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School Choice in Chile

Jose Correa, Rafael Epstein, Juan Escobar
Departamento de Ingenieria Industrial, Universidad de Chile

Ignacio Rios
Graduate School of Business, Stanford University

Bastian Bahamondes
Departamento de Ingenieria Industrial, Universidad de Chile

Carlos Bonet
Graduate School of Business, Columbia University

Natalie Epstein, Nicolas Aramayo
Departamento de Ingenieria Industrial, Universidad de Chile

Martin Castillo
Department of Politics, New York University

Andres Cristi, Boris Epstein, Felipe Subiabre
Departamento de Ingenieria Industrial, Universidad de Chile

Centralized school admission mechanisms are an attractive way of improving social welfare and fairness in large educational systems. In this paper we report the design and implementation of the newly established school choice system in Chile, where over 274,000 students applied to more than 6,400 schools. The Chilean system presents unprecedented design challenges that make it unique. On the one hand, it is a simultaneous nationwide system, making it one of the largest school choice problems worldwide. On the other hand, the system runs at all school levels, from Pre-K to 12th grade, raising at least two issues of outmost importance; namely, the system needs to guarantee their current seat to students applying for a school change, and the system has to favor the assignment of siblings to the same school. As in other systems around the world, we develop a model based on the celebrated Deferred Acceptance algorithm. The algorithm deals not only with the aforementioned issues, but also with further practical features such as soft-bounds and overlapping types. In this context, we analyze new stability definitions, present the results of its implementation and conduct simulations showing the benefits of the proposed innovations.

Key words: school choice, matching, two-sided market

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1. Introduction

According to the Duncan Index of segregation, Chilean schools are extremely socially segregated (Bellei 2013, Valenzuela et al. 2014). Several authors have shown that the costs of school segregation are important, including low social cohesion and lack of equal opportunities and social mobility (Villalobos and Valenzuela 2012, Wormald et al. 2012). While the drivers of school segregation include societal aspects well beyond school choice, social movements and politicians were probably right at blaming features of the admission system.

The School Inclusion Law marks a breaking point in the organization and functioning of the school system. The Law, promulgated in 2015, changed the old admission process drastically by (i) eliminating co-payments in publicly subsidized schools, (ii) forbidding publicly subsidized schools from selecting their students based on social, religious, economic, or academic criteria, and (iii) defining priorities that must be used to assign students to schools. [1]

In this paper we report the results of an ongoing collaboration with the Chilean Ministry of Education (MINEDUC) addressing the practical challenges of implementing the School Inclusion Law. To this end, we designed and implemented a centralized system that (i) provides information about schools—seats available, mission, values, educational project, among others—to help parents and students in building their preferences; (ii) collects families' preferences through an online platform, reducing the time and cost that visiting each school involved in the past; and (iii) assigns students to schools using a transparent, fair and efficient procedure.

One of the distinctive features of the new school choice system in Chile is its broadness, as it runs nationwide and throughout all school levels. Being nationwide, the system accommodates the needs of both urban and rural families. Since the system runs throughout all school levels, it needs to favor the joint allocation of siblings. These two aspects are unique to our problem and bring new design challenges for school choice.

At the core of the system is the assignment algorithm, which adapts the celebrated Deferred Acceptance (DA) algorithm—introduced in the seminal paper by Gale and Shapley (1962)—to incorporate all the elements required by law and by MINEDUC. In particular, the system considers a set of priority groups—students with siblings in the school, students with parents that work in the school and former students of the school—that are served in strict order of priority. The system also includes quotas for students (i) from disadvantageous environments, (ii) with special educational needs and disabilities, and (iii) with high academic performance. Within each priority group, ties are broken randomly.

The admission system runs throughout all educational levels from the highest (12th grade) to the lowest (Pre-K). This makes patent the need to secure their current enrollment to students that want to change to another school. Additionally, some families may have two or more of their children simultaneously applying to schools and may naturally want their children to attend the same school. Two features of our implementation favor the assignment of siblings to the same school. First, the tie-breaking lotteries are run over families rather than over students. Lotteries over families create correlation in the priority ranks of siblings applying to different levels in a given school. As our results and examples show, this correlation makes more likely siblings end up assigned to the same school. Second, families can express their willingness to have their children assigned to the same school by filling a *family application* (FA). A family application ensures that once the oldest child is assigned to some school, the application of the younger siblings are modified to put that school as their most preferred one.

In summary, our design presents at least four innovative features:

- It deals with multiple overlapping quotas.
- It allows currently enrolled students to apply to a different school while guaranteeing her current seat.
- It runs tie-breaking lotteries over families, significantly increasing the fraction of siblings that end up assigned to the same school.
- It adds a family application heuristic that improves the chances of having siblings assigned to the same school.

The results reported in this paper consider the current state of the system, which includes all regions, except the Metropolitan area of Santiago, reaching 274,990 students and 6,421 schools in the main round. In the current admission process—for students who started their academic year in March, 2019—students applied to 3.4 schools on average, and 59.2% of students were assigned to their top preference. Moreover, 82.5% of students were assigned to one of the schools in their application list, 8.6% were assigned by secured enrollment to their current schools, and only 8.9% resulted unassigned. In addition, there were 10,301 family applications involving 21,424 students and 65.3% of these were successful, i.e., siblings got assigned to the same school, while 3% were partially successful, i.e., only a subset of siblings got assigned together.[2] We also provide simulations evaluating different elements of our design.

Designing, implementing and improving the Chilean school choice system has resulted in many lessons that could be useful for other practitioners designing large-scale clearinghouses. From a theoretical standpoint, we contribute to the existing literature by introducing the notion of family applications. We show that a stable matching may not exist, and we provide heuristics that are successful at increasing the fraction of siblings assigned to the same school. Finally, our results show that having lotteries over families (considering students applying to a given school at all levels simultaneously) significantly increases the fraction of siblings assigned to the same school. From a practical standpoint, a key lesson is that having a continuous communication and collaboration with policy-makers is essential, as many aspects evolve over time and must be incorporated in the design. In addition, fragmenting the implementation in a given number of steps allowed us to gain experience, solve unexpected problems and continuously improve the system. As centralized procedures to assign students to schools are becoming the norm in many countries, we expect that the lessons and solutions offered in this work are deemed useful in other implementations.

The remainder of the paper is organized as follows. In Section 2 we describe the school choice problem in Chile, with the main features requested by law and by MINEDUC. In Section 3 we discuss how this paper relates to several strands of the literature. In Section 4 we present our model and describe its implementation. In Section 5 we present the results, focusing on the admission process of 2018. In addition, we evaluate the effects of (i) quotas for disadvantaged students and (ii) family applications via simulations. Finally, in Section 6 we conclude and provide directions for future work.

2. The problem in Chile

The Chilean school choice system considers fourteen levels, ranging from pre-kindergarten to 12th grade. There are five entry levels: pre-kindergarten, kindergarten, 1st, 7th and 9th grade, which are the levels where a school can start. Depending on their type of funding, schools can be classified in three types: (1) private, for those schools that are independent and privately funded; (2) voucher, where families make co-payments to complement state subsidies; and (3) public, for those schools that are fully funded and operated by local governments. Voucher and public schools, which are the focus of this paper, account for more than 90.3% of the total number of students in primary and secondary education (MINEDUC 2018).

Before the introduction of the School Inclusion Law, schools ran their admission processes independently, often selecting their students based on arbitrary rules, such as interviews with the students and their parents, results of unofficial admission exams, past academic records, among many others. Since the admission processes were not coordinated, in many cases parents were forced to strategically decide whether to accept an offer or to reject it and wait until other schools released their admission offers. Moreover, many schools used “first-come first-served” rules to prioritize students, resulting in many parents waiting in long overnight queues to secure a seat for their children. Overall, the freedom of schools to choose their students and the existence of voucher schools are considered among the main reasons that explain the polarization and segregation of the Chilean school system (Valenzuela et al. 2014).

To address these problems, the School Inclusion Law forbids any sort of discrimination in the admission processes of schools that receive (partial or full) government funding, and mandates schools to use a centralized system that collects families' and students' preferences and returns a fair allocation. In this system, students and families can access a platform where they can collect information—number of open seats, number of students per classroom and level, educational project, rules and values, co-payments required, among others—to build their preferences, and later they can use it to apply to as many schools as they want by submitting a strict order of preferences. The system collects all these applications and runs a mechanism that aims to assign each student to their top preference provided that there are enough seats available. More specifically, if the number of applicants is less than the number of open seats, the law requires that all students applying to that school are admitted, unless they can be allocated in a school they prefer over it. On the other hand, for schools that are over-demanded the law defines a set of priority groups that are used to order students. In particular, there are three priority groups, which are processed in strict order of priority:

1. *Siblings.* For students that have a sibling already enrolled at the school.
2. *Working parent.* For students that have a parent working at the school.
3. *Former students.* For students that were enrolled at the school in the past and were not expelled from it.

Within each priority group students are randomly ordered and each school uses a different random tie-breaker. In addition to these priorities, the law specifies three different types of quotas:

1. *Special needs.* This quota serves students with disabilities. It reserves at most two seats per classroom per school and it is processed before any other priority group or quota. The quota only applies to schools that have a validated special program.
2. *High-achieving.* This quota applies to students with high academic performance. It is processed right after the special needs quota and considers between 30% to 85% of the total number of seats depending on the school. Only a subset of pre-selected schools can implement this quota in 7th and 9th grade, and students can only be ranked based on an admission exam.
3. *Disadvantaged.* This quota prioritizes the most vulnerable students (bottom third in terms of income according to the Social Registry of Homes). At each level in every school, 15% of seats are reserved for disadvantaged students, and this group is processed right after the first priority group, i.e., students with siblings.

There are two additional features that are relevant in the design of the system. First, students that are currently enrolled at a school and apply in the system aiming to change their allocation have a *secured enrollment* in their current school, i.e., in case of not being assigned to a school of their preference they can keep their current assignment. Second, families having two or more children that participate in the centralized system can choose to *apply as a family*, which means

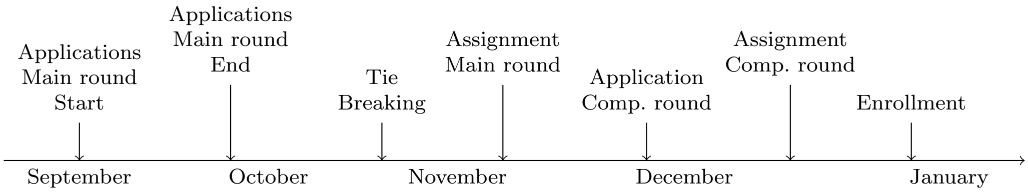

Figure 1 Timeline of the Admission Process

that they prioritize having their children assigned to the same school over a better school in their reported preferences.

The timing of the admission process is summarized in Figure 1. After collecting information, families submit their application between September and October. After all applications are received, the centralized mechanism generates the lotteries that will be used to order students in over-demanded schools and executes the main round of the process. For those students that result unassigned or did not apply in the main round, there is a complementary process where they submit a new application list that only includes schools with available seats. Finally, students that result unassigned in the complementary round are assigned to the closest school with available seats that does not charge a co-payment. We refer to this as *distance assignment*. In case that there are no schools with seats available within 17km, students remain unassigned and MINEDUC gives them a solution. The process ends in late December with the enrollment.

3. Literature

This paper is related to four strands of the literature: school choice, affirmative action, assignment of families and tie-breaking.

School Choice. In the last two decades, and starting from the theoretical formalization of the school choice problem by Abdulkadiroğlu and Sönmez (2003), there have been reforms to the school choice system in many places around the world. The first major reform was introduced in New York, where a variation of the Deferred Acceptance (DA) algorithm with restricted lists was implemented (Abdulkadiroğlu et al. 2005a). In 2005, the Boston Public School system decided to switch from the so called *Boston Mechanism* (BM), also known as *Immediate Acceptance* (IA) mechanism, to DA in order to address the strategic incentives introduced by the former algorithm (Abdulkadiroğlu et al. 2005b). Despite its lack of strategy-proofness, BM has been revisited in the last few years since it better captures cardinal preferences and therefore can lead to higher social welfare (Abdulkadiroğlu et al. 2011). Since then, other systems such as Barcelona (Calsamiglia and Güell 2018), Amsterdam (Gautier et al. 2016), New Orleans (Abdulkadiroglu et al. 2017), among others, have implemented centralized school choice systems using some variant of DA, BM or *top-trading cycles* (TTC). This paper contributes to this literature by adding a new case study with some additional features that have not been explored in previous literature, such as the admission of siblings in different levels and the secured enrollment problem. In addition, this is one of the first papers that describes the implementation of a system at a country level.

Priorities and Affirmative Action. Many school choice systems include affirmative action policies to promote diversity in the classrooms. Ehlers (2010) explores DA under type-specific quotas, finding that the student-proposing DA is strategy-proof for students if schools' preferences satisfy responsiveness. Kojima (2012) studies the implementation of majority quotas and shows that this may actually hurt minority students. Consequently, Hafalir et al. (2013) propose the use of minority reserves to overcome this problem, showing that DA with minority reserves Pareto dominates the one with majority quotas. Ehlers et al. (2014) extend the previous model to account for multiple disjoint types, and propose extensions of DA to incorporate soft and hard bounds. Other types of constraints are considered by Kamada and Kojima (2015), who study problems with distributional constraints motivated by the Japanese Medical Residency. Dur et al. (2016a) analyze the Boston school system and Dur et al. (2016b) analyze the impact of these policies using Chicago's system data. However, these models consider disjoint types of students. Kurata et al. (2017) study the overlapping types problem and show that, even in the soft-bound minority quotas scenario, a stable matching might not exist. As a solution, they propose a model in which stability is recovered by counting each student towards the seat type he was assigned to. We contribute to this literature by implementing a new case study and analyzing the effect of different minority quotas in the implementation. Moreover, we show that quotas depend on students truly being a minority and that there can also be some auto-segregation of minorities.

Assignment of families. The family application combines features of a many-to-many matching between families and schools and the existence of coalitions. A similar structure can be found in the labor market of medical residents, where couples prefer (in general) to be allocated in the same city. Roth (1984) shows that one cannot guarantee the existence of a matching without justified envy when couples have arbitrary preferences over pairs of hospitals. Kojima et al. (2013) show that a stable matching exists if the number of couples is relatively small and preference lists are sufficiently short relative to the size of the market. Another positive result is presented by Ashlagi et al. (2014), who introduce a new algorithm that finds a stable matching with high probability (in large matching markets) and where truth-telling becomes an approximate equilibrium for the induced game. We contribute to this literature by showing that a stable matching may not exist when there are family applications, and by introducing a new heuristic that can solve this problem.

Tie-breaking. A common approach to deal with ties in school choice is to use random tie-breaking rules, such as *single tie-breaking* (STB)—all schools use the same ordering for breaking ties—and *multiple tie-breaking* (MTB)—each school uses its own random order. Abdulkadiroğlu et al. (2009)

are the first to empirically compare these tie-breaking rules, and they find that there is no stochastic dominance in NYC. A similar pattern is found in Amsterdam, as described by De Haan et al. (2015). These findings are in line with the theoretical results by Ashlagi et al. (2019), who find that when there is low competition there is no stochastic dominance between random assignments. However, they also show that when there is a shortage of seats, STB almost dominates MTB and also leads to lower variance in students' rankings. Moreover, Arnosti (2015) shows that STB can lead to more matches when preferences are short and random. We contribute to this literature by introducing and studying the effect of family applications and breaking ties between families instead of students.

4. Model and Implementation

The Chilean school choice problem can be formalized as follows. Let K be the set of all levels, including pre-kindergarten and kindergarten in preschool, 1st to 8th grade in primary school and 9th to 12th grade in secondary school. Each school offers some or all of these levels, and students can only apply to schools that offer their level. For simplicity, we will first focus on a fixed level to define the basic setting, and later (in Section 4.3) we will introduce how levels interact with each other through the family application.

At a fixed level, let $S = \{s_1, ..., s_n\}$ be the set of students and $C = \{c_1, c_2, ..., c_m\}$ be the set of schools. Each school c has a capacity $q_c \in \mathbb{N}$ that accounts for the number of available seats. Students have a strict preference profile $\succ_S = (\succ_{s_1}, ..., \succ_{s_n})$ over schools, where $c \succ_s c'$ means that student s strictly prefers school c to c'. Students who rank a subset of schools implicitly declare that they prefer to be unassigned over being assigned to a school that is not in their preference list.

Schools, on the other hand, have a weak priority profile $\succsim_C = (\succsim_{c_1}, ..., \succsim_{c_m})$ over students, where $s \succsim_c s'$ means that at school c, student s has a higher or equal priority ranking than student s'. Students within a priority group are randomly ordered, creating a strict priority profile $\succ_C = (\succ_{c_1}, ..., \succ_{c_m})$ over students.

A **matching** μ is a function from the set $C \cup S$ to the subsets of $C \cup S$ such that:

(i) $\mu(s) \subseteq C, |\mu(s)| = 1$ or $\mu(s) = \emptyset$ for every student $s \in S$.

(ii) $\mu(c) \subseteq S$ and $|\mu(c)| \leq q_c$ for every school $c \in C$.

(iii) $c \in \mu(s)$ if and only if $s \in \mu(c)$, for every student $s \in S$ and school $c \in C$.

To simplify notation, for the case of a student $s \in S$ we write $\mu(s) = c$ to represent $\mu(s) = \{c\}$.

This definition formalizes that a feasible matching cannot assign more students to a school than its capacity, and it cannot assign a student to more than one school.

In what follows, we extend this model to address the particular features of the Chilean problem, namely: (1) secured enrollment, for students already enrolled at a school; (2) quotas for students of different types; (3) family applications; and (4) the tie breaking rule.

4.1. Secured enrollment.

The system allows students of any level to apply to schools of their choice, provided that those schools offer their level. In particular, some students may want to change to another school they prefer, but are better off at their current school than remaining unassigned or assigned to other schools they prefer less. The School Inclusion Law gives students the right to keep a seat at their current school in case they do not get a better assignment.

To address this requirement, we add to each student who seeks to change to another school a preference over their current school at the bottom of their preference list. In addition, we consider the secured enrollment of a student as a priority criterion with a higher priority than any other priority group. With this extra criterion, the complete list of priority groups is given by:

$$\text{Secured Enrollment} \succ_c \text{Siblings} \succ_c \text{Working Parent} \succ_c \text{Former Students} \quad \forall c \in C. \tag{1}$$

4.2. Quotas.

In order to promote diversity within schools, the School Inclusion Law includes affirmative action policies for financially disadvantaged students and children with special needs. Furthermore, a limited number of schools are allowed to reserve seats for students with high-achieving records.

Let $T = \{\textit{Special needs},\ \textit{High-achieving},\ \textit{Disadvantaged}, \textit{Regular}\}$ be the set of all possible types that students may belong to. In general, for each student, these types are school-dependent (as an exception, the disadvantaged type is school-independent) and may overlap. Each student belongs to at least one type, being *Regular* the default (i.e., *Regular* encodes the absence of type). We define a mapping $\tau : S \times C \to 2^T$ that maps students to their types on each school.

A function $p : C \times T \to \mathbb{N}$ defines type-specific quotas for each school, where p_{ct} represents a soft lower bound for school c, i.e., a flexible limit that regulates school c's priorities dynamically, giving higher priority to students of type t up to filling p_{ct} seats. Furthermore, we assume that in each school quotas can be met without violating its capacity, i.e., $\sum_{t \in T} p_{ct} \le q_c$ for every school $c \in C$.

As shown by Kurata et al. (2017), when student types overlap the general concepts of stability for a matching with soft lower bounds proposed in literature (Hafalir et al. (2013), Ehlers et al. (2014)) are insufficient to guarantee the existence of a stable matching. To overcome this difficulty, they propose a new model based on the framework of matching with contracts due to Hatfield and Milgrom (2005). In this model, schools provide distinct reserved seats for each student type, and assignments are interpreted as contracts that explicitly state that a student is assigned to a particular reserved seat at a school, in contrast to previous models where a student accounts for seats of multiple types.

Following this literature we extend our model as follows. Every student s has now strict preferences $\succ_s$ over contracts of the form $(c,t) \in C \times T$, and every school c has now a weak priority profile $\succsim_c$ over contracts of the form $(s,t) \in S \times T$. Then, a **matching** μ is a function from $(S \cup C) \times T$ to the subsets of $(S \cup C) \times T$ such that:

(i) $\mu(s) \subseteq C \times T, |\mu(S)| = 1$ or $\mu(s) = \emptyset$, for every student $s \in S$.

(ii) $\mu(c) \subseteq S \times T$ and $|\mu(c)| \leq q_c$ for every school $c \in C$.

(iii) $\mu(s) = \{(c,t)\}$ if and only if $(s,t) \in \mu(c)$, for every student $s \in S$, school $c \in C$ and type $t \in T$.

In other words, a student s is either unassigned or assigned to a seat of type t in school c, $\mu(c)$ is the set of students assigned at school c, each one to a type-specific seat, and student s is assigned to a seat of type t in school c if and only if school c's assignment contains s assigned to a seat of type t. Note that this definition does not require that type t students must be matched to seats of type t.

Let $\mu_t(c) := \{s \in S : \ \mu(s) \in \{c\} \times T \text{ and } t \in \tau(s,c)\}$ be the set of students of type t assigned to school c. Two well-known and desirable properties of a matching are to be fair (or justified envy-free) and non-wasteful. In our setting, a student s has **justified envy** towards a student s' with assignment $\mu(s') = (c', t')$ in matching μ if there exists a type $t \in T$ such that:

(i) $(c', t) \succ_s \mu(s)$,

(ii) $(s,t) \succ_{c'} (s', t')$,

(iii) and either $t' = t$ or $|\mu_{t'}(c')| > q_{c't'}$.

That is, s has justified envy towards s' assigned to school c' in a seat of type t' if either s prefers (c', t') to his assignment and is preferred by the school on that seat, or for some type $t \neq t'$, s prefers (c', t) to his assignment, school c' has exceeded the quota of type t' students and prefers s in a seat of type t to s' in a seat of type t'.

A student s **claims an empty seat** of a school c in matching μ if there exists $t \in T$ such that $(c,t) \succ_s \mu(s)$ and one of the following conditions hold:

(i) $|\mu(c)| < q_c$ or

(ii) $\mu(s) = (c, t')$ for some type $t' \in T$, $(s,t) \succ_c (s,t')$ and $|\mu_{t'}(c)| > q_{ct'}$.

A student s **claims an empty seat by type** in school c in matching μ if there exists $t \in T$ such that $(c,t) \succ_s \mu(s)$ and the following condition holds:

(iii) $|\mu_t(c)| < q_{ct}$.

Table 1 Weak priorities by type-specific seats. Lower numbers indicate higher priority.

Priority	$\succsim_{c,\text{special needs}}$	$\succsim_{c,\text{high-achieving}}$	$\succsim_{c,\text{disadvantaged}}$	$\succsim_{c,\text{regular}}$
1	Secured enrollment	Secured enrollment	Secured enrollment	Secured enrollment
2	Special needs	High-achieving	Siblings	Siblings
3	Siblings	Siblings	Disadvantaged	Working parent
4	Working parent	Working parent	Working parent	Former students
5	Former students	Former students	Former students	No priority
6	No priority	No priority	No priority	

Namely, s claims an empty seat of type t at c if s prefers that contract to her assignment, and either c has empty seats, or s is assigned to c in a seat that exceeded its quota and the school prefers having s in a seat of type t, or the quota of type t students has not been met at c.

When no student claims an empty seat or claims an empty seat by type at any school, we say the matching is **non-wasteful**. Finally, a matching is **stable** if it is non-wasteful and eliminates justified envy for all students. This notion of stability matches the one proposed by Kurata et al. (2017), even when we assume some students might remain unassigned.

In Table 1 we describe the schools' weak preference profiles over contracts $(s,t) \in S \times T$ for each fixed type $t \in T$. At every school c, students currently enrolled at the school have the highest priority in all types of seats. Then, for the special needs and high-achieving seats, students of the corresponding type are given the second highest priority, and the rest of the students are given priorities according to (1). As required by law, students that have siblings currently enrolled at the school have higher priority than disadvantaged students, even in seats reserved for that type.

Priorities of schools over contracts $(s,t) \in S \times T$ and preferences of students over contracts $(c,t) \in C \times T$ also need to be defined to fully state our model, even though neither schools nor students have real preferences over the type of seat defined by the contract. The way to break ties is not straightforward: as shown by Dur et al. (2016b), different tie-breaking rules might favor some type of students. To reduce over-representation of quotas, we break ties in a way that students and schools favor assignments of students to seats of one of their corresponding types.

4.3. Family application.

Families that have two or more children participating in the system are given the option to prioritize assigning them to the same school of their preference list to the detriment of better schools reported in each individual preference list. We refer to this as *family application*.

It is important to emphasize the different roles that siblings play in the system. On the one hand, there is a sibling's priority that gives special priority to students applying to schools where they have siblings already enrolled at. On the other hand, there is the family application, which addresses siblings (either in the same or in a different level) that are all participating in the admission process.

We implement family applications by defining an equivalence relation on S that captures the sibling relationship among students that are participating in the system. Thereby, families correspond to equivalence classes of size two or more induced by this relation. We only consider families that have at least one school in common in their members' preference lists.

Similar to the matching with couples problem, a stable matching might not exist in the school choice problem with family applications. To explain why, we must first define stability in this context. Consider the simplest version of the school choice problem: without families, types or quotas. A stable matching is an assignment μ where there is no pair student-school (s,c) such that $c \succ_s \mu(s)$ and $|\{s' \in \mu(c) : s' \succ_c s\}| \leq q_c - 1$. Now, a school c in level k has capacity q_c^k and a family A has preferences over the possible assignments of its members. In particular, we consider that a family A has a strict order of preferences $\succ_A$ over a set of acceptable schools, and that they strictly prefer assignments where $A \subseteq \mu(c)$ for some c, and the rest of comparisons are given by the Pareto partial ordering induced by $\succ_A$. Then, we say that a matching μ is stable if (i) there are no triplets of student, school and level s, c, k where s is not matched to the same school as the rest of the family $A \ni s$, such that $c \succ_s \mu(s)$ and $|\{s' \in \mu(c,k) : s' \succ_c s\}| \leq q_c^k - 1$; and (ii) there are no pairs A, c, such that $|\{s' \in \mu(c,k) : s' \succ_c A\}| \leq q_c^k - |A_k|$ for all $k \in K$ and either $c \succ_A \mu(A)$ if $|\mu(A)| = 1$ or c is just acceptable for A if $|\mu(A)| > 1$.

Proposition 1. *If there are family applications, a stable matching might not exist. This is true even in the case where the families have at most two siblings, the preferences of schools are over families, and students of the same family have the same preferences.*

In Figure 2 we present an instance with two levels, four families $\{A, B, C, D\}$ and four schools $\{c_1, c_2, c_3, c_4\}$. In this example, each school has one seat in each level, families A and C have only one child (in levels 1 and 2 respectively), and families B and D have two children, one in each level.

For each level we represent the instance as a graph, where each row corresponds to a family and each column to a school, and preferences are captured through arrows pointing towards more preferred options. Notice that each family is the most preferred one for some school, so no student can result unassigned and siblings must be assigned in the same school in any stable matching. However, we claim that there is no stable matching. To see this, suppose that there is a stable matching μ. Then, both children from family B must be assigned in the same school $c \in \{c_1, c_2\}$, which we denote by $B \in \mu(c)$. Hence, there are two cases:

1. if $B \in \mu(c_1)$, then family C will be assigned to c_2 (its top choice) in level 2, family D will take its favorite school—c_3—in both levels, and family A will take its favorite school—c_4—in level 1. As a result, no family with higher priority than B is assigned to c_2, and since c_2 is family B's top choice in both levels we conclude that μ is not stable.

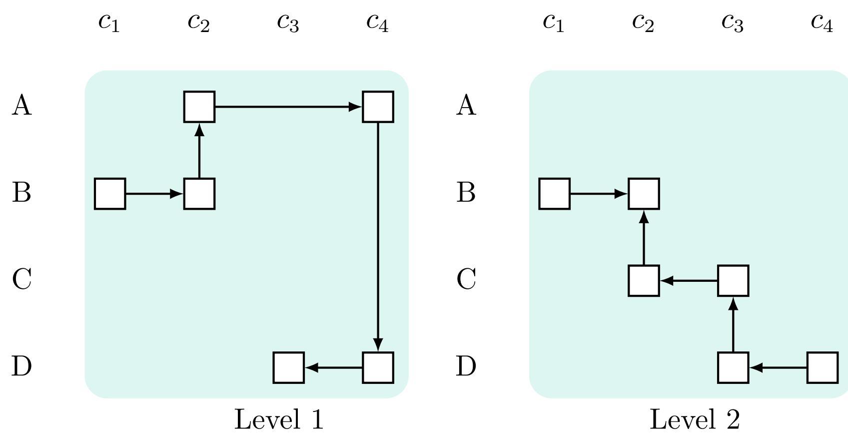

Figure 2 Instance with two levels where no stable matching exist because of the family applications.

2. if $B \in \mu(c_2)$, then family C is assigned to school c_3 in level 2, family D can only be accommodated in school c_4 (as both children must be assigned to the same school in any stable matching), and thus family A results unassigned. This contradicts the fact that all families should be allocated in any stable matching, and thus we conclude that there is no stable matching.

Due to this result, we implement the following heuristic to find a feasible solution to the problem with family applications.

1. Start from level $k = 12$, i.e., the highest level.
2. Obtain an assignment for level k. Call it μ_k.
3. Using μ_k, update the preferences of students whose siblings where assigned in Step (2) and that applied as a family, so that their top choice becomes the school where their elder sibling was assigned.
4. Update $k \leftarrow k - 1$, and go back to Step (2). If there are no levels left, stop.

4.4. Tie breaking rule.

As discussed earlier in this section, students within a priority group are randomly ordered to create a strict preference profile for each possible contract at each school. By law, schools are allowed to have their own lotteries, so we implement a variant of the multiple tie-breaking rule to account for this requirement.

As explained before, the system seeks to give siblings a higher chance of being assigned together. In order to do so, a second feature we implement is to break ties at the family level—which we call *family lotteries*—as opposed to having single student lotteries. Under this new approach, ties between families are broken first, and later a single student lottery is run within each family. In Proposition 2 we show that using family lotteries improves the probability that families are assigned together.

PROPOSITION 2. *(Informal statement) Consider a family with two children, Alice and Bob, applying to different levels. Fix the preference profiles of all students applying to these two levels. Also fix the priorities of all students on all schools except those of Alice and Bob. Then the probability that Alice and Bob get assigned to the same school is larger under family lotteries than under student lotteries.*

To ease exposition we defer the formal statement and proof of the latter proposition to Appendix A.2. Interestingly, the proof involves a novel lemma establishing that the cutoffs determining the allocation of any given student are independent of her preferences.

4.5. Example: Family Applications and Lotteries.

To illustrate the benefits of the family application and the lotteries by family we present the following example. Consider two schools, c and c', that have a single seat in levels H and L, and suppose that level H is processed first. In addition, suppose that there are four students: a_1 and a_2 who are siblings and apply to H and L respectively, b_1 who is a single student applying to H and d_2 who is a single student applying to L. Finally suppose all students prefer school c over school c'. We consider four scenarios: with or without family application, and with student or family lotteries. To illustrate the impact of the proposed policies, in each scenario we compute the probability that the family is assigned together.

(i) *Student lotteries, no family application.* The probability that the siblings are assigned together is equal to the probability that they are both assigned to either school c or c'. Since the probability that a_1 is assigned to c (or c') is $\frac{1}{2}$ and the probability that a_2 is assigned to c (or c') is also $\frac{1}{2}$, then the overall probability of the family being assigned together is $\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{2}$.

(ii) *Lotteries by family, no family application.* There are three possible lottery outcomes for the family in school c, namely, being ranked first, second or last. Each outcome has probability $\frac{1}{3}$. If the family is ranked first in c, a_1 and a_2 are assigned together in school c. If the family is ranked last in school c, both are assigned to school c'. Finally, if the family is ranked second in school c, one of the children is assigned to school c and the other to c'. Then, the overall probability that the family is assigned together is $\frac{1}{3}\cdot 1+\frac{1}{3}\cdot 1+\frac{1}{3}\cdot 0=\frac{2}{3}$.

(iii) *Lotteries by student, with family application.* Here the probability that the family is assigned together in school c is $\frac{1}{4}$ as in case (i). However, notice that the probability that the family is assigned to school c' is now $\frac{1}{2}$. The reason is that once student a_1 is assigned to c' (which happens with probability $\frac{1}{2}$), then a_2 updates her preferences and now prefers school c' and gets a seat for sure. Therefore the overall probability that the family is assigned together is $\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot 1=\frac{3}{4}$.

(iv) *Lotteries by family, with family application.* As in (ii), in school c the family may be ranked ranked first, second or last. Also, we know that the family is assigned together if it is ranked first or last. However, when the family is ranked second, either b_1 or d_2 are ranked first. In the former case the family is assigned together because in level H, a_1 is assigned to school c', and then a_2 updates her preferences also getting school c'. In the latter case the family is not assigned together because in level H, a_1 is assigned to school c, but in level L student a_2 is assigned to c'. As a result, the overall probability that the family is assigned together is $\frac{1}{3} \cdot 1 + \frac{1}{3} \cdot 1 + \frac{1}{3} \cdot \frac{1}{2} = \frac{5}{6}$.

4.6. Implementation.

Finally, we present the algorithm that determines the matching of students to schools. For each level $k \in K$, from the highest (12th grade) down to the lowest (Pre-K) do:

1. Update preferences of students in family application
2. Create the weak priority list for each school, updating sibling priority to applicants whose siblings were assigned in a higher level.
3. Create the strict priority list for each type of seats at each school, using random tie breaking by family.
4. Create student preferences over seats in each school.
5. Run the student-optimal *Deferred Acceptance* algorithm for overlapping types on all students and schools that belong to level k.

Our implementation of the Deferred Acceptance algorithm is based in the approach of directed graphs first proposed by Balinski and Ratier (1997) for one-to-one stable matchings and later extended by Baïou and Balinski (2004) for many-to-one matchings, which is efficient and allows us to find an assignment in a few seconds. At each level, we encode the instance as a graph in which every node represents a student applying to a type-specific seat in a school, and every directed edge connects either two prefences or two priorities, from the least preferred (or prioritized) one to the most. Then, the algorithm eliminates nodes that do not belong to any stable matching, until no further eliminations are possible. Finally, in order to find the student-optimal matching, we pick the top preference of each student that has at least one preference remaining in the graph. This algorithm allows us to solve all levels of the nationwide instance of the school admission problem in just a few seconds.

5. Results

In this section we report part of the implementation results. We start by describing how the system evolved from 2016 to 2018. Then, we focus on the current admission process and report the results of the main and complementary rounds in Sections 5.1 and 5.2 respectively. In Section 5.3 we

Table 2 Evolution of the System - Main Round

	2016	2017	2018
Regions	1	5	15
Schools	63	2,174	6,421
Students	3,436	76,821	274,990
% assigned 1st preference	58.0	56.2	59.2
% assigned any preference	86.4	83.0	82.5
% unassigned	9.0	8.7	8.9

analyze the effect of the quota for disadvantaged students. Finally, in Section 5.4 we study the impact of the family application.

In Table 2 we summarize how the admission system evolved. In 2016, we considered only the entry levels of the Magallanes region, located in the extreme south of the country. In 2017, the system was extended to all levels in Magallanes, and to four more regions considering only their entry levels. For the 2018 process all the levels of the aforementioned regions were added, and all the remaining regions (except for the Metropolitan area) were included at their entry levels. By 2020, the plan is to implement the system in the entire country and considering all levels, i.e., from Pre-K to 12th grade. As the table shows, most of the relevant performance metrics of the main round—fraction of students assigned to their top choice and unassigned—have remained stable over time.

5.1. Main Round.

In 2018, 274,990 students and 6,421 schools—divided in 32,198 sections, i.e., school-level pairs—participated in the system, with a total of 522,859 available seats (average of 16.2 seats per section). In Table 3 we classify students based on (1) their gender, (2) whether they have any type of priority in the schools they applied to, and (3) whether they were eligible for any quota in the schools of their choice. Notice that the percentage of disadvantaged students exceeds 50% of the total number of applicants. As the quota for this group is only 15%, an interesting design question is whether having a quota has any impact when the targeted population is relatively large. We analyze this in Section 5.3.

Analyzing the submitted preferences we observe that students apply on average to 3.4 schools. Among these applications, 73.1% are to public schools and 26.9% to voucher schools, although only 11% of the total seats available are of the latter type. Out of the 485,905 applications made by disadvantaged students, 22.0% were made to voucher schools, which is significantly less than the general population. These differences are not surprising considering that disadvantaged students have less resources, and therefore their willingness to pay is probably lower.

Table 3 Characterization of Applicants - Main Round

		# applicants	% of total applicants
Gender	Female	134,973	49.1%
	Male	140,016	50.9%
Priority	Siblings	66,743	24.3%
	Working parent	3,700	1.3%
	Former students	9,165	3.3%
Quota	Special needs	1,631	0.6%
	Academic excellence	6,534	2.4%
	Disadvantaged	150,287	54.7%

In Figure 3 we present the distribution of assignments by preference. We observe that 59.2% and 12.8% of the applicants are assigned to their first and second preference respectively. In addition, 8.6% are assigned to their current school via secured enrollment, and 8.9% are left unassigned (recall that these students —the unassigned— have the chance to participate in the complementary process, whose results are described in Section 5.2).

Figure 4 shows the fraction of students that (1) are assigned to one of their preferences, (2) are assigned to their current school by secured enrollment, and (3) result unassigned, conditional on the number of reported preferences. We observe that when the number of declared preferences increases so does the probability of being assigned, but the average preference of assignment also increases. Moreover, we find that students who result unassigned apply on average to fewer schools (3.36, with std. dev. 1.49) than those who result assigned (3.42, with std. dev. 1.83). Applicants assigned by secured enrollment usually submit even less preferences (3.05, with std. dev. 1.49), which is expected as they have a secured option.

5.2. Complementary Round.

A total of 46,698 students participated in the complementary round including unassigned students from the main round and new applicants. In Table 4 we characterize these students based on their gender, priority type and eligibility for disadvantaged quota—the other quotas are not considered in the complementary round. In general we observe that there are no significant differences relative to the main round.

In Figure 5 we present the distribution of preference of assignment in the complementary round. We observe that the results are not as good as in the main round, as 47% are assigned to their top choice, 28% are assigned by distance, and 3.6% resulted unassigned. This result can be explained by the number of submitted preferences, as students that get assigned apply to 3.49 (std. 1.83) schools on average, compared to 2.57 (std. 0.98) for students with secured enrollment, 3.28 (std. 1.47) for students assigned by distance, and 2.79 (std. 2.08) for unassigned students.

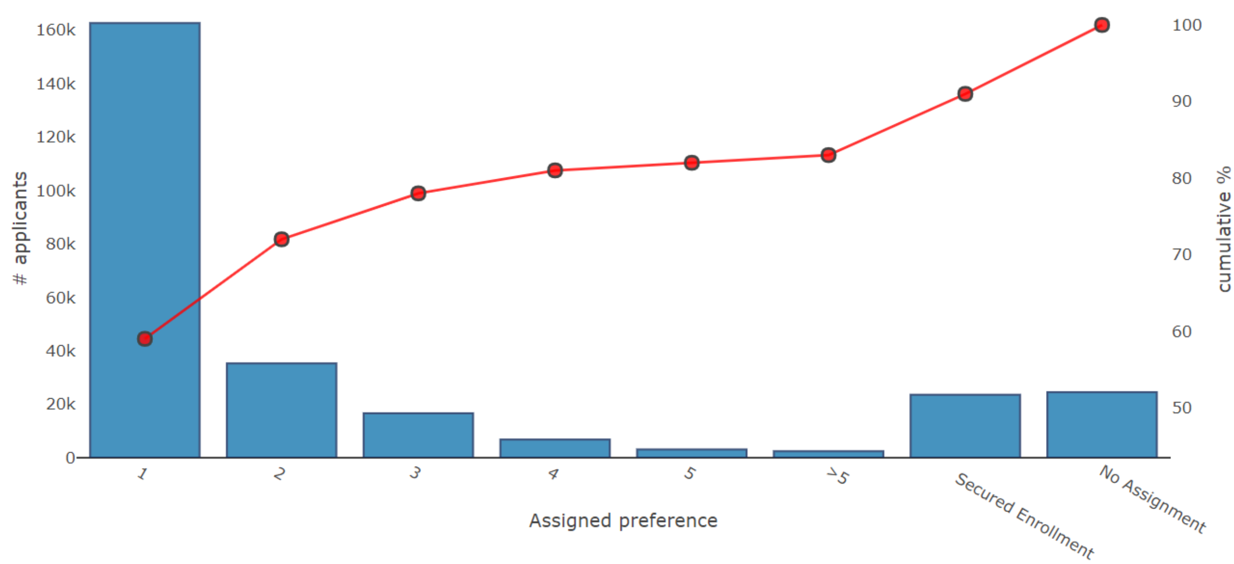

Figure 3 Number of assigned students according to their preferences and the cumulative percentage it represents, including students assigned by secured enrollment and students that were not assigned - Main Round

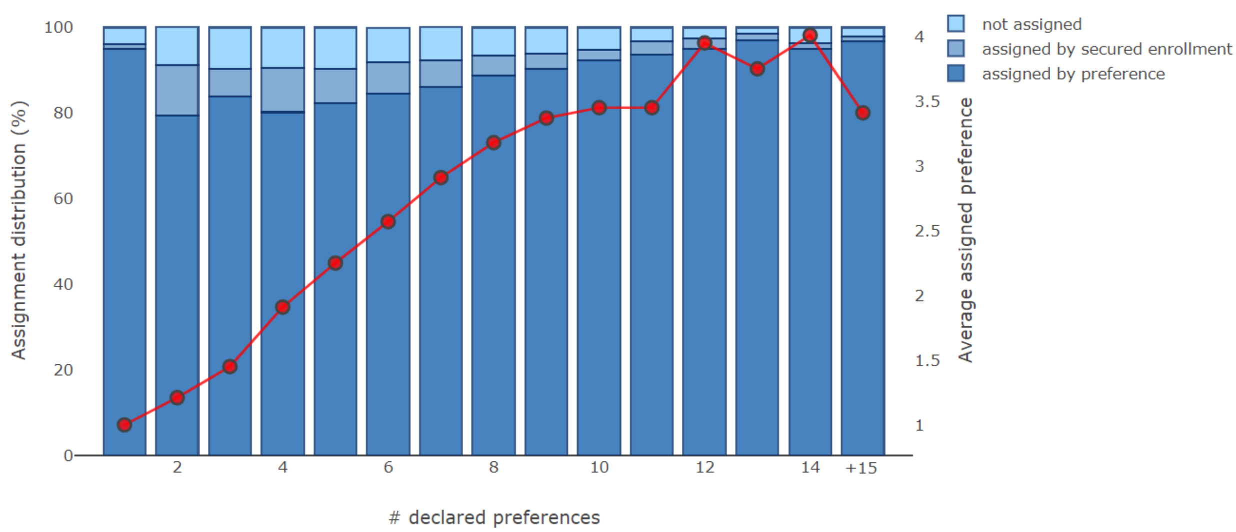

Figure 4 Assignment distribution and average rank distribution by number of declared preferences - Main Round

Table 4 Characterization of Applicants - Complementary Round

		# applicants	% of total applicants
Gender	Female	23,063	49.4%
	Male	23,635	50.6%
Priority	Siblings	5,443	11.7%
	Working parent	328	0.7%
	Former students	2,441	5.2%
Quota	Disadvantaged	23,414	50.1%

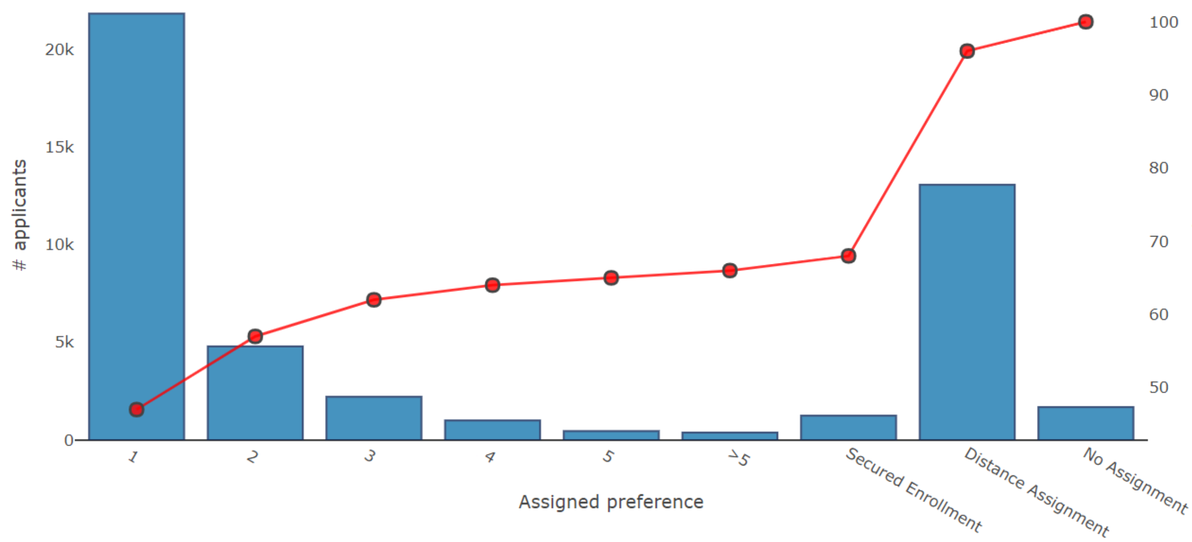

Figure 5 Number of assigned students according to their preferences and the cumulative percentage it represents, including students assigned by secured enrollment, distance and students that were not assigned - Complementary Round

Recall that those students who are not assigned to any of their preferences in the complementary round may be allocated to the nearest public school with remaining open seats within 17km, i.e., distance assignment. Indeed, 13,064 students were assigned by distance, and the average distance for these students was 2.17km, compared to 2.19km for those students who were assigned to one of their preferences in the complementary process and 3.35km for those assigned by secured enrollment. Finally, only 1,691 students—0.6% of the total number of applicants considering both rounds—resulted unassigned and were manually allocated by MINEDUC.

5.3. Quotas.

An important design question is whether quotas for disadvantaged students should be considered, especially when these students are the majority (54.7%) of those who participate in the system and when random numbers are used to break ties. In Table 5 we compare how well these students perform compared to those who are not eligible for this quota. The results for the other quotas, i.e., for students with special needs and for high-achieving students, are reported in Appendix B.1. As expected, students eligible for the quota perform better than those who are not, with more disadvantaged students being assigned to their top choice and less of them resulting unassigned.

In addition to helping students from disadvantageous environments, a second goal of the quota is to reduce segregation and achieve a higher level of socioeconomic heterogeneity in all the classrooms.

Table 5 Results for disadvantaged and non-disadvantaged students - Main Round

	Disadvantaged	Non disadvantaged
Number of applicants	150,287	124,703
1st preference	66.0%	51.0%
Other preference	21.1%	26.1%
Secured enrollment	7.3%	10.1%
Not assigned	5.7%	12.8%
Average rank	1.53	1.82
Average applications	3.02	3.37

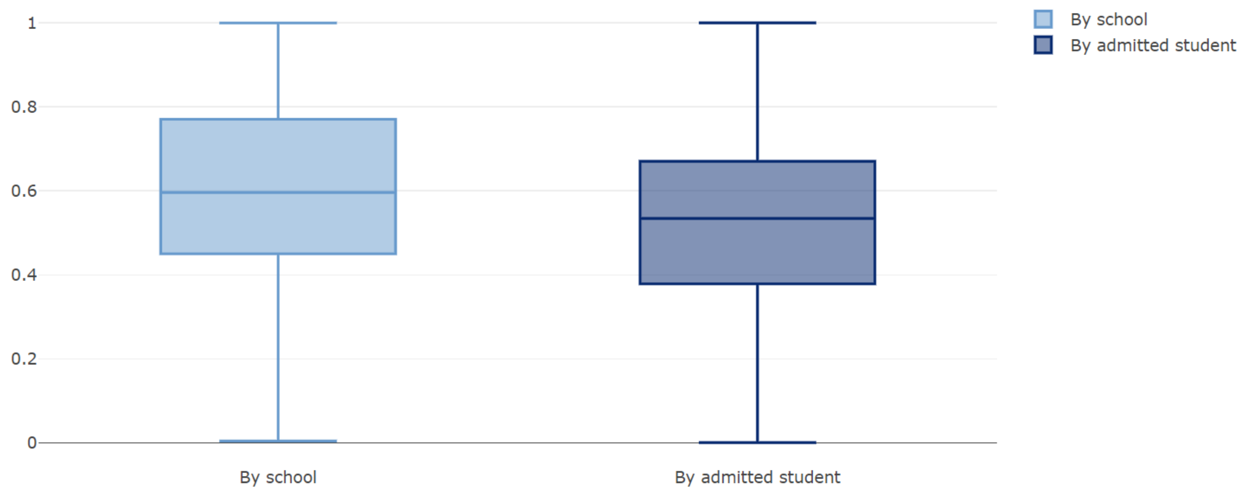

Figure 6 Distribution of the percentage of disadvantaged students in Pre-K - Main Round

To analyze this, in Figure 5.3 we show the distribution of the fraction of disadvantaged students assigned in pre-kindergarten (the lowest level). We focus on this level because there are no students with secured enrollment.

From this figure we observe that most school sections have a significant fraction of disadvantaged students, ranging from 45% to 77%. In addition, if we analyze this at the student level, we observe that 75% of students are assigned to a school that has between 38% and 67% of disadvantaged students. These results suggest that the disadvantaged quota has a positive effect in making diverse sections.

To complement this analysis, we conducted a simulation study to better understand the effect of having this quota. In particular, we performed 20,000 simulations, where half of them were done considering the quota of 15% and the other half were done eliminating this quota. In the latter case, seats were assigned regularly by priority groups.

Table 6 Results with and without socioeconomic quota - Simulations

	With quota		Without quota	
	Disadvantaged	Non-disadv.	Disadvantaged	Non-disadv.
1st preference	66.1%	50.1%	65.8%	51.3%
Other preference	21.1%	26.1%	21.0%	26.1%
Secured enrollment	7.2%	10.0%	7.4%	10.0%
Not assigned	5.6%	12.8%	5.8%	12.6%
Heterogeneity of schools				
	Mean	Std. Dev.	Mean	Std. Dev.
Fraction of disadv. classmates	0.5661	0.2173	0.5650	0.2198

In Table 6 we show the results of the simulations. As expected, disadvantaged students perform better when there is a quota, but the differences are not significant compared to the case with no quota. The fact that there is almost no change in performance when removing the quota could be due to auto-segregation, i.e., disadvantaged students apply to different schools than non disadvantaged students. Table 6 also shows the mean and standard deviation of the fraction of disadvantaged classmates that each student has (see to Appendix B.2 for details on the computation). We observe that the results are practically the same with and without quota, and therefore we conclude that having the quota has no major impact in the heterogeneity of schools.

5.4. Family Application.

Another distinctive feature of the Chilean school choice problem is the family application, which aims to increase the number of siblings that get assigned to the same school. In 2018, a total of 21,424 students were part of 10,301 family applications in the main round, with 2,869 (27.9%) formed by students that belong to the same level, and 7,432 (72.1%) having at least two students of different levels. For concreteness, we focus on the results for the main round.

Note that the overall number of siblings in the same level is less than 2%. However, family applications with siblings applying to the same level are over-represented mostly because (i) families can decide whether or not to apply as a family, and (ii) only PK, K, 1st, 7th and 9th grade are considered in the system, making it more likely to have siblings in the same level.

We say that a family application is successful if all of its members are assigned to the same school. Similarly, for families that include three or more students, we say that a family application is partially successful if at least two of its students, but not all of them, are assigned to the same school. Overall we observe that 6,725 (65.3%) family applications were successful, while 307 (3%) were partially successful. Figure 7 shows the success of family applications by size. As expected, larger families are less likely to be successful, as they require more students being allocated to the

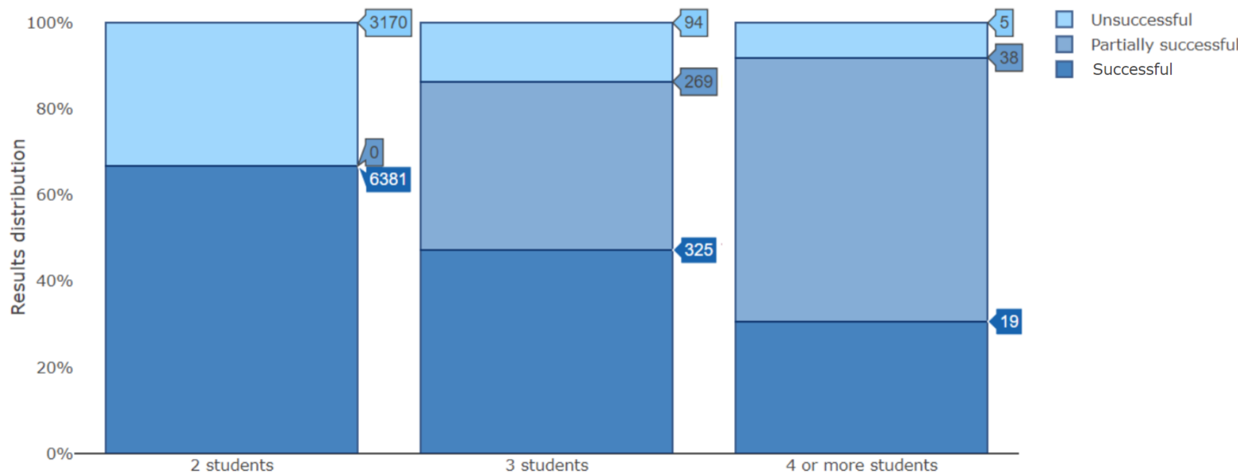

Figure 7 Number of successful and partially successful families by size - Main Round

same school. We refer to Appendix B.3 for results on family applications depending on the number of common schools a family declares.

As described in Section 4.3, to implement applications we consider lotteries by families and update students' preferences. However, there may be other approaches that could lead to more successful families. To explore the benefits of our approach, we conduct a simulation study to compare our current approach with three other alternatives: (i) lotteries by student in each school and updating preferences (ii) lotteries by families without updating preferences, and (iii) lotteries by student without updating preferences. Notice that the latter is the standard approach used in other school choice settings.

In Table 7 we report the results of 10,000 simulations for each approach. We observe that the fraction of successful families is larger when both components—lottery per family and updating preferences—are implemented. The mechanism cannot guarantee that all family applications will be successful. For example, siblings may apply to different schools, and younger siblings may not be eligible in some schools where their elder siblings are applying to (single-gender schools, schools with not all levels, among others). Moreover, the number of seats available may not be enough to allocate all students with sibling priority, as it is the case in non-entry levels. Finally, we observe that the percentage of partially successful family applications is around 3% in all four scenarios.

Our simulations show that family lotteries and updating siblings' priorities (step 2 in Section 4.6) play a different role. While updating siblings' priorities substantially increases the ranking of a young student in school c once her older sibling gets assigned to c, family lotteries correlate the siblings' rankings throughout all schools where both apply (as shown in Appendix A.2).

Table 7 Percentage of sucessfull family applications - Simulations

Scenario	Not updating preferences	Updating preferences
Lotteries by student	53%	62%
Lotteries by family	57%	66%

6. Conclusions

Centralized procedures to assign students to schools are becoming the norm in many countries. In this paper, we describe the design and implementation of the new school choice system in Chile, which expands previous applications in three main areas.

First, we introduce, analyze and evaluate the impact of features of our design intended to favor siblings in getting assigned to the same school. Concretely, we propose the use of two lotteries, one to order families and the other to break ties among siblings. In addition, our mechanism updates students' preferences to prioritize siblings getting assigned to the same school if they are part of a family application. Our results show that these features improve the fraction of siblings assigned in the same school by 13% compared to the standard approach of breaking ties at the student level. Second, we propose a multi-level mechanism that allows students to have a secured enrollment in their current school. This feature of the system eliminates the risk of ending up unassigned when trying to move to a new school. Finally, we implement a mechanism with multiple quotas and priority groups. We show via simulations that having quotas for disadvantaged students is not very effective when the majority of students are eligible and the quota is relatively small. Hence, other approaches should be considered if the goal is to really benefit this group.

The experience of implementing a large-scale nationwide system stresses the importance of having a continuous collaboration with policy makers, and the need of implementing changes in small steps. Having a gradual implementation not only allows to learn from the experience and continuously improve the system, but also gives time to the general public—and final users of the system—to get information, learn and understand the benefits of the new system. Overall, we will continue working to improve the system, increasing its efficiency and fairness to give equal opportunities to all students, regardless of their background.

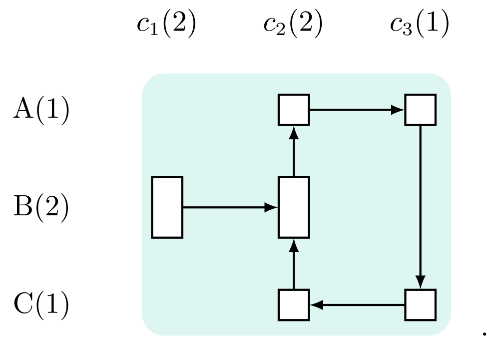

Figure 8 Example of instance where no stable matching exists. Each row is a family and each column is a school. In parenthesis is the size of the family or the number of available seats

Appendix A: Model

A.1. Family application

Regarding stability with family application, in Figure A.1 we show an instance with one level, three families and three schools, where no stable matching exist. Each row represents a family and each column represents a school, and in parentheses are the size of the family and the number of available seats. The arcs represent the preferences or priorities, pointing from the less preferred to the most preferred. In this case, each family is the most preferred for one school, so in any stable matching all students are matched. This also means that the two kids of family B must be assigned together. It is easy to check that none of the 2^3 possible combinations is a non-wasteful matching without justified envy.

A.2. Family lotteries vs. student lotteries

In this section we formally state and prove Proposition 2. Consider a family with two children, Alice (a) and Bob (b) applying to different levels, which we call H and L. As usual, the set of schools is C ($|C| = m$). School $c \in C$ has q_c^H, q_c^L available seats in levels H and L, respectively. For simplicity we will assume there are no specific quotas or priorities.[3] Also there are sets S_H, S_L of students applying to levels H and L, respectively. In particular $a \in S_H$ and $b \in S_L$.

Each student $s \in S_H \cup S_L$ has a preference profile over a subset $C_s \subseteq C$ denoted by $\prec_s$. For ease of presentation, the priorities of a student s in schools in C is given by a vector $u_s = (u_{s,c})_{c \in C} \in [0,1]^C$ so that the higher $u_{s,c}$, the higher the priority of student s in school c (in the random priority model these numbers can be thought to be i.i.d. $U[0,1]$ random variables, and in the following analysis we can obviate the null set where two students applying to the same level get equal lottery numbers in the same school).

The following result concerns only level H and is related to Lemma 4 of Abdulkadiroğlu et al. (2015).

LEMMA 1. *Given $\prec_s$ and u_s for all $s \in S_H \setminus \{a\}$ there exists a vector of cutoffs $(\tau_c)_{c\in C}$ such that for all preference profiles $\prec_a$ and all vectors u_a, if $\mu(a)$ denotes the assigned school of a in the DA mechanism, then $\mu(a) = c$ if and only if $u_{a,c'} < \tau_{c'}$ for all $c \prec_a c'$ and $u_{a,c} > \tau_c$.*

Proof. Recall that as proved by Dubins and Freedman (1981) the DA mechanism is truthful in the following sense: given the preferences of all students but a and all priorities, for all pairs of preference profiles $\prec_a$ and $\prec'_a$, if we denote as μ and μ' the assignments when student a declares $\prec_a$ and $\prec'_a$ respectively, then $\mu'(a) \preceq_a \mu(a)$.

As is also known from the standard literature on Deferred Acceptance, the target student-optimal assignment is unique and therefore independent from the order in which the student proposals are processed. Thus we may assume that an initial stable assignment has been reached without the participation of a, and then she is assigned her corresponding lottery number and inserted to run the process to completion.

For each $c \in C$ we define τ_c as the minimum value that $u_{a,c}$ can take to get a accepted to c if she were to apply to it as her first preference (note that they are well defined since the acceptance or rejection to c as a first preference does not depend on the next ones). In this case it is clear that any value of $u_{a,c}$ higher than τ_c would also result in acceptance to c, and a priority number lower than τ_c would result in rejection by construction.

We will now show that this same vector $(\tau_c)_{c\in C}$ also works for an arbitrary preference profile $\prec_a$.

First we claim that if $u_{a,c} < \tau_c$, then a cannot be accepted to c. Indeed, suppose by contradiction that $u_{a,c} < \tau_c$ and a is accepted to c. Noting that the definitions of the τ_c's do not depend on $\prec_a$ we can assume that the altered profile $\prec'_a$, given by restricting $\prec_a$ to start from school c, was the real preference profile and that $\prec_a$ is a deviation from the truth. By definition of τ_c, a will be rejected from c if she applies with profile $\prec'_a$, but accepted with profile $\prec_a$ by hypothesis, which contradicts the truthfulness of the mechanism.

Returning to the proof of the lemma, to prove the right implication suppose that a is assigned to c. The inequality $u_{c,a} > \tau_c$ follows from the previous claim. If $u_{c',a} > \tau_{c'}$ for some $c \prec_a c'$, then once again a could alter her preference profile to start from c' and by definition be accepted to her more preferred option c', contradicting the truthfulness of the mechanism.

For the left implication suppose by contradiction that the inequalities hold and there is a school $c' \neq c$ such that a would be assigned to c' instead. From the claim and the inequalities $u_{a,c'} < \tau_{c'}$ we get that it is not possible that $c \prec_a c'$. Also, if $c' \prec_a c$ we can once again consider the restricted preference profile starting from c and the inequality $u_{a,c} > \tau_c$ to contradict the truthfulness of the mechanism. $\square$

With this lemma at hand we want to compare the probability that a and b get assigned to the same school if on the one hand we draw u_a and u_b as vectors of i.i.d. uniform random variables $U[0,1]$, or on the other hand we draw u_a as a vector of i.i.d. random variables $U[0,1]$ and set $u_{b,c} = u_{a,c}$. To this end we call $\mathbb{P}_S$ the probability measure induced by the former situation (student lottery) and by $\mathbb{P}_F$ the one for the latter situation (family lottery).

Proposition 2. *(Formal statement) Given $\prec_s$ and u_s for all $s \in S_H \cup S_L \setminus \{a,b\}$ then $\mathbb{P}_S\left(\mu(a) = \mu(b)\right) \leq \mathbb{P}_F\left(\mu(a) = \mu(b)\right)$.*

Proof. We proceed by partitioning the event $\mu(a) = \mu(b)$ over the possible common school assignment $c \in C$. From Lemma 1 we know that the event $\mu(a) = c$ is equivalent to $u_{a,c'} < \tau_{c'}$ for all $c \prec_a c'$ and $u_{a,c} > \tau_c$. Therefore, since u_a is a vector of uniform i.i.d. random variables in $[0,1]$, conditional on the event $\mu(a) = c$, we have that u_a is a vector of independent random variables but with $u_{a,c} \sim U[\tau_c, 1]$, $u_{a,c'} \sim U[0, \tau_{c'}]$ for $c' \succ_a c$, and $u_{a,c'} \sim U[0,1]$ for $c' \prec_a c$.

If we apply Lemma 1 to level L, we get certain cutoffs $(\bar{\tau}_c)_{c\in C}$ such that $\mu(b) = c$ if and only if $u_{b,c'} < \bar{\tau}_{c'}$ for all $c' \prec_b c$ and $u_{b,c} > \bar{\tau}_c$. Now, since under family lotteries $u_{a,c'} = u_{b,c'}$ for all $c' \in C$, we have that $\mathbb{P}_F(u_{b,c'} < \bar{\tau}_{c'} | \mu(a) = c) \geq \mathbb{P}_S(u_{b,c'} < \bar{\tau}_{c'} | \mu(a) = c) = \mathbb{P}_S(u_{b,c'} < \bar{\tau}_{c'})$ for all $c' \neq c$ and $\mathbb{P}_F(u_{b,c} > \bar{\tau}_c | \mu(a) = c) \geq \mathbb{P}_S(u_{b,c} > \bar{\tau}_c | \mu(a) = c) = \mathbb{P}_S(u_{b,c} < \bar{\tau}_c)$. Then, because the variables in the vector u_b are independent, we can multiply the inequalities and get the following.

$$\mathbb{P}_F\left(\mu(b) = c | \mu(a) = c\right) \geq \mathbb{P}_S(\mu(b) = c).$$

Note that for a given school $c \in C$, the marginal probabilities $\mathbb{P}_F(\mu(a) = c)$ and $\mathbb{P}_S(\mu(a) = c)$ are equal since they concern level H only. Hence, we can multiply by $\mathbb{P}_F(\mu(a) = c)$ on both sides of the previous inequality and sum over all $c \in C$ to obtain that

$$\sum_{c\in C} \mathbb{P}_F\left(\mu(b) = c, \mu(a) = c\right) \geq \sum_{c\in C} \mathbb{P}_S\left(\mu(b) = c, \mu(a) = c\right),$$

and therefore, $\mathbb{P}_F(\mu(a) = \mu(b)) \geq \mathbb{P}(\mu(a) = \mu(b))$. □

Appendix B: Results

B.1. Other Quotas

Recall that students can belong to three quotas: (1) special needs, (2) high-achieving, and (3) disadvantaged. Students are indifferent to being assigned by any quota or non of them and schools only declare their total available seats and the mechanism calculates seats for the different types of quotas that are allowed by the system. In Table 8 we show the distribution of the 524,178 declared seats for the 2018 process.

Table 8 Total seats declared by schools - Main Round

Quota	# seats	% of total
Special needs	15,324	2.9%
Disadvantaged	43,336	8.3%
High-achieving	2,591	0.5%
Regular	462,927	88.3%

Figure 9 Results by quota - Main Round

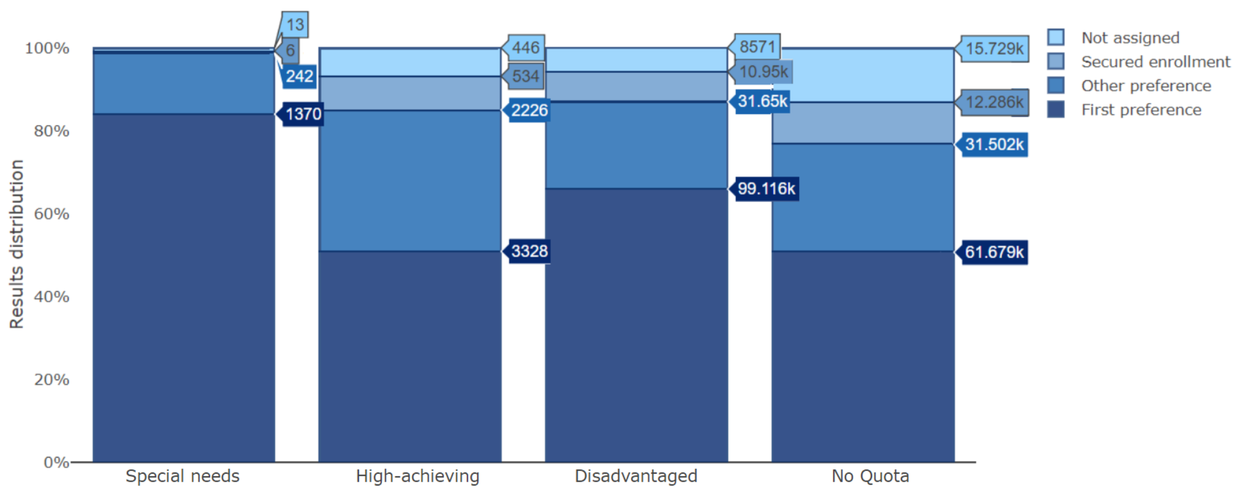

Figure 9 shows the results for students belonging to different quotas (recall a student may belong to more than one quota). It is clear to see that students belonging to special needs and disadvantaged quotas outperform students belonging to high-achieving quota or students that do not belong to any quota in both percentage of students assigned to their first choice and percentage of unassigned students. The low performance of the high-achieving quota compared to the other two quotas could be explained by the fact that high-achieving students apply to a subset of very over-demanded schools.

B.2. Heterogeneity Simulations

From the point of view of schools, MINEDUC seeks to have balanced and heterogeneous schools with respect to the socioeconomic composition. In this sense, we analyze in both scenarios (with and without the quota) the balance of disadvantaged students in schools. For this aim, we consider the following measure of heterogeneity: among all the students of the first round that get an assignment, we pick a student uniformly at random and count the fraction of disadvantaged students that are assigned to her course. This defines a random variable that depends both on the lottery used for the tie-breaking rule and on the selected student.

Let S_{dis} be the set of disadvantaged students that participate in the first round. Given an assignment μ, let $S(\mu) := \{s \in S : \mu(s) \neq \emptyset\}$ be the set of all students that get an assignment in μ, and similarly, let $S_{\text{dis}}(\mu) := S(\mu) \cap S_{\text{dis}}$ be the set of all the disadvantaged students that get an assignment in μ.

For a fixed lottery, let μ^{lottery} be the assignment obtained from its induced tie-breaking and s be a student chosen at random among all the students that get an assignment in μ^{lottery}. For a school $c \in C$ with $\mu(c) \neq \emptyset$, let $f^{\text{lottery}}(c) := \frac{|\mu^{\text{lottery}}(c) \cap S_{\text{dis}}|}{|\mu^{\text{lottery}}(c)|}$ be the fraction of disadvantaged students assigned to c in μ. Then, our random variable can be expressed as $f^{\text{lottery}}(\mu^{\text{lottery}}(s))$. Its conditional expectation given the lottery turns out to be the ratio of all the disadvantaged students assigned in μ^{lottery} to all the students assigned in μ^{lottery}, since

$$\begin{aligned}
\mathbb{E}\left[f^{\text{lottery}}(\mu^{\text{lottery}}(s)) \,|\, \text{lottery}\right] &= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{t \in S(\mu^{\text{lottery}})} f^{\text{lottery}}(\mu^{\text{lottery}}(t)) \\
&= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{c \in C : \mu^{\text{lottery}}(c) \neq \emptyset} f^{\text{lottery}}(c) |\mu^{\text{lottery}}(c)| \\
&= \frac{|S_{\text{dis}}(\mu^{\text{lottery}})|}{|S(\mu^{\text{lottery}})|}.
\end{aligned}$$

Its second moment, on the other hand, is given by

$$\begin{aligned}
\mathbb{E}\left[\left(f^{\text{lottery}}(\mu^{\text{lottery}}(s))\right)^2 \,|\, \text{lottery}\right] &= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{t \in S(\mu^{\text{lottery}})} \left(\left(f^{\text{lottery}}(\mu^{\text{lottery}}(t))\right)^2\right. \\
&= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{c \in C : \mu^{\text{lottery}}(c) \neq \emptyset} \left(f^{\text{lottery}}(c)\right)^2 |\mu^{\text{lottery}}(c)| \\
&= \frac{1}{|S(\mu^{\text{lottery}})|} \sum_{c \in C : \mu^{\text{lottery}}(c) \neq \emptyset} \frac{|\mu^{\text{lottery}}(c) \cap S_{\text{dis}}|^2}{|\mu^{\text{lottery}}(c)|}.
\end{aligned}$$

We estimate $\mathbb{E}\left[f^{\text{lottery}}(\mu^{\text{lottery}}(s))\right]$ by computing the average of $\mathbb{E}\left[f^{\text{lottery}}(\mu^{\text{lottery}}(s)) \,|\, \text{lottery}\right]$ over the results of the 10,000 simulations. Similarly, we estimate $\text{Var}\left[f^{\text{lottery}}(\mu^{\text{lottery}}(s))\right] = \mathbb{E}\left[\left(f^{\text{lottery}}(\mu^{\text{lottery}}(s))\right)^2\right] - \mathbb{E}\left[f^{\text{lottery}}(\mu^{\text{lottery}}(s))\right]^2$ by averaging $\mathbb{E}\left[\left(f^{\text{lottery}}(\mu^{\text{lottery}}(s))\right)^2 \,|\, \text{lottery}\right]$ over the 10,000 simulations and then subtracting the square of the estimator for $\mathbb{E}\left[f^{\text{lottery}}(\mu^{\text{lottery}}(s))\right]$. Finally, we calculate the standard deviation as the square root of the variance.

B.3. Family Application

We measure the success of family applications of size two as a function of the number of schools their students declare in common in their preference lists. Table 9 shows, as expected, that the success rate increases with the number of common preferences, having its greatest increment when the number of common schools grows from one to two. Furthermore, in both rounds, blocks (families) of size two of the same level were more successful in percentage than those of different levels.

Table 9 Results of family applications for blocks of size 2 by number of schools in common

	Main round		Complementary round	
# schools in common	# blocks	% of success	# blocks	% of success
1	1,291	38.5%	497	67.7%
2	3,216	69.8%	2,245	76.6%
3	2,441	71.7%	1,750	77.8%
≥4	2,603	72.6%	1,889	80.5%
Total	9,551		1,421	

Indeed, the main round has 2,832 blocks of size two of the same level and 6,719 of different levels, with success rates of 77,8% and 62.2% respectively. For the complementary round, there are 362 such blocks of the same level, with a success rate of 82.3%, and 1,059 of different levels, with a success rate of 70.7%.

Endnotes

1. The Law also radically changed the way in which families apply and are assigned to schools, which made the transmission of information essential to the implementation. The key to these challenges was gradualism. The system was first implemented in 2016 in the least populous of the sixteen regions in Chile. This allowed to gain practical experience to improve the system as more regions were subsequently added. The system will be fully in place by 2020.
2. This 3% corresponds to 307 partially successful family applications. However, only 750 FAs were of size 3 or more, therefore this represents 41% of the possibly successful FAs.
3. This is actually w.l.o.g.

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