abdulkadiroğlu and Tayfun Sönmez. 2003. School Choice: A Mechanism Design Approach. *American Economic Review* 93 (2003), 729 – 747.

[9] Nick Arnosti. 2015. Short Lists in Centralized Clearinghouses. In *Proceedings of the Sixteenth ACM Conference on Economics and Computation (EC '15).* ACM, New York, NY, USA, 751–751. https://doi.org/10.1145/2764468.2764533

[10] Itai Ashlagi, Mark Braverman, and Avinatan Hassidim. 2014. Stability in Large Matching Markets with Complementarities. *Operations Research* 62, 4 (2014), 713–732.

[11] Itai Ashlagi, Afshin Nikzad, and Assaf Romm. 2015. Assigning More Students to Their Top Choices: A Tiebreaking Rule Comparison. (2015).

[12] Mourad Baïou and Michel Balinski. 2004. Student admissions and faculty recruitment. *Theoretical Computer Science* 322, 2 (2004), 245–265.

[13] Michel Balinski and Guillaume Ratier. 1997. Of stable marriages and graphs, and strategy and polytopes. *SIAM review* 39, 4 (1997), 575–604.

[14] Cristian Bellei. 2013. El estudio de la segregación socioeconómica y académica de la educación chilena. *Estudios pedagógicos (Valdivia)* 39, 1 (2013), 325–345.

[15] Caterina Calsamiglia and Maia Güell. 2014. The illusion of school choice: empirical evidence from Barcelona. (2014).

[16] Cooperativa. 2017. Apoderados nuevamente hicieron fila para postular a colegio capitalino. https://www.cooperativa.cl/noticias/pais/educacion/colegios/apoderados-nuevamente-hicieron-fila-para-postular-a-colegio-capitalino/2017-06-12/074548.html

[17] Monique De Haan, Pieter A. Gautier, Hessel Oosterbeek, and Bas Van der Klaauw. 2015. *The performance of school assignment mechanisms in practice.* Technical Report. Institute for the Study of Labor (IZA), Bonn.

[18] Umut Dur, Scott Duke, Kominers Parag, Parag A Pathak, and Tayfun Sönmez. 2016. Reserve Design: Unintended Consequences and The Demise of Boston's Walk Zones. *Journal of Political Economy* (2016).

[19] Umut Dur, Parag A. Pathak, and Tayfun Sönmez. 2016. Explicit Vs. Statistical Preferential Treatment in Affirmative Action: Theory and Evidence From Chicago'S Exam Schools. *NBER Working Paper Series* (2016). https://doi.org/10.3386/w22109

[20] Lars Ehlers, Isa E. Hafalir, M. Bumin Yenmez, and Muhammed A. Yildirim. 2014. School choice with controlled choice constraints: Hard bounds versus soft bounds. *Journal of Economic Theory* 153 (2014), 648–683.

[21] David Gale and Lloyd S. Shapley. 1962. College admissions and the stability of marriage. *Amer. Math. Monthly* 69, 1 (January 1962), 9–15.

[22] Pieter A. Gautier, Monique De Haan, Bas Van Der Klaauw, and Hessel Oosterbeek. 2016. Eerste analyse matching en loting voortgezet onderwijs Amsterdam 2016*. *Evaluation Paper, URL: http://www. verenigingosvo. nl/wp-content/uploads/2013/09/Rapport-loting-en-matching-2016-beschrijvend-rapport. pdf, Retrieved June* 14 (2016), 2016.

[23] Isa E. Hafalir, M. Bumin Yenmez, and Muhammed A. Yildirim. 2013. Effective Affirmative Action in School Choice. *Theoretical Economics* 8 (2013), 325–363. https://doi.org/10.3982/TE1135

[24] John W. Hatfield and Paul R. Milgrom. 2005. Matching with contracts. *American Economic Review* 95, 4 (2005), 913–935.

[25] Andrés Hernando, Karina Niklitschek, and Daniel Brieba. 2014. Segregación escolar en Chile. *Santiago: Horizontal* (2014).

[26] Yuichiro Kamada and Fuhito Kojima. 2015. Efficient matching under distributional constraints: Theory and applications. *American Economic Review* 105, 1 (2015), 67–99.

[27] Fuhito Kojima. 2012. School choice: Impossibilities for affirmative action. *Games and Economic Behavior* 75, 2 (2012), 685–693. https://doi.org/10.1016/j.geb.2012.03.003

[28] Fuhito Kojima, Parag A. Pathak, and Alvin E. Roth. 2013. Matching with couples: Stability and incentives in large markets. *The Quarterly Journal of Economics* 128, 4 (2013), 1585–1632.

[29] Ryoji Kurata, Naoto Hamada, Atsushi Iwasaki, and Makoto Yokoo. 2017. Controlled school choice with soft bounds and overlapping types. *Journal of Artificial Intelligence Research* 58 (2017), 153–184.

[30] MINEDUC. 2018. Estadisticas de la Educacion 2017. hhttps://centroestudios.mineduc.cl/wp-content/uploads/sites/100/2018/12/ANUARIO-MINEDUC_VERSION-BAJA.pdf

[31] Alvin E. Roth. 1984. The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory. *Journal of Political Economy* 92, 6 (1984), 991–1016. http://www.jstor.org/stable/1831989

[32] T13. 2017. Apoderados hacen fila por tres dĂŋas para cupo en colegio. http://www.t13.cl/videos/nacional/video-apoderados-hacen-fila-tres-dias-cupo-colegio

[33] Juan P. Valenzuela, Cristian Bellei, and Danae De los Ríos. 2008. *Evolución de la Segregación Socioeconómica de los Estudiantes Chilenos y su Relación con el Financiamiento Compartido.* Technical Report. Ministerio de Educación.

[34] Juan P. Valenzuela, Cristian Bellei, and Danae De los Ríos. 2014. Socioeconomic school segregation in a market-oriented educational system. The case of Chile. *Journal of education Policy* 29, 2 (2014), 217–241.

[35] Juan P. Valenzuela, Cristian Bellei, A Osses, and A Sevilla. 2011. ¿ Por qué los jóvenes chilenos mejoraron su competencia lectora en la prueba PISA. *Evidencias para Políticas Públicas en Educación* (2011), 265–311.

[36] Cristóbal Villalobos and Juan P. Valenzuela. 2012. Polarización y cohesión social del sistema escolar chileno. *Revista de análisis económico* 27, 2 (2012), 145–172.

[37] Guillermo Wormald, Carolina Flores, Francisco Sabatini, María Paz Trebilcock, and Alejandra Rasse Figueroa. 2012. Cultura de cohesión e integración en las ciudades chilenas. *Revista invi* 27, 76 (2012), 117–145.